Supersymmetric Vortex Loops in 3d Gauge Theories

A. Kapustin, B. Willett, IY - arXiv:1211.2861

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Defects

- Operators in a QFT which are not field insertions
  - Specify some boundary conditions/singularities for the fundamental fields.
  - Have some forms belong to a non-trivial cohomology class (topological disorder).
  - Change the domain of the path integral in some other way.
- Some examples
  - Twist operators in 2d CFT.
  - The 't Hooft loop operator in 4d gauge theory.
  - Monopole operators in 3d CFT.

Motivation

- Why study defects?
  - Theory can be “incomplete” without them (they appear in OPEs).
  - Duality exchanges them with “normal” operators.
  - Study phases of gauge theories.
- Why supersymmetric defects?
  - Exact computation possible via localization even for strongly coupled theories.
  - Comparison to known field theory/gravity duals.
  - Supersymmetric versions of examples exist.

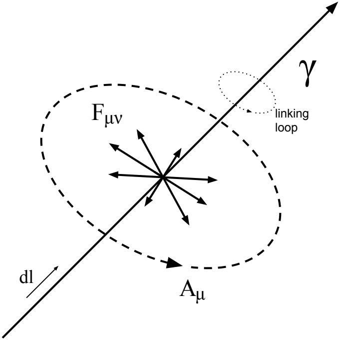
The vortex loop

- Vortices are solitons in 3d gauge theory
  - Classical ANO vortices have supersymmetric versions on the Higgs branches of $\mathcal{N} = 2$ theories.
  - Exchanged with elementary excitations in 3d mirror symmetry.
  - The (closed) worldline of a heavy vortex gives a loop operator analogous to a 4d 't Hooft loop for the monopole.
- Virtues of vortex loops
  - Easy to define in the UV even in the absence of gauge fields (use background gauging).
  - Gauge invariant operators that may be used to learn about phases of the theory.
  - Have supersymmetric versions.
  - Transform nicely under duality.

A vortex gauge field configuration

- Specify (Witten (1988), Seiberg, Moore (1989))
  - a loop γ
  - a symmetry group G (we take $U(1)$)
  - an element $\beta \in \mathfrak{g}$
- Gauge G with connection A (if it's already gauged then just add A)
  - A has holonomy β around a small linking loop.
  - equivalently $F_A = \beta \star [\gamma]$ (a delta function on the loop)
- Large gauge transformations imply (Gukov, Witten (2006))

$$q = \frac{\beta_{\text{abelian}}}{2\pi}, \qquad q \in \left(-\frac{1}{2}, \frac{1}{2}\right)$$



Witten's $SL(2,\mathbb{Z})$ action

Take $3d$ CFT with a conserved $U(1)$ current J

$$Z_{J,\alpha}[A] = \int \mathcal{D}\Phi e^{iS[\Phi]+i\int\sqrt{g}d^3xJ^\mu A_\mu+\ldots+\frac{i\alpha}{4\pi}\int A\wedge dA}$$

Define generators T, S

- T: shift the CS level

$$(T\cdot Z_{J,\alpha})[A] = Z_{J,\alpha+1}[A]$$

- S: use off-diagonal CS term to gauge $\star dA$ then make A dynamical.

$$(S\cdot Z_{J,\alpha})[A] = \int \mathcal{D}A'\mathcal{D}\Phi e^{iS[\Phi]+i\int\sqrt{g}d^3xJ^\mu A'_\mu+\ldots+\frac{i\alpha}{4\pi}\int A'\wedge dA'+\frac{i}{2\pi}\int A\wedge dA'}$$

and check[1]

$$S^2 = C, \qquad (ST)^3 = I$$

[1] Witten (2003)

Vortex via $SL(2,\mathbb{Z})$

Addition of an abelian Wilson loop

$$e^{iq\int_\gamma A} \rightarrow e^{\frac{i}{2\pi}\int \omega\wedge A} \rightarrow e^{\frac{i}{2\pi}\int A\wedge dA_\omega}$$

$$(W_\omega \cdot Z_{J,\alpha})[A] := \int \mathcal{D}\Phi e^{iS[\Phi]+i\int\sqrt{g}d^3xJ^\mu A_\mu+\ldots+\frac{i\alpha}{4\pi}\int A\wedge dA+\frac{i}{2\pi}\int A_\omega\wedge dA}$$

Globally $\omega = 2\pi q\delta_\gamma$ and locally $A_\omega = qd\theta$ (the connection of a defect!). Adding a defect amounts to

$$(D_\omega \cdot Z_{J,\alpha}[A]) = Z_{J,\alpha}[A + A_\omega]$$

Check that the algebra (up to global phases) is

$$[T, W_\omega] = 0, \qquad S^{-1}W_\omega S = D_\omega$$

Another perspective

The topological current of an abelian gauge theory

$$J_{top} = \frac{1}{2\pi} \star dA_{\text{dynamical}}$$

a defect in J_{top} is the usual dynamical Wilson loop

$$D_\omega \cdot Z_{J_{\mathrm{top}},\alpha}[A] = \left\langle e^{iq \int_\gamma A_{\mathrm{dynamical}}} \right\rangle$$

The "gauge defect" (a different operation)

$$A_{\mathrm{dynamical}} \to A_{\mathrm{dynamical}} + A_\omega$$

is equivalent in CS_k to a Wilson loop with charge $\propto k^2$

[2] Moore, Seiberg (1989)

Supersymmetry

$SL(2,\mathbb{Z})$ generalizes to theories with $\mathcal{N}=2$ supersymmetry

- CFT $\to$ SCFT
- Conserved current $J \to$ linear multiplet $\Sigma\,(\sigma, A, \lambda, D)$
- Connection $A \to$ vector superfield $V\,(\sigma, A, \lambda, D)$
- diagonal/off-diagonal abelian CS term $\to$

$$\begin{aligned} S_{\mathrm{BF}} &= \frac{k_{ij}}{4\pi}\int d^3x d^2\theta d^2\bar{\theta}\Sigma^i V^j \\ &= \frac{k_{ij}}{4\pi}\int d^3x \left(\varepsilon^{\mu\nu\rho} A^j{}_\mu \partial_\nu A^i{}_\rho - \frac{1}{2}\bar{\lambda}^j\lambda^i + D^i\sigma^j\right) \end{aligned}$$

Supersymmetric loops

An $\mathcal{N} = 2$ Wilson loop (1/2 BPS)

$$\exp\left(iq \oint_\gamma (A - i\sigma d\ell) \right)$$

has an associated defect operator (the *supersymmetric vortex loop*)

$$dA = 2\pi q \delta_\gamma, \quad \star D = -2\pi i q \delta_\gamma \wedge d\ell$$

which solves the BPS equation (here on S^3 with unit radius)

$$\delta\lambda = (-\frac{i}{2}\varepsilon^{\mu\nu\rho} F_{\mu\nu}\gamma_\rho - D + i\gamma^\mu D_\mu \sigma + \sigma)\varepsilon$$

They are $SL(2, \mathbb{Z})$ buddies!

The basics of localization

Deformation

- Identify an appropriate conserved fermionic charge: Q.
- Choose V such that $\{Q, V\}$ is a positive semi-definite functional (Q should square to 0 on V).
- Deform the action by a total Q variation $S \to S + t\{Q, V\}$. The resulting path integral is independent of t!
- Add some Q closed operators (Wilson loops, defect operators).

Localization

- Take the limit $t \to \infty$.
- The measure $exp(-S)$ is very small for $\{Q, V\} \neq 0$.
- The semi-classical approximation becomes exact, but there may be many saddle points to sum over ("the zero locus").
- Integrate over the zero locus of $\{Q, V\}$ $(+$ small fluctuations)

Ingredients for the matrix model

On S^3 we localize to a single matrix[3]

$$A_\mu = 0, \lambda = \lambda^\dagger = 0, D = -\sigma = a(const)$$

The integration measure for the matrix model is

$$\frac{1}{\mathrm{Vol}(G)} da\big|_{a \in \mathrm{Ad}(\mathfrak{g})} \tag{1}$$

The contribution of a level k Chern-Simons term

$$e^{-i\pi k \mathrm{Tr}\left(a^2\right)}$$

Insertion of a supersymmetric Wilson loop in a representation R gives a factor of

$$W(a) = \frac{1}{\dim(R)} \mathrm{Tr}_R\left(e^{2\pi a}\right) \xrightarrow[\text{abelian}]{} e^{2\pi q a}$$

[3] Kapustin,Willett,IY (2009)

Fluctuations

Every dynamical gauge multiplet contributes

$$Z_{1\text{ - loop}}^{\text{gauge multiplet}}(a) = \frac{\det \mathcal{O}_{\text{gauginos}}}{\sqrt{\det \mathcal{O}_{\text{vectors}}}} = \prod_{\rho \in \text{roots}(\mathfrak{g})} 2\sinh(\pi\rho(a))$$

and every dynamical chiral multiplet contributes

$$Z_{1\text{ - loop}}^{\text{chiral multiplet}}(a, \Delta) = \frac{\det \mathcal{O}_F}{\sqrt{\det \mathcal{O}_B}} = \prod_{\rho \in R} \exp\left(\ell\left(z(\rho(a), \Delta)\right)\right)$$

where ρ are the weights of R and

$$\ell(z) = -z \log\left(1 - e^{2\pi i z}\right) + \frac{i}{2}\left(\pi z^2 + \frac{1}{\pi}\text{Li}_2\left(e^{2\pi i z}\right)\right) - \frac{i\pi}{12}$$

$$z(\rho(a), \Delta) = i\rho(a) - \Delta + 1$$

Supersymmetric Deformations

Mass terms

Real mass terms are supersymmetric configurations for background flavor symmetry gauge fields $V_m \propto \theta\bar{\theta}m$

$$S_{mass} = -\int d^3x d^2\theta d^2\bar{\theta} \sum_{matter} (\phi^\dagger e^{2V_m}\phi + \tilde{\phi}^\dagger e^{-2V_m}\tilde{\phi})$$

in the matrix model this just shifts $\rho(a) \to \rho(a) + m$.

Fayet-Iliopoulos (FI) terms

Fayet-Iliopoulos (FI) terms for the $U(1)$ factors of the gauge group are equivalent to gauging topological symmetries $\hat{V}_{FI} \propto \theta\bar{\theta}\eta$

$$S_{FI} = Tr \int d^3x d^2\theta d^2\bar{\theta}\Sigma\hat{V}_{FI} \to e^{2\pi i\eta tr_f(a)}$$

Adding a defect

We should localize the theory in the presence of the background defect

$$dA = 2\pi q \delta_\gamma, \quad \star D = -2\pi i q \delta_\gamma \wedge d\ell$$

- Doesn't introduce new zero modes[4].
- Doesn't modify the classical contributions (the exception is a "gauge defect" in CS theory).
- Will modify the fluctuation determinant for the charged fields (in chiral multiplets). Evaluate by
    - Using the $SL(2, \mathbb{Z})$ definition.
    - Smearing out the defect.
    - Working with a regularized background.

[4] for generic q!

A quick evaluation using $SL(2,\mathbb{Z})$

Define the defect insertion as

$$D_q = S^{-1} W_q S$$

The SCFT depends on the modulus m (the vev of σ in the same supermultiplet as A)

$$Z(m) \to (S \cdot Z)(\eta) = \int dm Z(m) e^{2\pi i \eta m}$$

Insert the supersymmetric Wilson loop

$$(W_q S \cdot Z)(\eta) = e^{2\pi q \eta} \int dm Z(m) e^{2\pi i \eta m}$$

$$(S^{-1} W_1 S \cdot Z)(m') = \int e^{-2\pi i \eta m'} e^{2\pi q \eta} \int dm Z(m) e^{2\pi i \eta m}$$

hence (assuming the integral converges)

$$\boxed{(D_q \cdot Z)(m) = Z(m + iq)}$$

Working on S^3

We will work in toroidal coordinates on S^3

$$ds^2 = d\eta^2 + \sin^2\eta d\theta^2 + \cos^2\eta d\phi^2$$

- Surfaces of constant η are tori, which degenerate to great circles at $\eta = 0, \frac{\pi}{2}$.
- There is a Killing spinor and an associated Killing vector field

$$\nabla_\mu \varepsilon = \frac{i}{2}\gamma_\mu \varepsilon, \qquad v = \varepsilon^\dagger \gamma^\mu \varepsilon \partial_\mu = \frac{\partial}{\partial\theta} + \frac{\partial}{\partial\phi}$$

- S^3 is a $U(1)$ bundle over S^2 (Hopf fibration),v points along the fiber and the one form

$$v = \sin^2\eta d\theta + \cos^2\eta d\phi, \qquad dv = 2\star v$$

Smearing

Define a smeared (abelian) Wilson loop (the usual loop has a singular f)

$$\mathcal{O}_f = \exp\left(i \int_{S^3} \sqrt{g} d^3 x f(x)(v^\mu A_\mu - i\sigma) \right)$$

$$f(x) = \frac{q}{\pi} + f_o(x), \qquad \int_{S^3} \sqrt{g} d^3 x f_o = 0 \Rightarrow f_o = \nabla^2 g$$

We can prove that for f_0 independent of the fiber direction the smearing is gauge invariant and "Q exact":

$$\delta_\varepsilon\left(\int \sqrt{g} d^3 x \epsilon^\dagger \gamma^\mu (\partial_\mu g) \lambda\right) = \int \sqrt{g} d^3 x (\nabla^2 g)(v^\mu A_\mu - i\sigma)$$

The $SL(2,\mathbb{Z})$ dual of $f = \frac{q}{\pi}$ is a non-singular background

$$F = 2q \star v = q dv, \quad D = -2iq$$

Resolving the singular background

Introduce a smoothing function

$$f_2 = \frac{g(\eta/\epsilon)}{2\pi\epsilon \sin\eta \cos\eta} \xrightarrow[\epsilon\to 0]{} \delta(\eta)\,, \qquad G' = g$$

and use the non-singular configuration

$$F = 2\pi q\, f_2 \star v = \frac{q}{\epsilon} g(\eta/\epsilon) d\eta \wedge (d\theta - d\phi)$$

$$D = -2\pi i q f_2 = -\frac{iq\, g(\eta/\epsilon)}{\epsilon \sin\eta \cos\eta}$$

We still have a connection

$$dF = 0 \Rightarrow A = qG(\eta/\epsilon)d\theta - q(G(\eta/\epsilon) - 1)d\phi$$

The result

- The smeared loop leads to a determinant of a scalar Laplacian and a Dirac operator twisted by v (easy to compute).
- The resolved loop leads to complicated differential operators, but many modes cancel.
- Either method leads to

$$Z^{\text{chiral multiplet}}_{\text{1 - loop}}(a,\Delta) = \frac{\det \mathcal{O}_F}{\sqrt{\det \mathcal{O}_B}} \to Z^{\text{chiral multiplet}}_{\text{1 - loop}}(a+iq,\Delta)$$

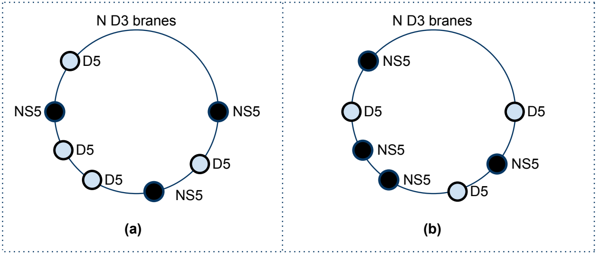
- At $|q| = 1/2$ we see a new bosonic zero mode of the charged fields emanating from $a = 0$.
- The result is naively periodic in q (one needs to continue canceling modes between bosons and fermions even for negative bosonic “masses”).

Some context

- The supersymmetric vortex loop is the reduction of the 4d Gukov-Witten surface operator[5].
- Works for non-abelian groups[6]
    - For a global non-abelian symmetry: rotate the data into the Cartan.
    - For a non-abelian gauge symmetry: the operator (partially) breaks gauge invariance at the loop.
    - For Chern-Simons theory: carefully quantize the charge.

[5] Gukov,Witten (2006); Drukker, Gomis,Young (2008)
[6] Drukker, Okuda, Passerini (2012)

Mirror Symmetry



Basics of 3d mirror symmetry

- Relates the IR limit (strong coupling!) of different supersymmetric quiver gauge theories.
- Can be realized with S-duality in type IIB brane construction.
- Duality exchanges the Higgs and Coulomb branches of the moduli space.
- Abelian mirror symmetry is closely related to $SL(2, \mathbb{Z})$.

Duality

- $U(1)$ $\mathcal{N}=4$ gauge theory with a single charged flavor is IR equivalent to a free twisted hypermultiplet[7]
  Exchanges $U(1)_J$ (topological) with $U(1)_{\text{flavor}}$ (and the FI term with the real mass)

$$\frac{1}{\cosh \pi\omega} \quad \underset{\eta\leftrightarrow\omega}{\longleftrightarrow} \quad \int d\sigma \frac{e^{2\pi i\sigma\eta}}{\cosh \pi\sigma}.$$

- Exchanges Wilson loops and (flavor) defect operators (no Wilson loops in the dual!)

$$W_q Z^{\mathcal{N}=4}_{U(1),N_f=1}(\eta) = \int d\lambda \frac{e^{2\pi i\eta\lambda}e^{2\pi q\lambda}}{2\cosh(\pi\lambda)}$$
$$= \frac{1}{2\cosh \pi(\eta - iq)} = D_q Z^{\mathcal{N}=4}_{\text{free twisted hyper}}(\eta)$$

[7] Kapustin, Strassler (1999)

More duality

- Knowing the mapping of global symmetries is enough to determine the mapping of (single) loop operators.
- There is an interesting algebra of Wilson loops in Chern-Simons-matter theories[8] and possibly of defects.
- Duality can mix the two objects and insertions need not commute[9].

[8] Kapustin,Willett (2013)

[9] In abelian mirror symmetry: Dimofte, Gaiotto, Gukov (2011)

Convergence

- Gauge theories with CS level $k = 0$ seem to support only Wilson loops with small enough charge. For $U(1), N_f = 1$

$$\int d\lambda \frac{e^{2\pi q\lambda}}{2\cosh(\pi\lambda)} < \infty \iff |q| < \frac{1}{2}$$

or for $N_f > 1$

$$\int d\lambda \frac{e^{2\pi q\lambda}}{2\cosh^{N_f}(\pi\lambda)} < \infty \iff |q| < \frac{N_f}{2}$$

- The defect description provides a natural analytic continuation past $q = N_f/2$ (it is periodic). Note

$$q_{\text{defect}} = \frac{q}{N_f}$$

- An addition bosonic zero mode for charged fields at $q_{\text{defect}} = \pm 1/2$ (fermionic zero modes at other values). This is hard to see in the mirror (Wilson loop) description.

Conclusions about vortex loops

- The definition
  - Simple UV definition even in the absence of gauge fields.
  - Associated to a conserved current and an important part of the $SL(2,\mathbb{Z})$ action.
  - Supersymmetric version available. The "data" is in a vector multiplet with non-standard reality conditions.
- Localization
  - All regularizations lead to the same result.
  - The answer is consistent with compactified 4d theories.
  - Implies that mirror symmetry exchanges Wilson loops with vortex loops.
  - Gives a continuation to Wilson loops with large charge (physical?).

Thank you

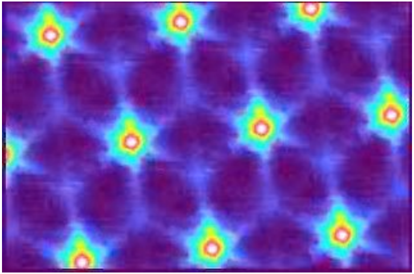


Figure: Abrikosov vortex lattice (H. Hess 1988)

Thank you!

Path integral localization

Deformation

- Identify an appropriate conserved fermionic charge: Q.
- Choose V such that $\{Q, V\}$ is a positive semi-definite functional (Q should square to 0 on V).
- Deform the action by a total Q variation $S \to S + t\{Q, V\}$. The resulting path integral is independent of t!
- Add some Q closed operators (Wilson loops, defect operators).

Localization

- Take the limit $t \to \infty$.
- The measure $exp(-S)$ is very small for $\{Q, V\} \neq 0$.
- The semiclassical approximation becomes exact, but there may be many saddle points to sum over ("the zero locus").
- Integrate over the zero locus of $\{Q, V\}$ (+ small fluctuations)

Supersymmetry on S^3

- We wish to compute all expectation values on S^3.
- After a conformal transformation
  1. All derivatives become covariant.
  2. Scalars with a kinetic term get a conformal mass (proportional to the Ricci scalar).
- (Covariantly) constant spinors exist only on ricci flat manifolds.
- Manifolds of constant curvature have Killing spinors satisfying $\nabla_\mu \varepsilon = \alpha \gamma_\mu \varepsilon$.
- On the three sphere $\nabla_\mu \varepsilon = \pm \frac{i}{2} \gamma_\mu \varepsilon$ (two of each).
- Actions with fermionic symmetries may be constructed using these spinors.

$\mathcal{N} = 2$ Vector multiplets

- The vector multiplet

σ, D (real)	λ_α (complex)	A_μ (real)

- Additional gauge fixing fields: c, $\bar{c}$ and b.
- The gaugino variation is (I have set the radius $r = 1$)

$$\delta\lambda = (-\frac{1}{2}\gamma^{\mu\nu}F_{\mu\nu} - D + i\gamma^\mu D_\mu\sigma - \sigma)\varepsilon$$

- We actually consider a combined supercharge

$$\tilde{Q} = Q_\varepsilon + Q_{BRST}, \qquad V = Tr\left(\{\tilde{Q}, \lambda\}^\dagger\lambda + \bar{c}\partial^\mu A_\mu\right)$$

Vector multiplet localization

- The localizing functional is similar to a normal Yang-Mills action

$$S_Q = t \int_{\mathcal{M}} \sqrt{g}\, Tr\Big(\frac{1}{2}F^{\mu\nu}F_{\mu\nu} + D^{\mu}\sigma D_{\mu}\sigma + (D+\sigma)^2 + i\lambda^{\dagger}\not{D}\lambda$$
$$+ i[\lambda^{\dagger},\sigma]\lambda - \frac{1}{2}\lambda^{\dagger}\lambda + \nabla^{\mu}\bar{c}D_{\mu}c + b\nabla^{\mu}A_{\mu}\Big)$$

- Localizing to

$$S_Q = 0 \Leftrightarrow A_{\mu} = 0, \lambda = \lambda^{\dagger} = 0, c = 0, \bar{c} = 0, D = -\sigma = \sigma_0(const)$$

  and with b unrestricted.

- Path integral reduces to

$$\int_{\sigma=const} S_{original}[\sigma = const]\,(\text{one loop determinant})$$

Gauge sector matrix model

- The fluctuation determinant is

$$\prod_{\alpha}\prod_{l=0}^{\infty}\frac{((l+i\alpha(a))(-l-1+i\alpha(a)))^{l(l+1)}}{((l+1)^2+\alpha(a)^2)^{l(l+2)}}=\prod_{\alpha\in\text{roots}}\frac{2\sinh(\pi\alpha(a))}{(\pi\alpha(a))}$$

- The supersymmetric Chern-Simons action becomes

$$\frac{ik}{4\pi}tr_f\int_{\mathcal{M}}\sqrt{g}(2D\sigma)\to\exp(-i\pi k tr_f(a^2))$$

- The supersymmetric Wilson loop

$$W_{1/2}\equiv\mathcal{P}\mathrm{Tr}_R\exp[\oint(iA_\mu dx^\mu+\sigma\,|\dot{x}|)]\to\mathrm{Tr}_R exp(2\pi a)$$

The Chern-Simons matrix model

The matrix integral

- The expectation value of the Wilson loop has been reduced to a matrix integral

$$\int da \underbrace{\frac{\exp(-ik\pi tr(a^2))}{\text{classical CS term}}} \frac{\det_{Ad} 2\sinh(\pi a)/(\pi a)}{\text{1 loop det}} \frac{tr_R \exp(2\pi a)}{\text{Wilson loop}}$$

Consistency checks

- The above matrix model was derived independently by other means for pure CS theory.
- Exact results for $U(N)$ are available and compare well with known results.
- The supersymmetric computation demands a specific "framing".

Matter fields

- Component fields and fermion transformations

ϕ,F (complex)	ψ_α (complex)

$$\delta\psi = (-i\gamma^\mu \nabla_\mu \phi - i\sigma\phi + \frac{1}{2}\phi)\varepsilon, \qquad \delta\psi^\dagger = \varepsilon^T F^\dagger$$

- The localizing term is

$$S_Q = t\int_{\mathcal{M}} \sqrt{g}\, Tr\Big[\nabla^\mu \phi^\dagger \nabla_\mu \phi + i\phi^\dagger v^\mu \nabla_\mu \phi + \phi^\dagger \sigma_0 \phi + \frac{1}{4}\phi^\dagger \phi$$

$$+ F^\dagger F + \psi^\dagger \Big(i\not{\nabla} - i\sigma_0 + \Big(\frac{1+\not{v}}{2}\Big)\Big)\psi\Big], \qquad v_\mu \equiv \varepsilon^\dagger \gamma_\mu \varepsilon$$

- No additional zero modes arise. All fields are set to 0.

The matter determinant

A self dual representation $R \oplus R^*$ (like a hypermultiplet)

$$Z_{1\ \text{loop}}^{\text{matter}} = \prod_{\rho \in \text{weights}} \frac{1}{\cosh(\pi\rho(a))}$$

A general chiral superfield of conformal dimension Δ

$$\delta\psi = (-i\gamma^\mu \nabla_\mu \phi - i\sigma\phi + \Delta\phi)\varepsilon, \qquad \delta\psi^\dagger = \varepsilon^T F^\dagger$$

$$Z_{1\ \text{loop}}^{\text{matter}} = \prod_{\rho \in \text{weights}} exp^{\ell(\frac{1}{2}+i\rho(a))}$$

where

$$\ell(z) = -zlog\left(1 - e^{2\pi iz}\right) + \frac{i}{2}\left(\pi z^2 + \frac{1}{\pi}Li_2\left(1 - e^{2\pi iz}\right)\right) - \frac{i\pi}{12}$$

is a solution to $\partial_z \ell(z) = -\pi zcot(\pi z)$

A Wilson loop in ABJM

A(harony)B(ergman)J(afferis)M(aldacena)

- A superconformal $\mathcal{N} = 6$ Chern-Simons matter theory.
- Gauge group $U(N) \times U(N)$ with CS levels $(k, -k)$.
- Two hypermultiplets in the $\left(N, \bar{N}\right)$ representation.
- Conjectured to be the low energy limit of $\mathcal{N} = 8$ SYM and holographically dual to M-theory on $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$.

An $\mathcal{N} = 2$ Wilson loop

- A loop operator preserving 2 real supercharges

$$W_{1/2} \equiv \mathcal{P}\mathrm{Tr}_R \exp[\oint (iA_\mu dx^\mu + \sigma |\dot{x}|)] \to \mathrm{Tr}_R exp(2\pi a)$$

- There is a 1/2 BPS (in the $\mathcal{N} = 6$ sense of ABJM) version in the same cohomology class (Drukker, Trancanelli)