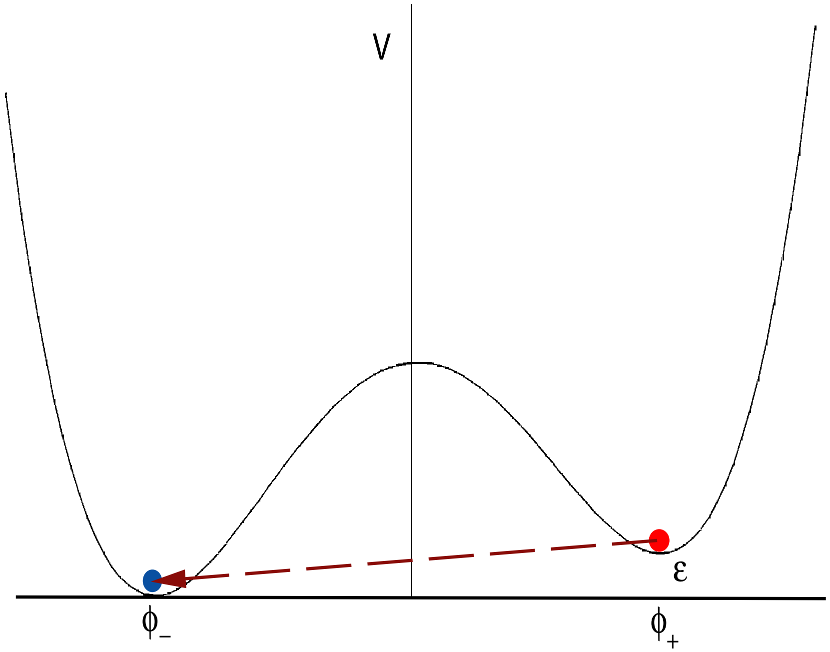
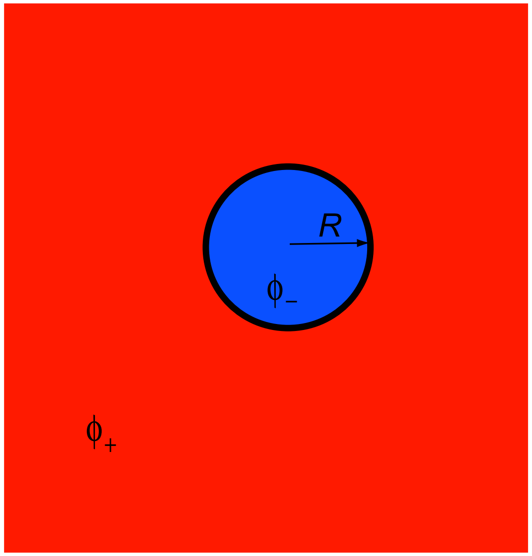
Spontaneous and induced decay of false vacuum

Mikhail Voloshin

FTPI, University of Minnesota

• Introduction. False vacuum.

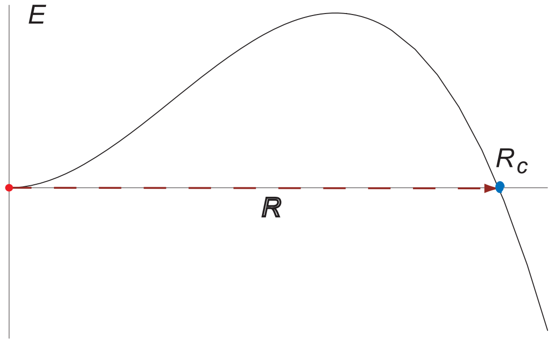




d space-time dimensions:

Gain in volume energy: -Volume$\times\epsilon \propto \epsilon\, R^{d-1}$

Loss in surface energy: Area$\times\mu \propto \mu\, R^{d-2}$



$R_c = (d-1)\,\frac{\mu}{\epsilon}$

The rate of critical bubble formation (per unit time×volume) $w_0 \sim \exp(-\text{Action})$.

- Euclidean-space calculation

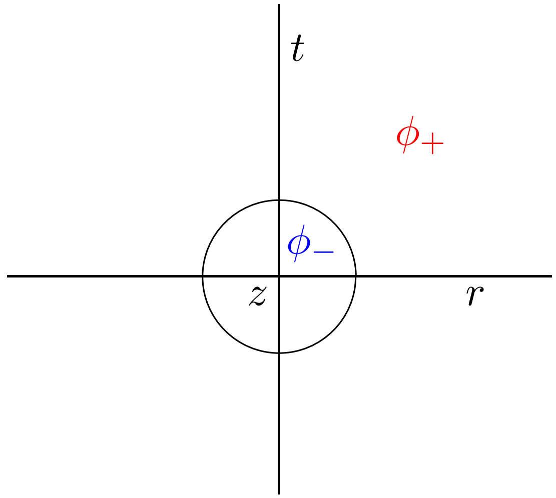
Decay rate = imaginary part ($\times(-2)$) of the false vacuum energy. The path integral

$$Z = \mathcal{N} \int e^{-S[\phi,\dots]}\, \mathcal{D}\phi \dots = \exp(-E_{\text{vac}} T)$$

$\Rightarrow w_0 = 2\,\text{Im}(\ln Z)/VT$.

- Bounce

Stationary configuration: $O(d)$-symmetrical solution of field equations with $\phi \to \phi_+$ at $r \to \infty$ and $\phi \approx \phi_-$ at $r = 0$.



$$w_0\, d^d z = \left| \frac{\det'(S^{(2)})}{\det(S_0^{(2)})} \right|^{-1/2} \exp(-S_B) \left(\frac{S_B}{2\pi}\right)^{d/2} d^d z$$

Pre-exponent

$$w_0 = \Gamma \exp(-S_{cl})$$

- $d = 2$

$\Gamma = \frac{\epsilon}{2\pi}$ in a model with no fermions

$\Gamma = 2^N \frac{\epsilon}{2\pi}$ for a model with N fermions having zero mode on the kink (in the $\epsilon \to 0$ limit). 2^N =the number of final states for kink-antikink.

Some details on $d = 2$:

γ — closed curve in $d = 2$.

Effective action: $S[\gamma] = \mu\, P[\gamma] - \epsilon\, A[\gamma]$.

$$Z_1 = \int \exp(-S[\gamma])\, \mathcal{D}\gamma$$

μ and ϵ are the renormalized parameters supplied by the 'microscopic' theory.

Stationary curve: circle with $R = \mu/\epsilon$, $S_{cl} = \pi\mu^2/\epsilon$.

Let $r(\theta)$ be the polar parametriztion of γ. Hamiltonian form:

$$Z_1 = \int \exp\left(-\int p\, dq + \int H\, d\theta\right) \frac{\mathcal{D}p\, \mathcal{D}r}{2\pi}$$

$$p = \mu\, \dot{r}/\sqrt{r^2 + \dot{r}^2} \qquad H = \frac{1}{2}\,\epsilon\, r^2 - r\,\sqrt{\mu^2 - p^2}$$

$|p| < \mu$ — no self-intersections of γ.

Canonical transform $(r, p) \to (q,\ p)$ with $q = r - \sqrt{\mu^2 - p^2}/\epsilon$:

$$H(q, p) = \frac{p^2}{2\epsilon} + \frac{\epsilon}{2}\, q^2 - \frac{\mu^2}{2\epsilon}$$

which up to the constant $-\mu^2/(2\epsilon)$ is a Euclidean-space Hamiltonian for a harmonic oscillator with the frequency $\omega^2 = -1$.

$\Rightarrow$ The path integral for Z_1 is Gaussian in a finite neighborhood of the stationary point $p = 0, q = 0$.

$\Rightarrow$ $w_0 = \frac{\epsilon}{2\pi} \exp(-\pi\mu^2/\epsilon)$ is exact up to higher exponents. (No power corrections in ϵ/μ^2)

One known exact case (M. Stone '76): Sine-Gordon staircase

$$\frac{1}{2}(\partial_\mu\phi)^2 + \frac{\alpha}{\beta^2}\cos\beta\phi + J\phi \leftrightarrow i\bar{\psi}\gamma\cdot\partial\psi - \frac{1}{2}gj^\mu j_\mu + m\bar{\psi} + eA^0 j^0$$

$\beta^2/4\pi = (1 + g/\pi)^{-1}$, $\partial_1 A^0 = J$, $e = 2\pi/\beta$.

Schwinger process: pair creation in external field E. At $\beta^2 = 4\pi$ the Thirring model is free, $g = 0$, $\Rightarrow$ exact result

$$w_0 = -\frac{\epsilon}{2\pi}\ln\left[1 - \exp\left(-\pi\frac{\mu^2}{\epsilon}\right)\right]$$

• $d = 3$ The 'low-energy' effective action for 2-d closed surface γ in 3-d

$$S = \mu\, Area[\gamma] - \epsilon\, Volume[\gamma]$$

is not renormalizable. Still some universality remains:

$$w_0 = \frac{A}{\epsilon^{7/3}} \exp(-S_{cl})$$

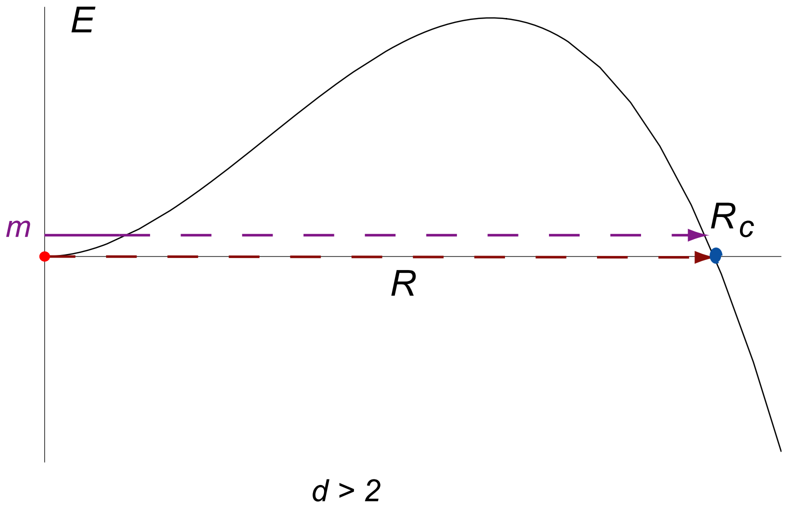
A depends on the parameters (masses, couplings) of the 'microscopic' theory, but not on ϵ.
Specific ϕ^4 model: G. Münster and S. Rotsch '00, general case MV '04

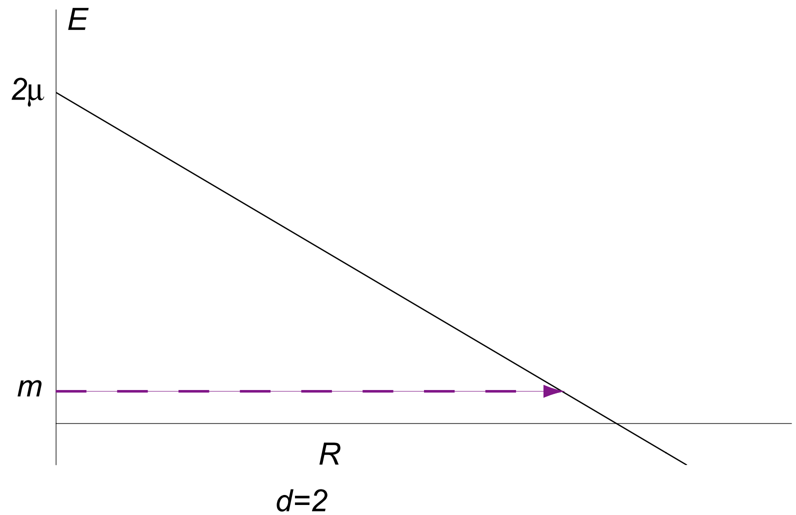
• $d = 4$ Any universality of Γ is totally lost — essential dependence on details of the model. Latest work: G.Dunne and H. Min, Phys.Rev.D72:125004,2005. hep-th/0511156.

• Catalysis by presence of a particle. (Particle decay → true vacuum.)

Energy transfer when the particle field has zero mode on the wall. (Boson of the master field, or fermion.)

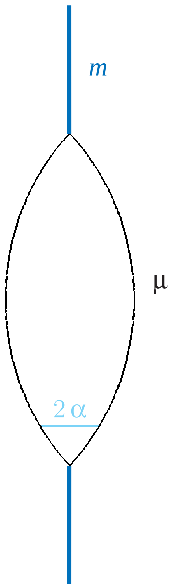
Initial state $\delta E = m$ (the particle mass). Final state: $\delta E = 0$ (particle 'rides' as a bound state on the bubble wall).





Decay rate: $\Gamma = K\, w_0$. K - catalysis factor.

$d = 2$

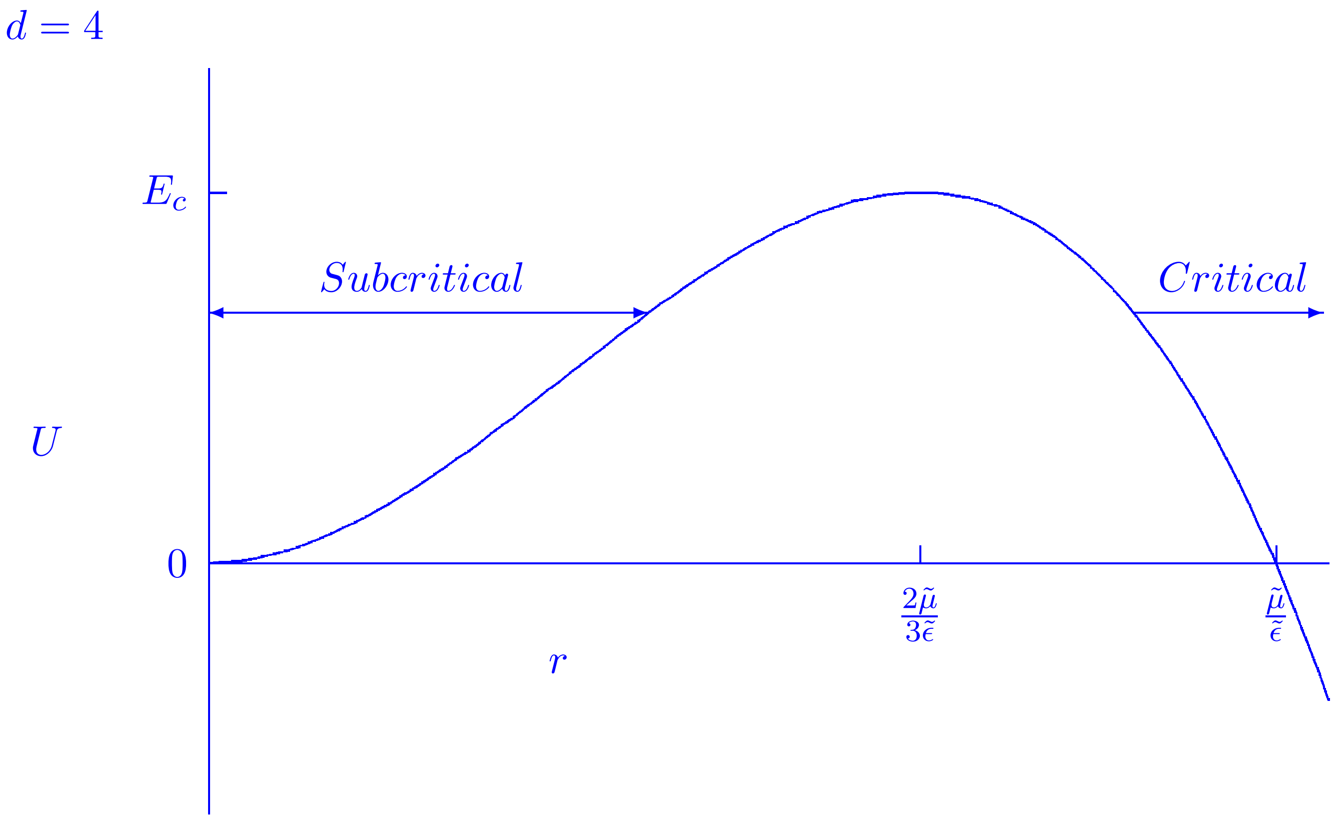


Capillarity problem. $2\mu\cos\alpha = m$

$$S_{eff} = \frac{\mu^2}{\epsilon}\left[2\,\arccos\left(\frac{m}{2\,\mu}\right) - \frac{m}{\mu}\sqrt{1-\frac{m^2}{4\,\mu^2}}\right]$$

Tunneling through the barrier $2\mu - 2\epsilon R$ at energy $E = m$.

Same applies to collisions at energy E.



$$|A_+| \sim \exp\left[-\frac{\tilde{\mu}^4}{\tilde{\epsilon}^3}\left(b(E)+c(E)\right)\right]$$

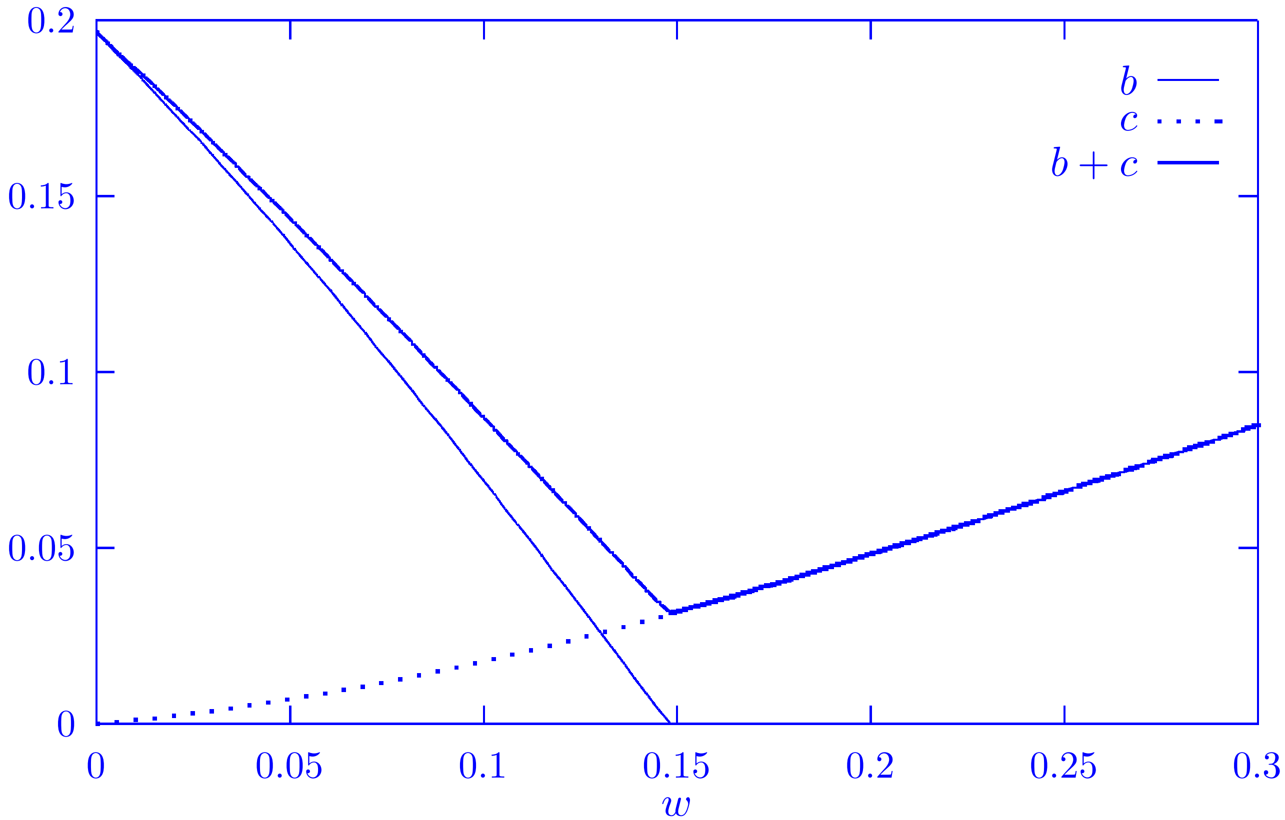


Figure 1: The barrier penetration function $b(E)$, the excitation function $c(E)$, and their sum vs. $w = E\,\tilde{\epsilon}^2/\tilde{\mu}^3$. At the point $w_c = 4/27$ and beyond the barrier disappears, hence $b(E) = 0$ and the sum coincides with $c(E)$.

For $mR \ll S_B$, but still $mR \gg 1$: $K = C \cdot \exp(2mR)$.

Calculation of C is the subject.

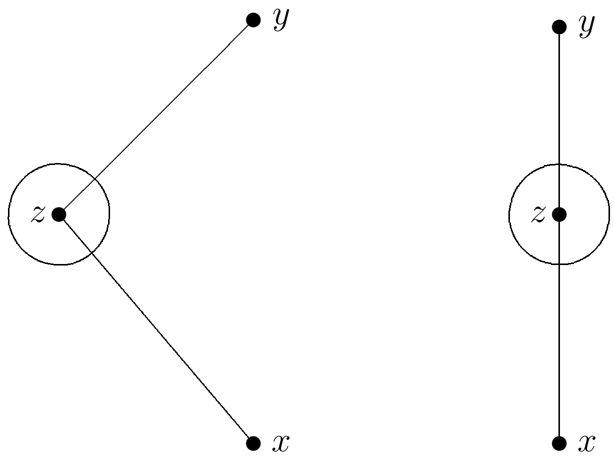
- $mR \ll S_B$ - arbitrary d.
- $m \approx 2\mu$ in $d = 2$.

• Boson

Consider the particle propagator in the ϕ_+ vacuum ($\sigma(x) = \phi(x) - \phi_+$):

$$D(x,y) = \frac{1}{Z} \int \sigma(x)\sigma(y)\, e^{-S[\phi,\ldots]}\, \mathcal{D}\phi \ldots$$

Bounce contribution at $L = |x - y| \gg R$



a) b)

$$\delta D(x,y) = \frac{i}{2}\, w_0 \int d^d z\, F(x-z, y-z)\, D_0(x-z)\, D_0(y-z) \ ,$$

$D_0(x)$ - free progator. Use saddle point in the $dz_\perp$ integration. Then at $L \gg R$

$$\delta D(x,y) = \frac{i}{2}\, w_0\, F_0 \int d^d z\, D_0(x-z)\, D_0(y-z) \ ,$$

with F_0 given by the alignment in Figure b). Compare with effect of $m^2 \to m^2 + \delta m^2$:

$$\delta_m D(x,y) = -\delta m^2 \int d^d z\, D_0(x-z)\, D_0(y-z)$$

• $\delta m^2 = -(i/2)\, w_0\, F_0$, which corresponds to the particle decay rate $\Gamma = F_0 w_0/(2m) \Rightarrow$ the catalysis factor K is found as

$$K = \frac{F_0}{2m}$$

For the bosons of the master field F_0 is found from asymptotic form of (classical) bounce field profile $\phi(r) - \phi_+ \to -2v \, \exp[-m(r-R)] \to C D_0(r)$.

$$D_0(r) = \frac{m^{d/2-1}}{(2\pi)^{d/2}\, r^{d/2-1}} K_{\frac{d}{2}-1}(mr)$$

$$\Rightarrow \quad C = -4\,(2\pi)^{d/2-1}\, m^{(3-d)/2}\, R^{(d-1)/2}\, v\, e^{m\,R}$$

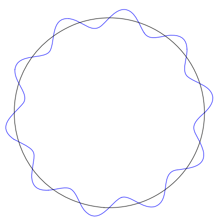
$F_0 = C^2 \Rightarrow$

$$K = 2^{d+1}\, \pi^{(d-3)/2}\, \Gamma\left(\frac{d+1}{2}\right) m^{2-d}\, v^2\, V_{d-1}\, e^{2\,m\,R}$$

$V_{d-1} = \pi^{(d-1)/2}\, R^{d-1}/\Gamma[(d+1)/2]$ is the spatial $(d-1$ dimensional) volume of the critical bubble.
Enhancement of K: classical $\exp(2mR)$ and an extra factor $m^{2-d}\, v^2$ - inverse of the (small) coupling constant for ϕ.

• Infrared complication in $d = 2$

Soft modes: distorsion of the shape of the bounce. Eigenvalues $\lambda_n = c_n/R^2$.



$d = 2$ normalized eigenmode ($\mu \sim mv^2$):

$$\sigma_n(r) \sim \frac{m\,v}{\sqrt{\mu R}}\, e^{-m(r-R)} \sim \sqrt{\frac{m}{R}}\, e^{-m(r-R)}$$

$\Rightarrow$ mode contribution to $\delta D(r_1, r_2)$:

$$\frac{R^2}{c_n}\, \sigma_n(r_1)\sigma_n(r_2) \sim c_n^{-1}\,(m\,R)\, e^{2mR}\, e^{-m(r_1+r_2)}$$

Classical part: $\sim v^2\, e^{2mR}\, e^{-m(r_1+r_2)} \Rightarrow$

$$\frac{\text{Mode contrib.}}{\text{Classical}} \sim \frac{m\,R}{v^2}$$

Although, as expected, suppressed by the small coupling $1/v^2$, is infrared unstable at large R.

• Solution

Effective action for the soft modes (polar-coordinate parametrization of the bounce shape, $(r(\theta), \theta)$)

$$S = \int_0^{2\pi} \left(\mu \sqrt{r^2 + \dot{r}^2} - \frac{1}{2}\, \epsilon\, r^2 \right) d\theta = \frac{\pi\, \mu^2}{\epsilon} + \int_0^{2\pi} \frac{\epsilon}{2} \left(\dot{\rho}^2 - \rho^2 \right) d\theta + O(\rho^4)$$

$R = \mu/\epsilon$, $\rho(\theta) = r(\theta) - R$, $\dot{\rho} = d\rho/d\theta$.

Modes: $\lambda_n \propto (n^2 - 1)$.

One negative mode: $\rho_0 = 1/\sqrt{2\pi}$.

Two types of $n > 0$ modes:

$$\rho_n^{(1)} = \frac{1}{\sqrt{\pi}} \cos n\theta \ , \quad \text{and} \quad \rho_n^{(2)} = \frac{1}{\sqrt{\pi}} \sin n\theta \ ; \qquad (n = 1, 2, \ldots)$$

Only the fluctuations of the vertical size of the bounce contribute to $\delta D(x, y)$:

$$\langle [\rho(0) + \rho(\pi)]^2 \rangle \propto \sum_n \frac{[\rho_n(0) + \rho_n(\pi)]^2}{n^2 - 1}$$

The sum $\rho(0) + \rho(\pi)$ is not vanishing only for the negative mode and for the positive modes of the first type, $\rho_n^{(1)}$, with even n, i.e. $n = 2k$. Thus

$$\langle [\rho(0) + \rho(\pi)]^2 \rangle \propto -\frac{1}{2} + \sum_{k=1}^{\infty} \frac{1}{4\, k^2 - 1} = 0$$

The theory cures itself from the inrared problem by cancellation between one negative and the sum over the positive modes.

• Fermionic case. $d = 2$.

If a fermion field is present in the theory (no actual fermions in the false vacuum), such that the fermion mass $m(\phi)$ changes sign between ϕ_+ and ϕ_-, the fermion has a zero mode on the bubble wall and affects the spontaneous decay rate of the false vacuum: in $d = 2$ it makes $w_0 \to 2\, w_0$. Two (degenerate) final states: F(kink,antikink)=(+1/2,-1/2) and F(kink,antikink)=(-1/2,+1/2).

• Fermion present in the false vacuum.

Consider the bounce contribution to the fermion Green function $G(x, y) = \langle\psi(x)\overline{\psi}(y)\rangle$.

Fermion zero mode, $[\sigma_i \partial_i + m(\phi)]\, \psi_0 = 0$

$$\psi_0(r, \theta) = C_f \sqrt{\frac{R}{r}} \exp\left\{-\int_R^r m[\phi(r')]\, dr'\right\} \chi(\ell) \begin{pmatrix} e^{-i\,\theta/2} \\ e^{i\,\theta/2} \end{pmatrix}$$

$\chi(\ell)$ is a one-dimensional fermion field living on the bounce boundary and (nominally) depending on the length parameter $\ell = R\theta$ along the boundary. Classical equation for χ: $\dot{\chi} = 0$. C_f is the normalization factor:

$$2\, C_f^2 \int \exp\left[-2 \int_R^r m(\phi)\, dr'\right] dr = 1\ .$$

Switch to $\tilde{C}_f$:

$$C_f \exp\left\{-\int_R^r m[\phi(r')]\, dr'\right\} \to \tilde{C}_f \exp\left[m_f(R - r)\right]$$

Generally

$$\tilde{C}_f^2 = \frac{m_f}{2}\, f\left(\frac{m_f}{m}\right)$$

f - dimensionless function of m_f/m with m standing for other mass parameters in the false vacuum.

For $m_f/m \to 0$, $f(m_f/m) \to 1$. In a ϕ^4 theory with $m =$ the boson mass

$$f(u) = \frac{2^{2u}}{\sqrt{\pi}}\,\frac{\Gamma(u+1/2)}{\Gamma(u+1)}$$

Contribution of the zero mode to $G(x,y)$:

$$\delta G(x,y) = -\frac{i}{2}\,\frac{w_0}{2}\,d^2z\,\tilde{C}_f^2\,e^{2\,m_f\,R}\,R\,\frac{e^{-|x-y|}}{\sqrt{|x-z|\,|y-z|}}\,(1+\sigma_1)\,g(0,\pi R)$$

with $g(\ell_1,\ell_2) = \langle\chi(\ell_1)\chi^\dagger(\ell_2)\rangle$ the propagator of χ.
$g(\ell_1,\ell_2) = (1/2)\,\mathrm{sign}(\ell_1-\ell_2) \Rightarrow g(0,\pi R) = -1/2$
Compare with $m_f \to m_f + \delta m_f$:

$$\delta_m G(x,y) = -\delta m_f\,d^2z\,G_0(x-z)\,G_0(z-y) \to -\delta m_f\,d^2z\,\frac{m}{4\pi}\,(1+\sigma_1)\,\frac{e^{-|x-y|}}{\sqrt{|x-z|\,|y-z|}}$$

$G_0(x,y) = \frac{1}{2\pi}\,(-\sigma_i\partial_i + m)\,K_0(m_f\,|x-y|) \Rightarrow$

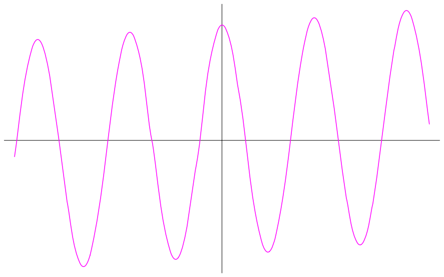
$$\Gamma_f = \frac{\pi}{2}\,f\left(\frac{m_f}{m}\right)\,R\,w_0\,\exp(2\,m_f\,R) = \frac{\mu}{2}\,f\left(\frac{m_f}{m}\right)\,\exp\left(-\frac{\pi\,\mu^2}{\epsilon}+2\,m_f\,R\right)$$

$w_0 = (\epsilon/\pi)\,\exp(-\pi\,\mu^2/\epsilon)$ and $R = \mu/\epsilon$
For the fermion indeed $K_f \sim R\,\exp(2m_fR)$.

Meson decay in sine-Gordon model

- Weak coupling

$$L_{SG} = \frac{1}{2}(\partial\phi)^2 + \frac{\alpha}{\beta^2}\cos(\beta\phi) + (\epsilon\,\beta/2\pi)\,\phi$$



The catalysis factor for a boson is

$$K = \frac{32}{\beta^2}\,\frac{\mu}{\epsilon}\,e^{2\,m_b\mu/\epsilon}$$

m_b =boson mass.

- Strong coupling

Equivalent: Thirring model in external electric field

$$L_{Th} = i\bar{\psi}\partial_\nu\gamma^\nu\psi - \frac{1}{2}\,g\,j^\nu j_\nu + \mu\,\bar{\psi}\psi + A_0\,j_0$$

$\frac{\beta^2}{4\pi} = (1+\frac{g}{\pi})^{-1}$, $j_\nu = \bar{\psi}\gamma_\nu\psi$, μ= soliton mass in the sine-Gordon model, $\partial_x A_0 = \epsilon$.
Small g: the SG boson is a shallow bound state of fermion-antifermion, $m_b = 2\mu - \mu g^2$.
The near-threshold dynamics of the fermion-antifermion (soliton-antisoliton) pair can be described by the nonrelativistic Hamiltonian

$$H = \frac{p^2}{\mu} - \epsilon\,x - 2g\,\delta(x)$$

Boson-induced vacuum decay = ionization of the bound state in $U(x) = -2g\delta(x)$ by ext. electric field ϵ.

Equation for the Green function ($E = -\kappa^2/\mu$):

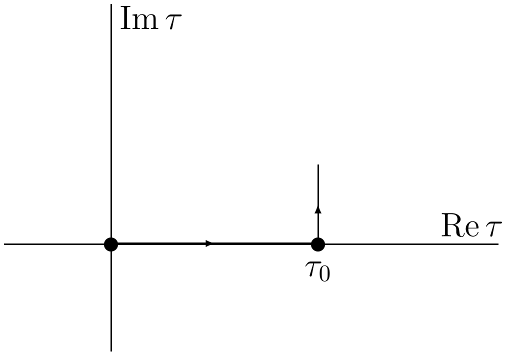
$$G(0,0;-\kappa^2/\mu) = \frac{G_\epsilon(0,0;-\kappa^2/\mu)}{1 - 2g\,G_\epsilon(0,0;-\kappa^2/\mu)}$$

with $G_\epsilon(x,y;E)$ the Green function in linear potential $(-\epsilon x)$

$$G_\epsilon\left(0,0;-\frac{\kappa^2}{\mu}\right) = \int_0^\infty \sqrt{\frac{\mu}{4\,\pi\,\tau}}\ \exp\left(\frac{\epsilon^2}{12\,\mu}\,\tau^3 - \frac{\kappa^2}{\mu}\,\tau\right)\,d\tau$$

The pole (in κ) is determined by

$$2g\,G_\epsilon\left(0,0;-\frac{\kappa^2}{\mu}\right) = 1$$



$\tau_0 = 2\kappa/\epsilon$. The decay rate (due to ionization):

$$\Gamma = 2\,\mu\,g^2\ \exp\left(-\frac{4}{3}\,g^3\,\frac{\mu^2}{\epsilon}\right)$$