Solving the 3D Ising Model with the Conformal Bootstrap

based on

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Outline

CFT's: why bother?

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  - in higher D: buy 1 get D free!
- AdS/CFT and Supersymmetry excellent tools, but some questions cannot be addressed.
- We would like to have a more general technique to deal with any CFT.

Ising model

Model to describe critical phenomena (ex: phase transition in ferromagnetism).

Based on a spin lattice with nearest-neighbors interactions:

$$H = -\frac{1}{T} \sum_i \sum_{j \sim i} \sigma_i \sigma_j$$

Continuum limit: iteratively sum the spins in a block of size n and replace σ_i with the average value.

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At the critical temperature the QFT flows to an IR fixed point. How can we deal with such a theory?

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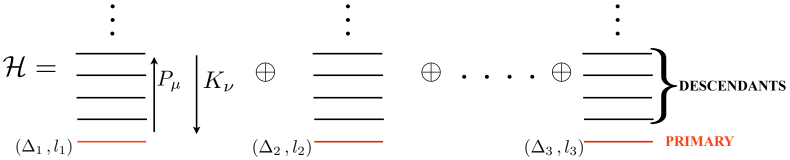
Can we do better?
Can we characterize a CFT without flowing to it from something else?

Conformal Algebra

In D dimensions : $M_{\mu\nu}\,,P_\rho,D\,,K_\sigma \simeq SO(D|2)$



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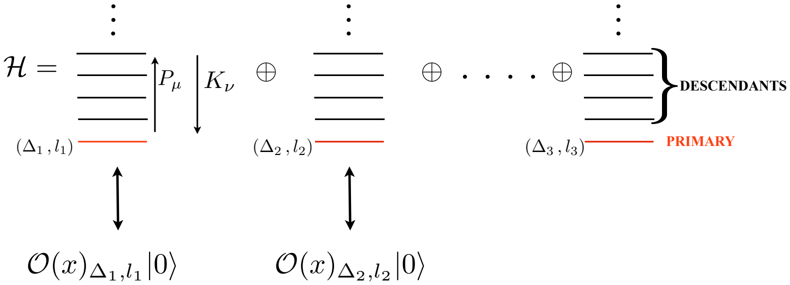

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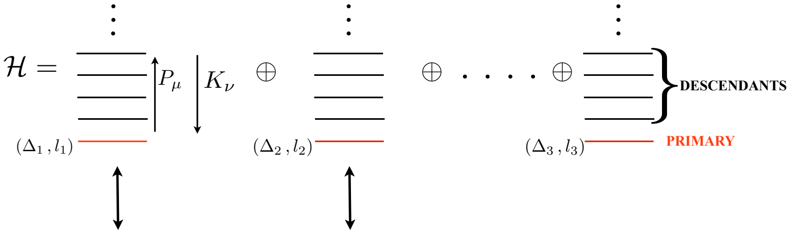

$$\mathcal{O}(x)_{\Delta_1,l_1}|0\rangle \qquad \mathcal{O}(x)_{\Delta_2,l_2}|0\rangle$$

Completeness of States $\Rightarrow$ Operator Product Expansion

$$\mathcal{O}(x)_{\Delta_1,0} \times \mathcal{O}(0)_{\Delta_2,0} = \frac{1}{|x|^{\Delta_1+\Delta_2}} \sum_{\Delta,l} C_{\Delta,l}(\mathcal{O}(0)_{\Delta,l} + \text{descendants})$$

The power of conformal invariance

Two point function: completely fixed

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Use OPE to reduce higher point functions to smaller ones

Four point functions

Recall the OPE

$$\mathcal{O} \times \mathcal{O} = \sum_{\mathcal{O}'_{\Delta,l}} C_{\Delta,l} (\mathcal{O}'_{\Delta,l} + \text{descendants})$$

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Conformal Blocks

$$g_{\Delta,l}(u,v) = \langle O'_{\Delta,l} O'_{\Delta,l}\rangle + \text{descendants}$$

They sum up the contribution of an entire representation

More on Conformal Blocks

Old idea (70's) but none could use them for long time, until..

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- external scalar fields

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'12: El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin, AV

- closed form for any dimension $l = 0, 1$ (but u, v related)
- Taylor expansion for any dimension and any l

The Bootstrap program

Which expansion is the right one?

$$\langle \mathcal{O}(x)\mathcal{O}(y)\mathcal{O}(z)\mathcal{O}(w)\rangle \quad \text{vs} \quad \langle \mathcal{O}(x)\mathcal{O}(y)\mathcal{O}(z)\mathcal{O}(w)$$

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Crossing symmetry $\Rightarrow$ Sum Rule

$$\sum_{\Delta,l} C^2_{\Delta,l} \underbrace{\frac{v^d g_{\Delta,l}(u,v) - u^d g_{\Delta,l}(v,u)}{u^d - v^d}}_{F_{d,\Delta,l}} = 1$$

- $F_{d,\Delta,l}$ known functions
- $C^2_{\Delta,l}$ unknown coefficients

Geometric interpretation

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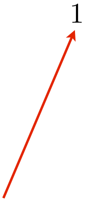

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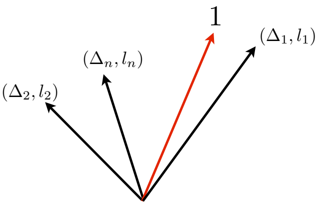

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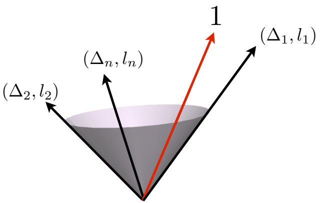

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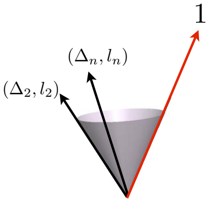

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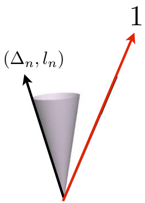

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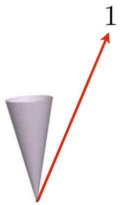
- A cone too narrow can't satisfy crossing symmetry: inconsistent spectrum

Geometric interpretation

How can we distinguish feasible spectra from unfeasible ones?



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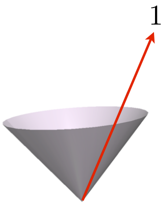

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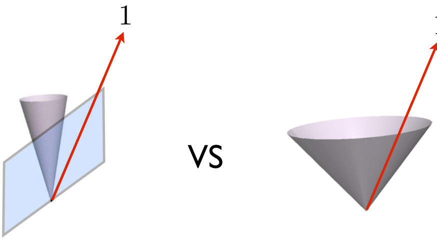

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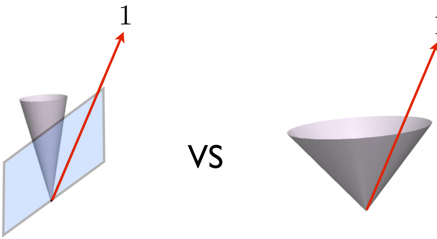

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More formally...

Look for a Linear functional

$$\Lambda[F_{d,\Delta,l}] \equiv \sum_{n,m}^{N_{\text{max}}} \lambda_{mn} \partial^n \partial^m F_{d,\Delta,l}$$

such that

$$\Lambda[F_1, \Delta, l] > 0 \qquad \text{and} \qquad \Lambda[1] < 0$$

Which spectrum?

Give me a spectrum and I'll tell you if it respects crossing symmetry

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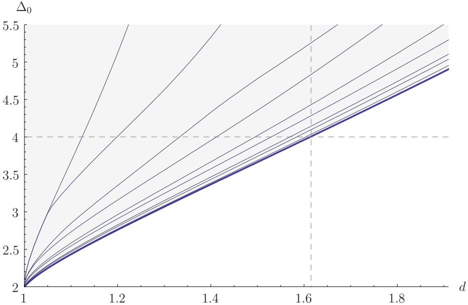

When $d \lesssim 1.6$, no CFT exists without relevant operator in $\phi \times \phi$

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Same story with OPE coefficients

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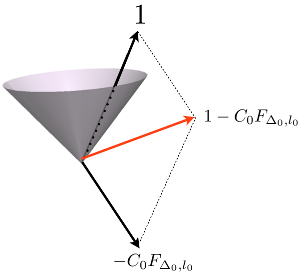

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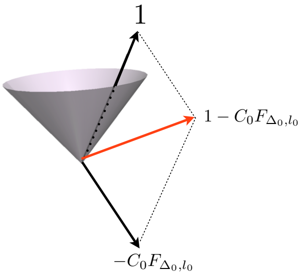

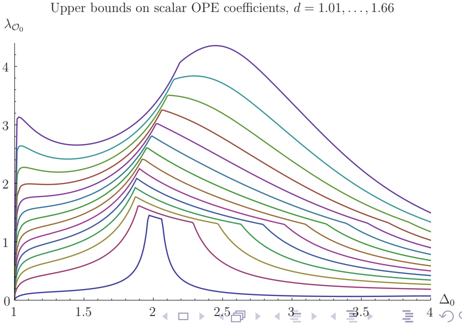

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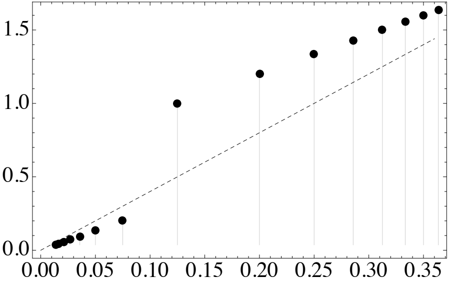

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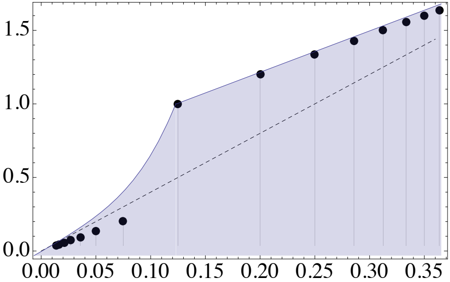

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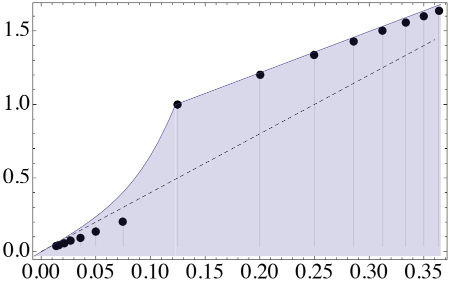

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A kink signals the presence of the Ising Model

(Re)discovering 2D Ising

Experimentally only one parameter must be tuned to reach the critical point
$\Rightarrow$ only 1 relevant scalar

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*3 instead of 2 to exclude generalized free theories. In Ising $\Delta_{\epsilon'} = 4$

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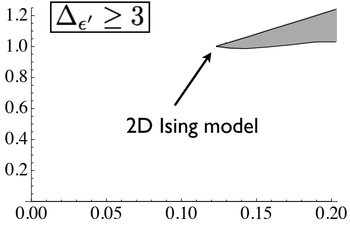

Tip ending at Ising Model: Ising first CFT with only one relevant operator!

Important

No use of Virasoro algebra. Extend the method to 3D right away

*3 instead of 2 to exclude generalized free theories. In Ising $\Delta_{\epsilon'} = 4$

3D Ising Model

Some notation:

$$\begin{array}{lcll} \sigma \times \sigma & \sim & 1 + \epsilon + \epsilon' + \epsilon'' + ... & L = 0 \\ & + & T_{\mu\nu} + T' + ... & L = 2 \\ & + & C + C' + ... & L = 4 \end{array}$$

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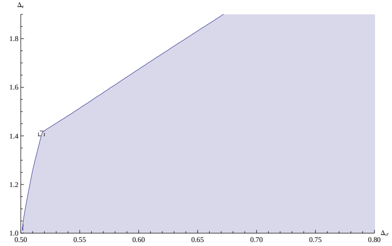

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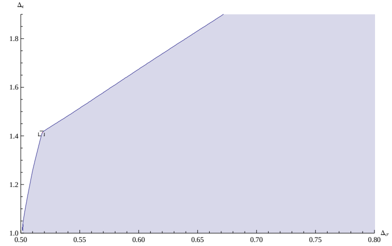

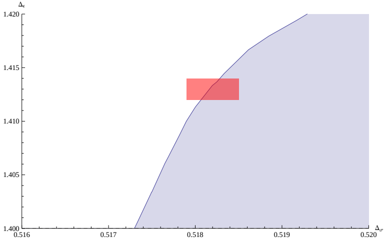

Already excluding part of $\epsilon-$expansion prediction

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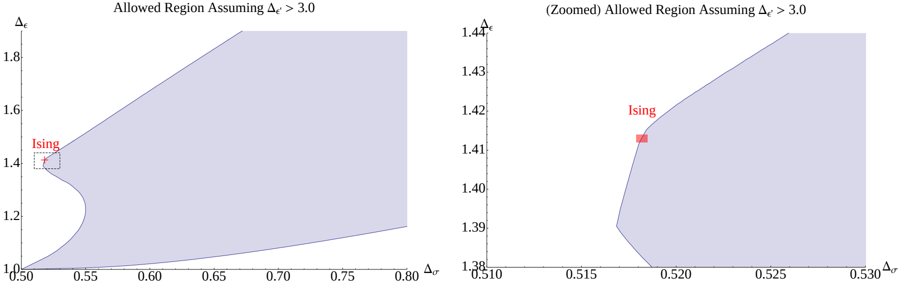

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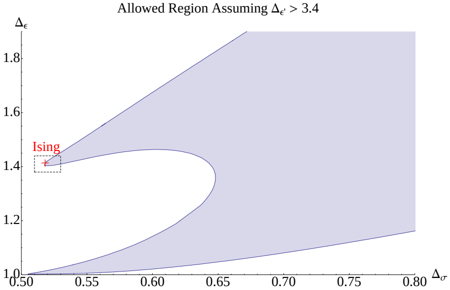
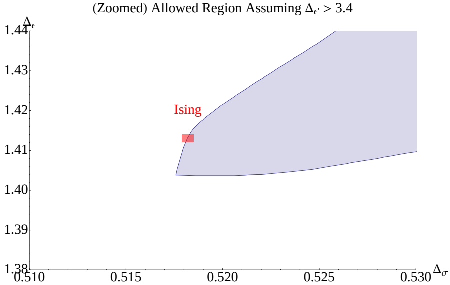
Allowed regions for in Δ_σ, Δ_ϵ plane if $\Delta_{\epsilon'} \geq 3.4$?





3D Ising Model

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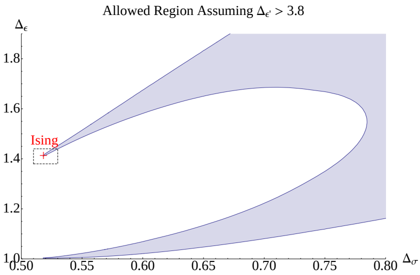
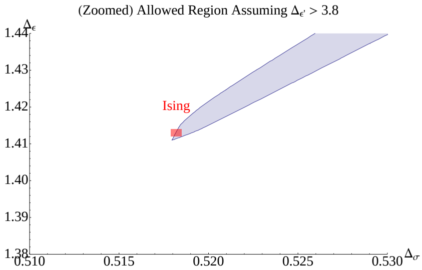
Allowed regions in Δ_σ, Δ_ϵ plane if $\Delta_{\epsilon'} \geq 3.8$?





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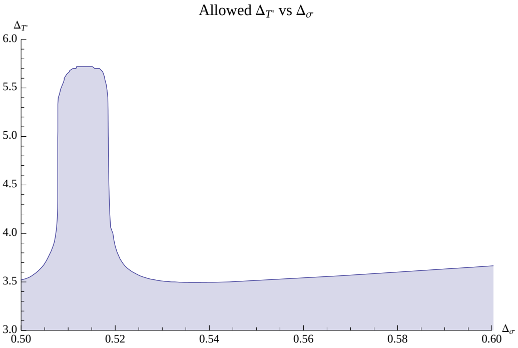
What about L=2 ?



- Energy momentum $T_{\mu\nu}$ tensor preset and $\Delta_T = 3$.
- What about $\Delta_{T'}$?

3D Ising Model

Assumptions

- Conformal symmetry at the fixed point
- safe assumptions on $\Delta_{\epsilon'}$
- safe assumptions on $\Delta_{T'}$

$\Rightarrow \quad (\Delta_\sigma, \Delta_\epsilon)$ predicted with good accuracy

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- $\Delta_\sigma,\ \Delta_\epsilon$ are the best measured quantities:
- $\Delta_\sigma^{exp} = 0.5183(4), \qquad \Delta_\epsilon^{exp} = 1.412(1)$
- one would like to assume them and predict the others

Back to 2D Ising Model

Compute bounds on OPE coefficients assuming $\Delta_\sigma = 1/8$, $\Delta_\epsilon = 1$

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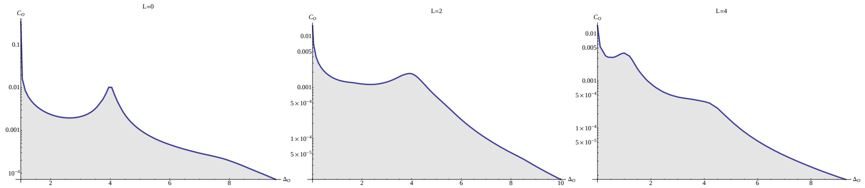

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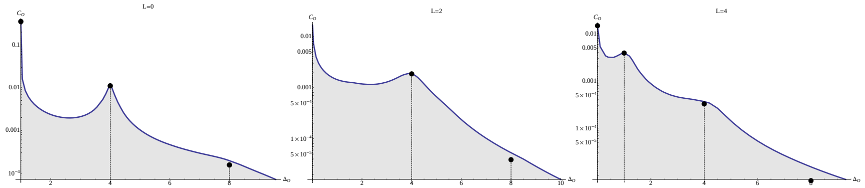

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Position determines the dimension Δ_O operators entering the $\sigma \times \sigma$ OPE

Height determines the OPE coefficient C_O of operators entering the $\sigma \times \sigma$ OPE

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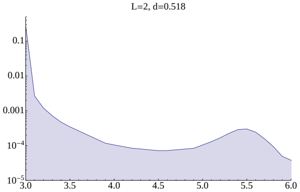

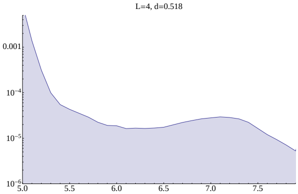

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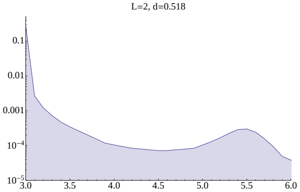

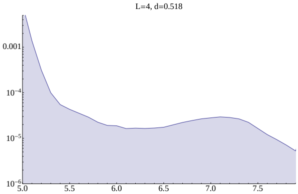

Again evidence of peaks:

L=2

Energy momentum tensor, $\Delta_T = 3$
$\Delta_{T'} \sim 5.5?$

L=4

$\Delta_C \sim 5.\ ?$
$\Delta_{C'} \sim 7.3?$

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Allowed values of c_T as function of Δ_σ:

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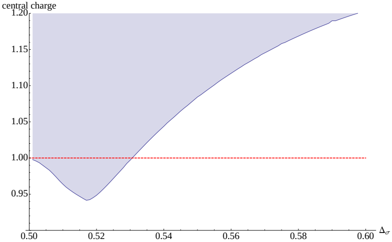

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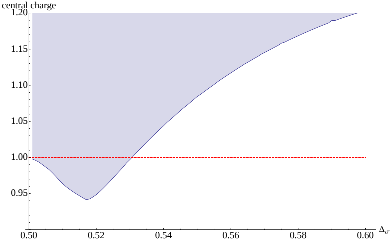

The minimum is in correspondence of Ising. It predicts $c_T^{\text{Ising}}/c_T^{\text{free}} \sim 0.94 - 0.95$

No accurate measurement nor calculation to compare with. $\epsilon-$expansion at first order gives $c_T^{\text{Ising}}/c_T^{\text{free}} \sim 0.98$

Note on 2D

Similar methods give $c_T^{\text{Ising}} \sim 0.4999$ and $c_T^{\text{exact}} = 0.5$.

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- Hunting for 4D Ising model?

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- Compare with AdS/CFT techniques.
- Bootstrap in Mellin space?