FRW holography from uplifted AdS/CFT

Gonzalo Torroba

SLAC, Stanford University

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with X. Dong, B. Horn, S. Matsuura, E. Silverstein

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At present, we lack a complete theoretical framework for cosmology.

Central goal: formulate quantum gravity on cosmological spacetimes and understand the degrees of freedom that describe these solutions.

Important progress in quantum gravity:

- AdS/CFT correspondence;
- entropy/area laws (black holes, de Sitter)

$$\mathcal{S} = \frac{\text{Area}}{4G_N}$$

In some examples, $\mathcal{S}$ understood microscopically in terms of dual gauge theory.
[Maldacena; Strominger-Vafa; Callan-Maldacena; ...]

Formulate cosmology holographically?

Problems in generalizing AdS/CFT to cosmology:

- Absence of non-fluctuating time-like boundary; no spatial infinity.
- Dynamical gravity also present in the dual theory
  (see e.g. dS/CFT, dS/dS, FRW/CFT)
- Cosmological horizons, observer dependent. Microstates?

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- Cosmological horizons, observer dependent. Microstates?

Goal: find cosmological solutions in string theory that admit a brane interpretation.

Basic strategy: “uplift” AdS/CFT adding **magnetic flavor branes**.

See [Polchinski, Silverstein] for AdS case, and [DHST] for dS

We will find a holographic duality

$$\begin{pmatrix}\textbf{FRW}_d\ \textbf{solutions with}\\ \textbf{scale factor}\ a(t)=ct\end{pmatrix} \Longleftrightarrow \begin{pmatrix}\textbf{Time-dependent QFT}_{d-1}\\ \textbf{w/magnetic flavors}\end{pmatrix}$$

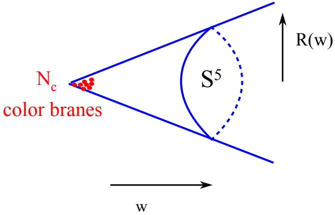
Road map for FRW holography

1. Uplifting AdS/CFT to cosmology
2. FRW sols sourced by magnetic flavors
3. Degrees of freedom in FRW holography
4. Dynamics of particles and branes

1. Uplifting AdS/CFT to cosmology

Let's re-formulate AdS/CFT in a way that can be extended to cosmology.

Consider N_c color D3 branes at the tip of the cone over S^5 (i.e. $\mathbb{R}^6$).



Supergravity ansatz:

$$
\begin{aligned}
S &= (M_{10})^8 \int \sqrt{-g^{(10)}} \left[e^{-2\phi}(\mathcal{R}^{(10)} + (\nabla\phi)^2) - \frac{1}{2}|F_5|^2 \right] + \dots \\
ds^2 &= e^{2A(w)}\, \eta_{\mu\nu} dx^\mu dx^\nu + dw^2 + R(w)^2 d\Omega_5^2 \ , \quad F_5 \propto \epsilon_{S^5}
\end{aligned}
$$

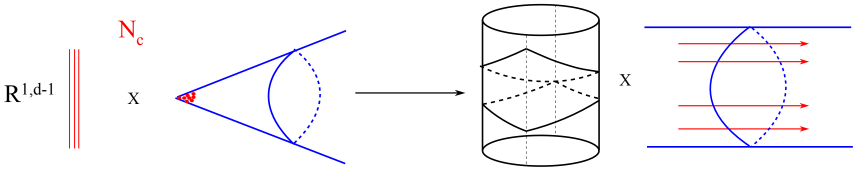
- 1) Probe limit: ignore backreaction ($e^{2A} = 1$, $F_5 = 0$).

$$\text{10d Einstein's eqs:} \quad \frac{(dR/dw)^2}{R^2} \sim +\frac{1}{R^2} \quad \Rightarrow \quad R(w) = w$$

- 2) Near horizon limit adding flux backreaction:

$$\frac{(dR/dw)^2}{R^2} \quad \sim \quad +\frac{1}{R^2} - \frac{N_c^2 g_s^2}{R^{10}} \quad \Rightarrow \quad R^4 = g_s N_c$$

$$e^{2A} \quad \sim \quad e^{2w/R} \quad , \quad F_5 = N_c \, \epsilon_{S^5}$$



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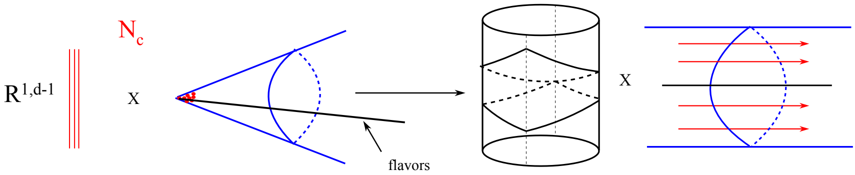

Compactification and effective potential

Compactify 10d theory on S^5, and treat g_s and R as fields in a 5d theory. In Einstein frame,

$$\mathcal{U}_{eff} = (M_5)^5 \left(\frac{g_s^2}{R^5}\right)^{5/3} R^5 \left(-\frac{1}{g_s^2}\frac{1}{R^2} + \frac{N_c^2}{R^{10}}\right) = (M_5)^5 \left(-\eta^2 + N_c^2 \eta^5\right)$$

$$\text{where} \quad \eta \equiv \frac{1}{R}\left(\frac{g_s^2}{R^5}\right)^{1/3} .$$

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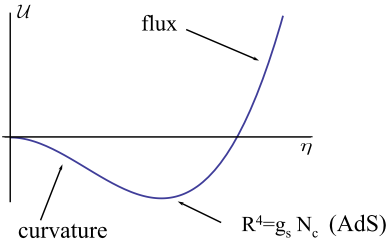
$$\text{where} \quad \eta \equiv \frac{1}{R}\left(\frac{g_s^2}{R^5}\right)^{1/3} .$$

$$\text{Minimum}: \quad \eta^3 = \frac{1}{N_c^2}$$

$$\Rightarrow \quad R^4 = g_s N_c$$

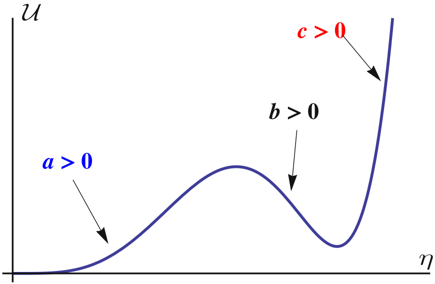


Uplifting AdS/CFT to de Sitter

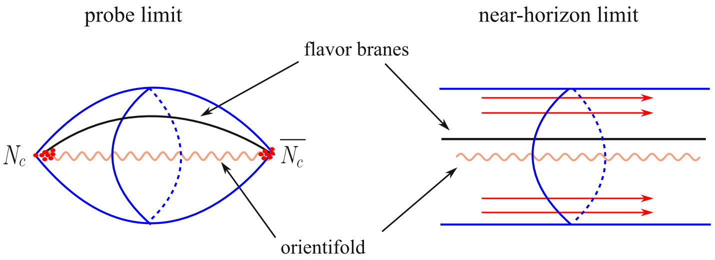
d-dim de Sitter in string theory using classical sources:

$$\mathcal{U} = (M_{P,d})^d \left(a(\sigma)\eta^2 - b(\sigma)\eta^{(d+2)/2} + c(\sigma)\eta^d \right)$$

- 1) Branes with tension $1/g_s^2$ that ‘uplift’ curvature and give $a > 0$
- 2) Orientifold to provide the negative $\eta^{(d+2)/2}$ term
- 3) Fluxes from color-branes that give η^d with $c > 0$



• String realization: brane/antibrane system w/magnetic flavors [DHST]



$$\frac{R'(w)^2}{R^2} \sim -\frac{1}{R^{n_1}} + \frac{g_s}{R^{n_2}} \quad \Rightarrow \quad \frac{R'(w)^2}{R^2} \sim -\frac{1}{R^{n_1}} + \frac{g_s}{R^{n_2}} - \frac{g_s^2 N_c^2}{R^{2n}}$$

plus dynamical gravity ...

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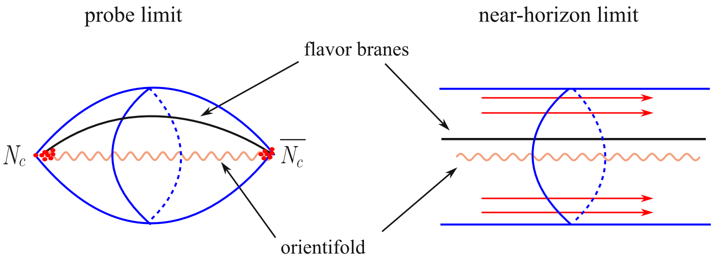

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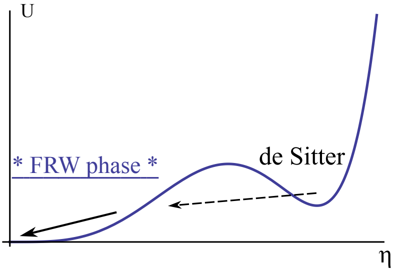
plus dynamical gravity ...

- Macroscopics: dS/dS correspondence [Silverstein et al]

$ds^2_{dS_d} = dw^2 + \sin^2 w \, ds^2_{dS_{d-1}} \quad \Rightarrow$ two throats, dual on dS_{d-1} +gravity

2. FRW sols sourced by magnetic flavors

Now focus on the FRW phase, where the ‘uplifting’ branes give the dominant contribution:



For concreteness, consider $AdS_5 \times S^5$ case.
[View S^5 as S^1 Hopf fibration over $\mathbb{P}^2$]

$\rightsquigarrow$ **(p,q) 7-branes** play role of uplifting ingredient.

They wrapp $AdS_5 \times S^1_f \times \Sigma_2(\subset \mathbb{P}^2)$

♦ (p,q) 7-branes compete with curvature: they are codimension 2 and have tension $T_7 \sim 1/g_s^2$.

E.g. 24 7-branes exactly cancel the curvature of $\mathbb{P}^1$.

For n 7-branes,

$$\mathcal{U}_R \propto \frac{\Delta n}{R^2} \quad , \quad \Delta n \equiv n - n_{\text{flat}}$$

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♦ They add electric and magnetic flavors to the gauge theory on D3s. Field theory well-understood for $\Delta n < 0$.
[Sen; Banks, Douglas, Seiberg; Argyres, Douglas; ...]

- $\Delta n < 0$: AdS/CFT sols, studied by [Aharony, Fayazzudin, Maldacena; Polchinski, Silverstein]
- $\Delta n \geq 0$: no static sol. $\rightsquigarrow$ **cosmology!**

We will present time-dependent cosmologies for $\Delta n > 0$, and study the holographic duals.

Cosmological 10d solution

Late time solution (string frame):

$$ds_s^2 = -dt_s^2 + \frac{t_s^2}{c^2} dH_4^2 + \frac{t_s^2}{c^2} dB_4^2 + dx_f^2 \quad , \quad c^2 = \frac{7}{3}$$

with B_4 is a compact 4-dim hyperbolic space.

- Arises at late times in the D3-(p,q)7 system with $\Delta n > 0$.
  - Color flux and metric flux from S^1 fiber subdominant.
  - 5d spacetime is open FRW (instead of AdS_5). B_4: uplifted $\mathbb{P}^2$.
- It is also an exact Ricci flat vacuum sol to Einstein's eqs.
- More general set of FRW solutions:

$$\begin{aligned} ds_s^2 &= -dt_s^2 + \frac{t_s^2}{c^2} dH_{d-1}^2 + \frac{t_s^2}{\hat{c}^2} dB_{2m}^2 + dx_f^2 \\ c^2 &= \frac{d+2m-2}{d-2} \, , \; \hat{c}^2 = \frac{d+2m-2}{2m-1} \end{aligned}$$

(Earlier work on a different class of sols: [Kleban, Redi])

Compactify to d-dimensions. Einstein frame metric: $t \sim t_s^{c^2}$,

$$ds_E^2 = -dt^2 + c^2 t^2\, dH_{d-1}^2 \quad , \quad dH_{d-1}^2 = d\chi^2 + \cosh^2\chi\, dH_{d-2}^2$$

This is sourced only by magnetic flavors: $\Delta n > 0 \iff c > 1$.
($c = 1$ is Minkowski space in Milne coords.)

Goal: using this concrete solution, set up the holographic dictionary.

When does a given a gravity solution admit a holographic dual? How do we construct it?

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Basic requirement: $\exists$ warped region that redshifts energies.[Maldacena]

E.g.: redshift near core of D3 branes

$$-g_{00} = e^{2A} = \left(1 + \frac{R^4}{r^4}\right)^{-1/2} \quad \Rightarrow \quad E = \sqrt{-g_{00}}\, E_p \ll M_P$$

Warped solution

Exhibit warped region in our cosmological solution.

Change variables: $t = T\left(1 - \frac{w^2}{T^{2/c}}\right)^{c/2}$, $\chi = \frac{1}{2}\log\frac{1 + w/T^{1/c}}{1 - w/T^{1/c}}$

$$\begin{aligned} ds_d^2 &= -dt^2 + c^2 t^2 \left(d\chi^2 + \cosh^2\chi \, dH_{d-2}^2\right) \\ &= c^2\left(T^{2/c} - w^2\right)^{c-1} dw^2 + \left(1 - \frac{w^2}{T^{2/c}}\right)^{c-1} \left(-dT^2 + c^2 T^2 \, dH_{d-2}^2\right) \end{aligned}$$

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- radial direction w, with warping for $c > 1$
- $(d-1)$ dual lives on $ds_{d-1}^2 = -dT^2 + c^2 T^2\, dH_{d-2}^2$
- UV slice $w = 0$, corresponding to $\chi = 0$ [here $t = T$]
- two IR regions $w \to \pm T^{1/c}$, corresponding to $|\chi| \gg 1$

GR redshift: $E(w,T) = \left(1 - \frac{w^2}{T^{2/c}}\right)^{\frac{c-1}{2}} E_{pr} \ll M_P$ as $w \to \pm T^{1/c}$

Basic properties

• Warp factor and redshifting are both time and radially dependent. Dual to time-dep couplings and nontrivial RG flow.

• Masses of KK modes of the base and fiber, strings, and 7-7 junctions, in string and Einstein frame: $t = T(1 - w^2/T^{2/c})^{c/2}$

$$m_{KK} \sim \frac{1}{t_s} \to \frac{1}{t} \quad , \quad m_f \sim m_{str} \sim const \to \frac{1}{t^{1-1/c^2}} \quad , \quad m_{77} \sim t_s \to \frac{1}{t^{1-2/c^2}}$$

• IR region arises in the absence of flux, suggesting that magnetic flavor branes support degrees of freedom in the IR.

• Planck mass in $(d-1)$-dimensional theory:

$$(M_{d-1})^{d-3} \sim (M_d)^{d-2} \int_0^{T^{1/c}} dw \sqrt{g}\, g^{00} \sim (M_d)^{d-2}\, T$$

At finite times, dual has propagating gravity, but as $T \to \infty$ gravity decouples. Precise QFT dual?

How much does the IR region (EFT) contribute to M_{Pl}?

- First do this for RS, with UV brane at $r = r_{UV}$ and

$$ds^2 = \frac{r^2}{R_{AdS}^2}(-dt^2 + d\vec{x}^2) + \frac{R_{AdS}^2}{r^2} dr^2$$

$$\Rightarrow \quad M_4^2 \sim M_5^3 \int_0^{r_{UV}} \sqrt{g} g^{00} \sim M_5^3 \frac{r_{UV}^2}{R_{AdS}} \sim \tilde{N}_{dof} \Lambda_c^2$$

where $\tilde{N}_{dof} = N^2$ and the QFT cutoff $\Lambda_c = r_{UV}/R_{AdS}^2$.

$\therefore$ Planck mass induced by QFT.

EFT contribution when $r_{UV} \to \infty$: define IR region $r_{IR} = \varepsilon r_{UV}$

$$\frac{M_{Pl,UV}}{M_{Pl,IR}} = \frac{\int_{r_{IR}}^{r_{UV}} \sqrt{g} g^{00}}{\int_0^{r_{IR}} \sqrt{g} g^{00}} \sim \frac{1 - \varepsilon^2}{\varepsilon^2} = \text{const}$$

so the IR region contributes a leading piece to M_4.

• Analyze this question in our FRW sol: define an IR region

$$E_{IR} = \varepsilon\, E_{UV} \quad \rightsquigarrow \quad w_{IR} = T^{1/c}\sqrt{1-\varepsilon^{2/(c-1)}}$$

$$(M_{d-1})^{d-3} \quad \sim \quad \int_0^{w_{IR}} \left(1-\frac{w^2}{T^{2/c}}\right)^{3(c-1)/2} + \int_{w_{IR}}^{T^{1/c}} \left(1-\frac{w^2}{T^{2/c}}\right)^{3(c-1)/2}$$

$$\Rightarrow \qquad \frac{M_{Pl,UV}}{M_{Pl,IR}} = const \;\; \text{as} \;\; T \to \infty$$

So, as in RS, M_{Pl} dominated by EFT region.

$\rightsquigarrow$ Possibility of a precise QFT description of FRW physics at late times.

3. Holographic degrees of freedom

In a QFT with a cutoff Λ_c, denote $\tilde{N}_{dof} \equiv$ number of field theory degrees of freedom per lattice point.

• Compute $\tilde{N}_{dof}$ for the holographic dual of the FRW sol., both from the gravity and QFT sides.

$\surd$ Basic estimates

$(d-1)$ Planck mass induced dominantly by the EFT:

$$(M_{d-1})^{d-3} \sim (M_d)^{d-2}\, T \sim \tilde{N}_{dof} \Lambda_c^{d-3} .$$

The holographic dual lives on an FRW metric with $H_{d-1} = 1/T$.

$$\text{FRW eq:} \;\; H_{d-1}^2 \sim G_N\, \rho \;\; \Rightarrow \;\; \frac{1}{T^2} \sim \frac{1}{(M_d)^{d-2}\, T} \tilde{N}_{dof} \Lambda_c^{d-1}$$

Combining these eqs $\Rightarrow$ $\tilde{N}_{dof} \sim (M_d)^{d-2}\, T^{d-2} \;\; , \;\; \Lambda_c \sim \dfrac{1}{T}$

So the system accumulates degrees of freedom per lattice point, but has a finite cutoff. Another way of measuring the number of degrees of freedom:

$\surd$ Covariant entropy bound [Bousso; Banks, Fischler; ...]

Compute entropy passing through an observer's past light sheet.

Change to spherical coords:

$$ds^2 = -dt^2 + c^2 t^2 \left(\frac{dr^2}{1+r^2} + r^2 d\Omega_{d-2}^2 \right)$$

with observer at the origin $r = 0$ at time t.

Past light cone at $t_1 < t$: delimits a sphere of size

$$\rho(t_1) = r(t_1)ct \;\; \text{with} \;\; \int_0^r \frac{dr}{\sqrt{1+r^2}} = -\int_t^{t_1} \frac{dt}{ct}$$

For $c > 1$, the sphere grows to max size $\rho_{max} \sim t$ and then begins to shrink as we go back in time.

Entropy bound: area of maximal sphere in Planck units,

$$\mathcal{S} \sim (M_d)^{d-2} t^{d-2}$$

Identifying $\mathcal{S} \sim \tilde{N}_{dof} \Lambda_c^{d-2}\, Vol_{d-2}$ where $Vol_{d-2} \sim t^{d-2}$, gives $\tilde{N}_{dof} \sim t^{d-2}$, consistent with the previous estimate (at late times $T \sim t$).

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Lessons from these calculations:

• They suggest a complete non-gravitational dual at late times, given by a cutoff field theory.

• Growth of $\tilde{N}_{dof}$ associated to a QFT with time-dependent masses and couplings.

Microscopic origin of $\tilde{N}_{dof} \to \infty$?

Estimate using the brane construction ...

Microscopic count of degrees of freedom

On the brane side we have $\Delta n = n - n_{flat} > 0$ (p,q) 7-branes.

We argue that the time dependent growth of $\tilde{N}_{dof}$ is given in terms of 7-7 string junctions.

Some preliminary remarks:

- Such a count may not work in a simple way, when interpolating between weak and strong coupling.
- 7-branes source warping in themselves and dominate at late times. We expect that fields of the dual theory that live at their intersection may account for $\tilde{N}_{dof}$.
- Bringing together $\Delta n > 0$ 7-branes in a *static way* leads to infinite dimensional algebras. [DeWolfe, Hauer, Iqbal, Zwiebach; ...]

Let's count string junctions up to a cutoff from backreaction and topology.

Parametrize a state by:

- the number n_{str} of strings stretching among the 7-branes
- the winding number n_f on the fiber circle
- the momentum number k_f on the fiber circle

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• 1) Bound n_{str} requiring that the core size of the strings does not exceed R, to avoid strong backreaction.

Size of the core determined by gravitational potential $1/r^{d_\perp - 2}$.

Since fiber $\ll$ R at late times, strings are effectively codim 7.

$$\frac{n_{str}}{r_{core}^5} \sim 1 \quad . \quad \text{Then} \quad r_{core} < R \quad \Rightarrow \quad \mathbf{n_{str}} < \mathbf{t^{15/7}}$$

- 2) Bound n_f because the fiber circle is contractible

If strings wind $R/R_f \sim t_s \sim t^{3/7}$ times around the fiber circle, they can detect that it is contractible. So cut off

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• 3) Tower of momentum modes k_f/R_f does not continue forever, but has a UV cut-off (e.g. giant graviton effect of [McGreevy, Susskind, Toumbas])

View states as bound states of KK gravitons and string junctions:

a) when $k_f/R_f \ll R$, energy of state $\sim R$, and the gravitons are well bound to the strings
b) when $k_f > RR_f$, gravitons no longer strongly bound to strings. Don't count as fundamental.

$$\Rightarrow \ \mathbf{k_f < t^{3/7}}$$

• 4) There could be additional group theory factors from the algebra generated by the junctions.

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Infinite dimensional algebras realized by static 7-branes with $\Delta n > 0$ studied by [DeWolfe, Hauer, Iqbal, Zwiebach; ...]

Approach: start from a set of branes that generates a finite dimensional algebra $\mathcal{G}_0$ and then add an extra set Z of 7-branes.

Junctions satisfy $\lambda \cdot \lambda = -\mathbf{J}^2 + n_Z \left(f(p,q) - 1 \right)$

- λ: weight vector under $\mathcal{G}_0$.
- $\mathbf{J}^2 \geq -2$: self-intersection.
- $f(p,q)$: function of asymptotic charges. Generic $f(p,q) > 1$

RHS becomes large and positive by increasing n_Z; longer and longer vectors λ allowed.

$\rightsquigarrow$ Infinite dimensional algebra!

Our assumption: there are no multiplicities which grow with n_{str} in the tower of available states.

Ultimately, the main point to understand is: What matter reps are physically realized?

This is an important and difficult question, and more work is needed.

Combining these results, number of available states:

$$\tilde{N}_{dof} \sim (n_{str} n_f k_f)_{max} \sim t^3 = t^{d-2} \quad \text{for} \quad d = 5$$

- $\surd$ Microscopic count of $\tilde{N}_{dof}$ agrees with the gravity side.
- $\surd$ Dominant contribution given by string junctions from magnetic flavor branes.

4. Dynamics of particles and branes

Finally, we study the dynamics of particles and branes in the infrared region $w \to \pm T^{1/c}$ of

$$ds^2 = \left(1 - \frac{w^2}{T^{2/c}}\right)^{c-1} \left(-dT^2 + c^2 T^2 \, dH_{d-2}^2\right) + c^2 \left(T^{2/c} - w^2\right)^{c-1} dw^2$$

Goals:

1) check whether the IR degrees of freedom in the warped throat are stable;
2) understand the role of the color sector (D3 branes);
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More generally, it is important to understand what additional criteria need to be satisfied in order to obtain a holographic dual.

Physical criterion: behavior of the warp factor be such that light particles remain in the IR region.

Dynamics of particles

Calculations are easier in the string frame metric

$$ds_s^2 = -dt_s^2 + \frac{t_s^2}{c^2}(d\chi^2 + \cosh^2\chi\, dH_3^2) + \frac{t_s^2}{c^2}dB_4^2 + dx_f^2$$

Consider a massive particle moving along χ:

$$S_{massive} = -\int dt_s\, m(t_s)\sqrt{1 - t_s^2\dot{\chi}^2/c^2}$$

with $m_{KK} \sim \frac{1}{t_s}$, $m_f \sim m_{str} \sim 1$, $m_{77} \sim t_s$

Using the conserved momentum,

$$p = \frac{m(t_s)\dot{\chi}t_s^2/c^2}{\sqrt{1-\dot{\chi}^2 t_s^2/c^2}} \Rightarrow \dot{\chi} = \frac{cp}{t_s\sqrt{p^2 + m(t_s)^2 t_s^2/c^2}}$$

$\therefore$ Massive particles remain in the IR region ($\dot{\chi} \to 0$ at late times.)

Color D3 branes

For a D3 brane extended along $\mathbb{H}_3$, the DBI action is

$$S_{D3} = -T_3 \int dt_s \, \frac{t_s^3}{c^3} \cosh^3 \chi \sqrt{1 - \dot{\chi}^2 t_s^2 / c^2}$$

So there is an extra force $\cosh^3 \chi$ that pushes the brane up to $\chi \to 0$.

$\rightsquigarrow$ D3 branes are not stable in the IR and move up the throat.

"Motion sickness".

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"Motion sickness".

• Unitarity problems? Familiar examples with color branes responsible for warping, where unitarity is not lost when color branes are pushed towards the UV.

• Here magnetic branes support warping, and the color sector is subdominant. So holographic dual built from d.o.f. living on 7-branes.

• Since $|\dot{\chi}| < c/t_s$, the ejection of branes takes longer and longer.

So motion sickness does not appear to be fatal in our system.

Massive propagators

♦ In AdS/CFT, massive propagators in the bulk turn into power-law correlators in the dual.

How does this happen? A massive propagator would usually be exp suppressed (e.g. in flat space) ...

This effect is due to the strong radial dependence of the warp factor: geodesics become shorter along the radial direction, leading to power law behavior rather than exp.

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♦ Compute 2-pt function for field with mass $m(t)$ in our geometry

$$ds_d^2 = -dt^2 + c^2 t^2 \left(d\chi^2 + \cosh^2 \chi \left[d\tilde{\chi}^2 + \sinh^2 \tilde{\chi}, d\Omega_{d-3}^2 \right] \right)$$

We want propagator between $(t, \chi, \tilde{\chi})$ and $(t, \chi, \tilde{\chi} + \Delta\tilde{\chi})$.

$$G(t, \chi; \Delta\tilde{\chi}) \sim \exp\left[iS_{WKB} \right]$$

$$S = -\int d\lambda\, m(t) \sqrt{\dot{t}^2 - c^2 t^2 (\dot{\chi}^2 + \cosh^2 \chi\, \dot{\tilde{\chi}}^2 + \ldots)}$$

For a KK mode $m(t) = n_{KK}/t$, the propagator in the UV $\chi = 0$ is

$$G(\Delta\tilde{\chi}) \sim \exp[-n_{KK}\, c\, \Delta\tilde{\chi}]$$

Geodesic distance along $(d-1)$ space: $\Delta x \sim e^{c\Delta\tilde{\chi}/2}$

$$G(\Delta x) \sim \frac{1}{(\Delta x)^{2n_{KK}}} \quad \Rightarrow \quad \textbf{power law correlator for KK modes!}$$

In the IR $|\chi| \gg 1$ there is an additional suppression factor

$$G(\chi; \Delta x) \sim e^{-2cn_{KK}\chi} \frac{1}{(\Delta x)^{2n_{KK}}} \sim \frac{(1 - w/T^{1/c})^{cn_{KK}}}{(\Delta x)^{2n_{KK}}}$$

This is characteristic of a strongly coupled theory with power-law wavefunction renormalization.

Modes with $m \sim 1/t^{\alpha}$, $\alpha < 1$, have exp suppressed correlators.

5. Conclusions and future directions

- Starting from AdS/CFT dual pairs, we constructed simple FRW solutions sourced by magnetic flavor branes.
- Time-dependent warped metric w/ redshifted region $\Rightarrow$ holographic description in terms of a cutoff field theory. At finite times: propagating gravity and finite $\tilde{N}_{dof}$.
- At late times gravity decouples, and $\tilde{N}_{dof} \to \infty$. Holographic degrees of freedom dominated by string junctions. Precise QFT dual of FRW cosmology at late times.

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- Develop further the holographic description, with time-dependent and running couplings.
- Distinction between $\Delta n < 0$ and $\Delta \geq 0$? Relations between unitarity and time-dependence?
- More general relevance of magnetic flavors and infinite algebras to cosmological solutions with holographic duals.
- Conditions for the existence of a holographic duality.