Warped Models in String Theory

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(Work in collaboration with B.S.Acharya and F.Benini)

Appearing soon

Introduction

5D warped models in a slice of AdS_5 very interesting from a phenomenological point of view:

- Based on *Randall-Sundrum* model $\Rightarrow$ they solve the Hierarchy problem through a warp factor.
- First proposal: all the SM localised on a 3-brane, where the warp factor is not negligible, but it is not necessary $\Rightarrow$ matter can come from 5D fields.
- Turning on bulk masses $\Rightarrow$ localization of fermions in extradim.
- Realization of Yukawa hierarchy through different overlap of fermion profiles with the Higgs.

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String realization of models presenting analogous characteristics:

- Warp factor $\rightsquigarrow$ Warped String Compactification.
- 5D matter fields $\rightsquigarrow$ Matter living on 8D *D7*'s worldvolume.
- Masses (localiz) $\rightsquigarrow$ Instanton background(localiz).
  $\Rightarrow$ Yukawa hierarchy.

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Outline

1. 5D Models
2. String Theory Warped Models
   - String realization of warped models
   - Fermion zero modes
   - Two realizations of Yukawa couplings
3. Conclusions

5D Warped Models [*Randall-Sundrum*]

RS1 model:

- The spacetime is 5 dimensional.
- The fifth dimension y is compactified on $\mathcal{S}^1/\mathbb{Z}_2$.
- Two orbifold fixed points: at $y = 0$ and at $y = \pi R \rightarrow$ boundaries.
- Two 3-branes at the boundaries:
  UV-brane at $y = 0$ and *IR*-brane at $y = \pi R$.
- The metric is not factorizable and is the form:

$$ds^2 = e^{-2\kappa y} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2 \,.$$

$\Rightarrow$ The spacetime between the two 3-branes is simply a slice of AdS_5 geometry. κ^{-1} is the AdS radius.

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5D Warped Models [*Randall-Sundrum*]

- The 4D reduced Planck mass M_4 is given by:

$$M_4^2 = \frac{M_5^3}{\kappa}(1 - e^{-2\pi\kappa R})$$

  it depends only weakly on R. The exponential has very little effect in determining the Planck scale.

- The warp factor plays an important role in determining the 4D masses on the IR-brane:

The HIERARCHY PROBLEM is addressed.

Generic mass scale M in 5D theory are scaled down to $e^{-\pi\kappa R} M$ on IR-brane. So, if the Higgs is localized at $y = \pi R \Rightarrow M_H \sim M_4\, e^{-\pi\kappa R}$.

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Fermion Zero Modes

- Matter fields not necessary on *IR*-brane [*Gherghetta-Pomarol*].
- 5D matter action with bulk mass terms ⇒ EOM's:

$$(g^{MN}\gamma_M D_N + m_\Psi)\Psi = 0$$

$m_\Psi = c\,\kappa\,\epsilon(y)$.

- D_N contains the warp factor:

$$\begin{aligned} e^{\kappa y}\eta^{\mu\nu}\gamma_\mu\partial_\nu\Psi_{(-)} + \partial_5\Psi_{(+)} + (m_\psi - 2k)\Psi_{(+)} &= 0 \\ e^{\kappa y}\eta^{\mu\nu}\gamma_\mu\partial_\nu\Psi_{(+)} - \partial_5\Psi_{(-)} + (m_\psi + 2k)\Psi_{(-)} &= 0 \end{aligned}$$

where $\Psi = \Psi_{(+)} + \Psi_{(-)}$ and $\gamma_5\Psi_{(\pm)} = \pm\Psi_{(\pm)}$.

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Fermion Zero Modes

- KK expansion of the fermions:

$$\Psi(x,y) = \sum_n \chi^{(n)}(x)\psi^{(n)}(y) \quad \text{with} \quad \eta^{\mu\nu}\gamma_\mu\partial_\nu\chi^{(n)} = m_n\chi^{(n)}$$

$\Rightarrow$ zero modes:

$$\psi^{(0)}(y) \propto e^{(\frac{1}{2}-c)\kappa y}$$

c is the bulk mass parameter

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Yukawa Couplings

- The Higgs is a 4D field, localized on the *IR*-brane.
- The Standard Model Yukawa coupling (YC) interactions are promoted to 5D interactions in the warped bulk:

$$\int d^4x \int dy \sqrt{-g}\, \lambda^{(5)}_{ij} \bar{\Psi}_i(x,y)\, \Psi_j(x,y)\, H(x)\, \delta(y - \pi R)$$

$$\Rightarrow \qquad \lambda_{ij} \sim \begin{cases} \lambda^{(5)}_{ij} \kappa \; e^{(1-c_i-c_j)\pi\kappa R} & c_{i,j} > 1/2 \\ \lambda^{(5)}_{ij} \kappa & c_{i,j} < 1/2 \end{cases}$$

Fermions mass hierarchy [*Grossman-Neubert*]

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5D models - Summary

The HIERARCHY PROBLEM is addressed.

The Higgs is localized on the *IR*-brane.

Fermions localized around different points in the bulk.

It is realized through a bulk mass term.

Yukawa hierarchy

It is generated by separating fermions from the Higgs.

Outline

Hierarchy in String Theory

In String Theory: warped compactifications

$$ds^2 = e^{-4A(z)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{4A(z)}\tilde{g}_{mn}dz^m dz^n$$

In type II there are solutions with non-trivial warp factor.

- Both Dbranes and Fluxes are sources of non-trivial warping.
- Regions where $e^{-4A} \ll 1$ are called throats:
  - $\Rightarrow$ The 4D energy of phenomena localized in these regions is redshifted by a factor of e^{-2A}.
- Local geometry in throats is tipically $\sim AdS_5 \times K_5$:

$$ds^2 = e^{-2y/L}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 + L^2 ds^2_{K_5} \qquad (L \text{ is } AdS_5 \text{ radius.})$$

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$D3$-branes background on 6D torus

Take a T^6 compactification of Type IIB [H.Verlinde].

N $D3$-branes (on $z = 0$) backreact on the gemetry, giving:

$$ds^2 = \frac{1}{f(z)^{1/2}} ds^2_{3,1} + f(z)^{1/2} dz^2$$

Define $r \equiv |z|$ and $L^4 \equiv 4\pi N g_s \alpha'^2$.

- When $r \gtrsim L \Rightarrow f(z) \sim 1$;
  $\hookrightarrow$ The geometry reduces to $\mathbb{R}^{3,1} \times T^6$.
- When $r \lesssim L \Rightarrow f(z) \sim \frac{L^4}{r^4}$.
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- The 4D reduced Planck mass M_4 is given by:

$$M_4^2 = M_{10}^8 V_6^w \quad \text{with} \quad V_6^w \equiv \int d^6 z f(z)$$

- The warp factor generate hierarchy of 4D scales:

$$S_H^{(p)} = -\frac{1}{2}\int d^4x \int d^{p-4}z \sqrt{-g} f^{1/2}[(\partial H)^2 + \frac{1}{f^{1/2}} M^2 H^2]$$

if a scalar field is localized in a region with warp factor $f_0^{-1/2}$, then its mass is suppressed (wrt a 10D mass M) to $f_0^{-1/4} M$.

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Warped compactifications on CY

In IIB compactifications on conformal CY [Giddings-Kachru-Polchiski].

$$ds^2 = e^{-4A(z)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{4A(z)}ds^2_{CY}$$

- Take CY with conical sing and deform it. [Klebanov-Strassler] Put M fluxes of RR F_3 and K of NS H_3 on the 3-cycles of the conifold. Near the sing, the geometry is $AdS_5 \times T^{1,1}$ (up to log-corrections) with $L^4 = 4\pi g_s MK\alpha'^2$ and:

$$e^{-4A_{\min}} \sim e^{-2\pi M/3Kg_s}$$

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Relation to 5D models

The throat resembles the situation in a slice of AdS_5:

- There is a warp factor depending on extra dimensional coordinate and that generates hierarchy of 4D scales.
- The role of UV-brane is played by the bulk compact manifold (where $e^{-4A} \sim 1$).
- There are string mechanisms to end the throat at $r_0 > 0$, avoiding divergent warp factor. The IR-brane is associated with $r = r_0$.

Can the other features of 5D models be found in a string setup?

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The Model - Introduction

- We take a *D*3 background to generate the warping.
- We put the matter on *D*7 branes $\longrightarrow$ 4 extra dimensions.
- We simulate the bulk mass terms by turning on instantons background on the 4dim euclidean extradim space
$\longrightarrow$ the fermion zero modes are localized around the instantons positions.
- Yukawa hierarchy generated by different overlap of the fermion profiles with the Higgs.

The Model

- We put N $D3$-branes in flat 10D flat spacetime $\Rightarrow$ warping and splitting of 10D metric.
- Interest in what happens in the throat $\leftrightarrow$ we consider regions where $r < L$.
- To introduce matter fields: $D7$-branes with SYM theory living on the 8-dim worldvolume:
  - ⋆ The metric is induced by the $D3$ one $\rightarrow$ warped product of (3,1) minkowski space and (4) euclidean space:

$$ds^2 = \frac{1}{f(r)^{1/2}} \eta_{\mu\nu} dx^\mu dx^\nu + f(r)^{1/2} \delta_{\alpha\beta} dz^\alpha dz^\beta \qquad f(r) = \frac{L^4}{r^4}$$

with $(\alpha, \beta = 1, ..., 4)$ and $r^2 = z_1^2 + ... + z_4^2$.

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with $(\alpha, \beta = 1, ..., 4)$ and $r^2 \equiv z_1^2 + ... + z_4^2$.

⋆ We are left with an 8D SYM theory with bosonic action:

$$S_{D7} = -\frac{1}{2g^2} \int d^8X \sqrt{-G}\, \mathrm{Tr}\left(F \wedge *_8 F - F \wedge F \wedge C_4\right)$$

G_{MN} and C_4 are the $D3$ background induced on the $D7$ worldvolume.

⋆ We turn on background gauge field, living only in (4)-space

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EOM's of the 8D warped theory gives 4D gauge field:

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Change of the radial coordinate.

- In the throat the $D7$ metric is given by:

$$ds^2 = \frac{r^2}{L^2} dx^2 + \frac{L^2}{r^2}(dr^2 + r^2 d\Omega_3)$$

- Make the change of coordinate: $r = Le^{-y/L}$:

$$ds^2 = e^{-2\kappa y} dx^2 + dy^2 + L^2 d\Omega_3$$

The warp factor is $e^{-2\kappa y}$.

- r coord $\rightarrow$ the extradim metric is conformally flat and the instanton results can be used.
- y coord $\rightarrow$ the metric resembles that one of the RS 5D models.

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Fermion zero modes in "warped instanton solution"

In the $D7$-brane spectrum $\rightarrow$ 8dim fermions satisfying:

$$\not{D}_8 \Psi = 0$$

Under the splitting of the 8D space, it becomes:

$$(\not{D}_{3,1} + \not{D}_4) \sum_k \chi_k(x) \otimes \psi_k(y)$$

Written in terms of (flat) $\tilde{\not{D}}_{3,1}$ and $\tilde{\not{D}}_4$, and of the warp factor:

$$\not{D}_8 = f^{1/4} \tilde{\not{D}}_{3,1} + \frac{1}{f^{1/4}} \tilde{\not{D}}_4 - \frac{1}{8 f^{1/4}} \frac{f'}{f} \gamma_r$$

Massless fermions in $(3,1)$dim $\leftrightarrow$ zero modes of $(\tilde{\not{D}}_4 - \frac{f'}{8f}\gamma_r)$:

$$\psi \propto f^{1/8} \tilde{\psi} \, .$$

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Fermion zero modes in "warped instanton solution"

Fermion zero mode profile:

$$\psi = d_\psi f^{1/8} \tilde{\psi}$$

- $\tilde{\psi}$ is the zero mode of the operator $\tilde{\not{D}}_4$, *i.e.* the instanton fermion zero modes.
- d_ψ is a constant, set by requiring 4D canonically normalized kinetic term.

Instanton fermion zero modes $\tilde{\psi}$

Instanton solutions found with ADHM construction. Take $SU(2)$ inst.

- The $SU(2)$ gauge field is written in terms of a $(2+2k)\times 2$ matrix $v(z)$:

$$A_\mu(z) = v(z)^\dagger \partial_\mu v(z)$$

- One gets self-dual field strength if $v(z)$ satisfies the algebraic equations:

$$v(z)^\dagger v(z) = 1$$
$$v(z)^\dagger \Delta(z) = 0$$

where $\Delta(z) \equiv a - b\mathbf{z}$ and a, b are $(2+2k)\times 2k$ matrices that contain the moduli of the instantonic configuration $(8k)$, and

$$\Delta(z)^\dagger \Delta(z) = s^{-1}(z) \otimes \mathbf{1}_2$$

Instanton fermion zero modes $\tilde{\psi}$

't Hooft solution.

- Particular form for a and b:

$$a = \begin{pmatrix} \rho_i \mathbf{1}_2 \\ \delta_{ji} \mathbf{Z}_i \end{pmatrix} \qquad b = \begin{pmatrix} 0 \\ \mathbf{1}_{2k} \end{pmatrix}$$

- This reduces the number of moduli to $5k$: Z_i^m and ρ_i ($i = 1, ..., k$ and $m = 1, ..., 4$).
- There are asymptotic regions of the parameters space where the multi-instanton configurations can be identified as being composed of well-separated single instantons, *i.e.* when

$$\rho_i \rho_j \ll (Z_i - Z_j)^2 \qquad \forall i \neq j$$

$\Rightarrow$ Z_i's become the positions of the k instantons, while the ρ_i's are their sizes.

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Instanton fermion zero modes $\tilde{\psi}$

- In the 't Hooft solution, the fermion zero modes are given by:

$$(v^\dagger b\, s)_i(z) = \left[1+\sum_{\ell=1}^{k}\frac{\rho_\ell^2}{(z-Z_\ell)^2}\right]^{-3/2}\frac{\rho_i}{(z-Z_i)^2}\times$$
$$\times\left\{\left[1+\sum_{\ell=1}^{k}\frac{\rho_\ell^2}{(z-Z_\ell)^2}\right]\frac{\mathbf{z}-\mathbf{Z}_i}{(z-Z_i)^2}-\sum_{j=1}^{k}\frac{\rho_j^2}{(z-Z_j)^4}(\mathbf{z}-\mathbf{Z}_j)\right\}$$

where $i = 1, ..., k$.

- In the limit of well separated k instantons:

$$\tilde{\psi}_i(z) \sim \frac{\rho_i}{(\rho_i^2+(z-Z_i)^2)^{3/2}}$$

Localization!

The profile $\tilde{\psi}_i$ is localized around Z_i in a region of size ρ_i.

Instanton fermion zero modes $\tilde{\psi}$

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&\times\left\{\left[1+\sum_{\ell=1}^{k}\frac{\rho_\ell^2}{(z-Z_\ell)^2}\right]\frac{\mathbf{z}-\mathbf{Z}_i}{(z-Z_i)^2}-\sum_{j=1}^{k}\frac{\rho_j^2}{(z-Z_j)^4}(\mathbf{z}-\mathbf{Z}_j)\right\}
\end{aligned}
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  where $i=1,\dots,k$.

- In the limit of well separated k instantons:

$$\tilde{\psi}_i(z) \sim \frac{\rho_i}{(\rho_i^2+(z-Z_i)^2)^{3/2}}$$

Localization!

The profile $\tilde{\psi}_i$ is localized around Z_i in a region of size ρ_i.

Normalization Constant d_ψ

- Consider the 8D kinetic term:

$$-\int d^8X\sqrt{-G}G^{\mu\nu}\bar{\Psi}\Gamma_\mu\partial_\nu\Psi + \dots$$

$$= -\int d^4z\, f(z)^{1/4}\psi(z)^\dagger\psi(z)\int d^4x\,\eta^{\mu\nu}\bar{\chi}(x)\gamma_\mu\partial_\nu\chi(x) + \dots$$

Canonically normalized (3,1)D kinetic term implies:

$$d_\psi^2\int d^4z\, f^{1/2}(z)\tilde{\psi}(z)^\dagger\tilde{\psi}(z) = 1 \quad \text{where} \quad \psi = d_\psi f^{1/8}\tilde{\psi}$$

- In regions of large warping and in the limit of well separated inst:

$$d_\psi = \sqrt{\frac{\rho^2}{L^2} + \frac{|Z|^2}{L^2}} \quad \sim \quad \frac{|Z|}{L} \equiv e^{-\kappa Y_\psi}$$

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Outline

1. 5D Models
2. String Theory Warped Models
   - String realization of warped models
   - Fermion zero modes
   - Two realizations of Yukawa couplings
3. Conclusions

Yukawa Couplings. A simple example.

Higgs field

The simplest choice for the Higgs $\rightarrow$ (3,1) field localized at some point Z_H in the extradim
$\Rightarrow$ whatever its string nature is, it can be effectively represented with a δ-function in the (4)-space.

- This choice is similar to the simplest 5D proposal for the Higgs.
- The large ratio between the Higgs and the Plank masses is realised putting the Higgs in the throat:

$$e^{-\kappa Y_H} = \frac{M_H}{M_{Pl}} \quad \Longleftrightarrow \quad \kappa\, Y_H \sim 37$$

Yukawa Couplings. A simple example.

Normalization of the Higgs.

- Take 8D kinetic term:

$$-\int d^8x\sqrt{-\hat{G}_{3,1}}\,G^{\mu\nu}\,d_H^2\,\partial_\mu H(x)\partial_\nu H(x)\,\delta(\vec{z}-\vec{Z}_H)=$$

$$=-d_H^2\,f(|\vec{Z}_H|)^{1/2}\int d^4x\,\partial_\mu H(x)\partial^\mu H(x)$$

- Requiring canonically normalized kinetic term:

$$d_H = e^{\kappa\, Y_H}$$

Yukawa Couplings. A simple example.

4D Yukawa coupling obtained by dim reduction of the 8D one:

$$\int d^8x \sqrt{-\hat{G}_{3,1}}\, \lambda^{(8)}\, d_H\, \bar{\Psi}\Psi H\, \delta(\vec{z} - \vec{Z}_H) =$$

$$= \lambda^{(8)} d_H f(|\vec{Z}_H|)^{-1} \psi(\vec{Z}_H)^2 \int d^4x\, \bar{\chi}(x)\chi(x)H(x)$$

$$\Rightarrow \qquad \lambda = \lambda^{(8)} d_H \left. \frac{\psi^2}{f} \right|_{Z_H}$$

Yukawa Couplings. A simple example.

- Substitute the expressions for ψ, f and d_H:

$$\lambda = \lambda^{(8)} e^{-2\kappa(Y_H+Y_\psi)} \frac{\rho^2}{\left[\rho^2 + (\vec{Z}_H - \vec{Z}_\psi)^2\right]^3}$$

- To get the maximal Yukawa coupling (top) $\Rightarrow \vec{Z}_\psi = \vec{Z}_H$:

$$\lambda = \frac{\lambda^{(8)}}{\rho^4} e^{-4\kappa Y_H}$$

Notice: ρ is the parameter controlling the instanton size in flat (4)-space, but is not a physical distance in the actual extradim space.

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Yukawa Couplings. A simple example.

Physical size:

$$\rho_{\text{phys}} = \int_{|\vec{Z}_\psi|-\frac{\rho}{2}}^{|\vec{Z}_\psi|+\frac{\rho}{2}} ds = \int_{|\vec{Z}_\psi|-\frac{\rho}{2}}^{|\vec{Z}_\psi|+\frac{\rho}{2}} f^{1/4}(r)dr \simeq e^{\kappa Y_\psi}\rho$$

The last result is obtained in the limit $\rho \ll |Z_\psi|$.

Substituting in expression for λ:

The Yukawa coupling for fermions on "*IR* brane":

$$\lambda = \frac{\lambda^{(8)}}{\rho^4_{\text{phys}}}$$

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The Yukawa coupling for fermions on "*IR* brane":

$$\lambda = \frac{\lambda^{(8)}}{\rho^4_{\mathbf{phys}}}$$

Take $\lambda^{(8)} \sim \ell^4$.

- When $\rho_{phys} \sim \ell \Rightarrow \lambda \sim 1$: *top* Yukawa coupling.
- When $\rho_{phys} > \ell \Rightarrow \lambda < 1$: smaller Yukawa couplings.

Yukawa Couplings. A simple example.

Hierarchically smaller Yukawa couplings.

- Move the fermion zero mode far from the Higgs: $Y_\psi < Y_H$.
- Keep $\rho \lesssim X$, where we defined $X \equiv |Z_\psi - Z_H|$.
- Impose these conditions on $\lambda = \lambda^{(8)} e^{-2\kappa(Y_H+Y_\psi)} \frac{\rho^2}{\left[\rho^2+(\vec{Z}_H-\vec{Z}_\psi)^2\right]^3}$
- We get:

$$\lambda = \lambda^{(8)}\, e^{-2\kappa(Y_H+Y_\psi)}\, \frac{\rho^2}{X^6}$$

We can rewrite it in a more readable form

$$\downarrow$$

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Yukawa Couplings. A simple example.

The Yukawa coupling for fermions far from "*IR* brane":

$$\lambda \quad = \quad \frac{\lambda^{(8)}}{\rho^4_{phys}} \frac{\rho^6}{X^6} e^{-2\kappa(Y_H - Y_\psi)}$$

- It is hierarchically smaller than the top Yukawa:
  - $\hookrightarrow$ Electron Yuk $\lambda_e \sim 10^{-6}$
    - *e.g.* $\frac{\rho}{X} \lesssim 1$ and $\kappa Y_\psi \sim 30$;
    - *e.g.* $\frac{\rho}{X} \sim \frac{1}{10}$ and $Y_\psi \sim Y_H$.
  - $\hookrightarrow$ Neutrino Yuk $\lambda_\nu \sim 10^{-16}$
    - *e.g.* $\frac{\rho}{X} \sim \frac{1}{10}$ and $\kappa Y_\psi \sim 22$;
    - *e.g.* $\frac{\rho}{X} \lesssim 1$ and $\kappa Y_\psi \sim 15$. $\qquad (\kappa Y_H \sim 37)$

The Higgs as a vector zero mode

- Take the 8D kinetic term: $\int d^8X\sqrt{-G}\bar{\Psi}\not{D}\Psi$.
- It contains the term

$$g\int d^8X\sqrt{-G}\bar{\Psi}\,\delta\not{A}\Psi \quad\supset\quad \int d^4x\,\bar{\chi}_i(x)\chi_j(x)H_k(x)\times g\int d^4z\,\psi_i^\dagger(z)\,\delta\not{a}_k(z)\psi_j(z)$$

where we split the fermions as above and the vectors as:

$$A(x,z)_m dy^m = A_{bkg}(z)+\sum_k H^k(x)\delta a_k(z)$$

The Higgs is a zero mode of the vector field.

The effective Yukawa coupling in the (3,1)-theory is:

$$\lambda_{ij} = g\; d_{\psi_i}d_{\psi_j}\int d^4z\,\tilde{\psi}_i^\dagger(z)\Phi_H(z)\tilde{\psi}_j(z)\,.$$

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The Higgs as a vector zero mode

Come back to instantons in flat (4)-space.

- Choose the zero mode associated with the translation of one of the k instantons ('t Hooft solution).
- Take the approximation of well separated instantons.
- Write the zero mode around its absolute maximum:

$$\tilde{\Phi}_H(z) \sim \frac{\rho_H^2}{(\rho_H^2 + (z - Z_H)^2)^2}$$

- We see again localization.

The Higgs as a vector zero mode

Put the Higgs inst in warped background.

Normalization.

$\hookrightarrow$ From 8D kinetic term:

$$\int d^8X\sqrt{-G}G^{\mu\nu}G^{mn}\partial_\mu\delta A_m\partial_\nu\delta A_n \quad\rightarrow\quad d_H^2\int d^4z\,\tilde{\Phi}_H(z)^2\int d^4x(\partial H)^2$$

$\Rightarrow$ $d_H \sim 1$ (not affected by warping) and $\Phi_H \sim \tilde{\Phi}_H$.

The Higgs as a vector zero mode

The Yukawa couplings are given by

$$\lambda_{ij} = g\, d_{\psi_i} d_{\psi_j} \int d^4 z\, \tilde{\psi}_i^\dagger(z) \Phi_H(z) \tilde{\psi}_j(z)\,.$$

- Take $k = 2$.
- The fermions are zero modes associated with one instanton, while the Higgs with the other one.
- Substitute the expressions for zero modes of $k = 2$ instanton background ('t Hooft).
- Parameters: ρ_H, ρ_ψ, Z_1 and Z_H. Define $X = |Z_\psi - Z_H|$.

The Higgs as a vector zero mode

- To get maximal Yukawa coupling (top) $\Rightarrow Z_\psi = Z_H$ and $\rho_\psi \sim \rho_H$:

$$\lambda \sim \frac{g}{\rho_H^2} e^{-2\kappa Y_H}$$

- Substitute the physical size:

The Yukawa coupling for fermions on "*IR* brane":

$$\lambda = \frac{\mathbf{g}}{\rho_{\mathbf{H}\mathrm{phys}}^{\mathbf{2}}}$$

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The Higgs as a vector zero mode

- Yukawa hierarchy → obtained by varying the instanton parameters to have different overlaps of the zero modes.
- One can approximately compute the integral giving the Yukawa's in other asymptotic regions of the parameter space:

limits	$g\int d^4z\, \tilde{\psi}_i^\dagger(z)\Phi_H(z)\tilde{\psi}_j(z)$
$\rho_H \sim \rho_\psi \ll X$	$\frac{g}{\rho_H^2} \left(\frac{\rho_H}{X}\right)^3$
$\rho_H \ll \rho_\psi \sim X$	$\frac{g}{\rho_H^2} \left(\frac{\rho_H}{X}\right)^2$
$\rho_H \ll \rho_\psi \ll X$	$\frac{g}{\rho_H^2} \left(\frac{\rho_H}{X}\right)^2 \left(\frac{\rho_\psi}{X}\right)^2 \left[1 + \frac{X}{\rho_\psi}\left(\frac{\rho_H}{\rho_\psi}\right)^2\right]$
$\rho_H \ll X \ll \rho_\psi$	$\frac{g}{\rho_H^2} \left(\frac{\rho_H}{\rho_\psi}\right)^4 \left[1 + \left(\frac{X}{\rho_H}\right)^2 \left(\frac{X}{\rho_\psi}\right)^2\right]$
$X \lesssim \rho_H \ll \rho_\psi$	$\frac{g}{\rho_H^2} \left(\frac{\rho_H}{\rho_\psi}\right)^4$

The Higgs as a vector zero mode

- Yukawa hierarchy $\longrightarrow$ obtained by varying the instanton parameters to have different overlaps of the zero modes.
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$\rho_H \ll \rho_\psi \sim X$	$\frac{g}{\rho_H^2}$	$\left(\frac{\rho_H}{X}\right)^2$
$\rho_H \ll \rho_\psi \ll X$	$\frac{g}{\rho_H^2}$	$\left(\frac{\rho_H}{X}\right)^2\left(\frac{\rho_\psi}{X}\right)^2\left[1+\frac{X}{\rho_\psi}\left(\frac{\rho_H}{\rho_\psi}\right)^2\right]$
$\rho_H \ll X \ll \rho_\psi$	$\frac{g}{\rho_H^2}$	$\left(\frac{\rho_H}{\rho_\psi}\right)^4\left[1+\left(\frac{X}{\rho_H}\right)^2\left(\frac{X}{\rho_\psi}\right)^2\right]$
$X \lesssim \rho_H \ll \rho_\psi$	$\frac{g}{\rho_H^2}$	$\left(\frac{\rho_H}{\rho_\psi}\right)^4$

The Higgs as a vector zero mode

Similar result as for the previous case:

The Yukawa coupling for fermions on "*IR* brane":

$$\lambda = \frac{g}{\rho^2_{\psi \text{phys}}} \left(\frac{\rho_H}{\rho_\psi} \right)^2$$

For $\rho_\psi \sim \rho_H \Rightarrow \lambda_{top} \sim 1$.

The Yukawa coupling for fermions far from "*IR* brane":

$$\frac{X}{\rho_\psi} \frac{\rho_H^2}{\rho_\psi^2} \gg 1 \quad \rightarrow \quad \lambda = \frac{g}{\rho^2_{\psi \text{phys}}} \left(\frac{\rho_\psi}{X} \right)^4$$

$$\frac{X}{\rho_\psi} \frac{\rho_H^2}{\rho_\psi^2} \ll 1 \quad \rightarrow \quad \lambda = \frac{g}{\rho^2_{\psi \text{phys}}} \left(\frac{\rho_\psi}{X} \right)^3 e^{-2\kappa(Y_\psi - Y_H)}$$

Hierarchically smaller Yukawa couplings.

Conclusions

The HIERARCHY PROBLEM is addressed.

Warped Compactification. The Higgs is localized deeply in the throat.

Localization of zero modes in the bulk

It is realized through an instanton background.

Fermion mass hierarchy

It is generated by varying instanton parameters (more than 5D):

- top Yukawa coupling $\longrightarrow$ top localized near the Higgs;
- very small Yukawa couplings $\longrightarrow$ fermion far from the Higgs.

Open problems and future directions

- How to get chiral spectrum (magnetic fluxes?).
- Stabilization of instanton moduli.
- Introduction of SUSY.
- String nature of 4D Higgs (simple example).
- Higgs as a scalar instanton zero mode.
- Change the setup (e.g. CY instead of T^6).