New string vacua from simple topologies

Alessandro Tomasiello

[Rutgers, March 11th, 2008]

Introduction

Supersymmetry must be spontaneously broken.

- Superpartners?
- $\Lambda > 0$ incompatible with unbroken supersymmetry.

[for susy vacua: $V \sim (|\not{D}W|^2 - |W|^2)$] (superpotential)

Strategy:

Step 1. Start from vacuum with $\Lambda < 0$

Step 2. $\begin{cases} \text{break susy} \\ \text{lift to } \Lambda > 0 \end{cases}$

Examples:

[Kachru,Kallosh,Linde,Trivedi'03]

Step 1. $AdS_4 \times CY_6$ in IIB

using

- quantum corrections (brane instantons)
- O3, D3, D7

Step 2. using D3-$\overline{\text{D3}}$ pairs

{deWolfe,Giryavets,Kachru,Taylor'05}

Step 1. $\mathrm{AdS}_4 \times \mathrm{CY}_6$ in IIA

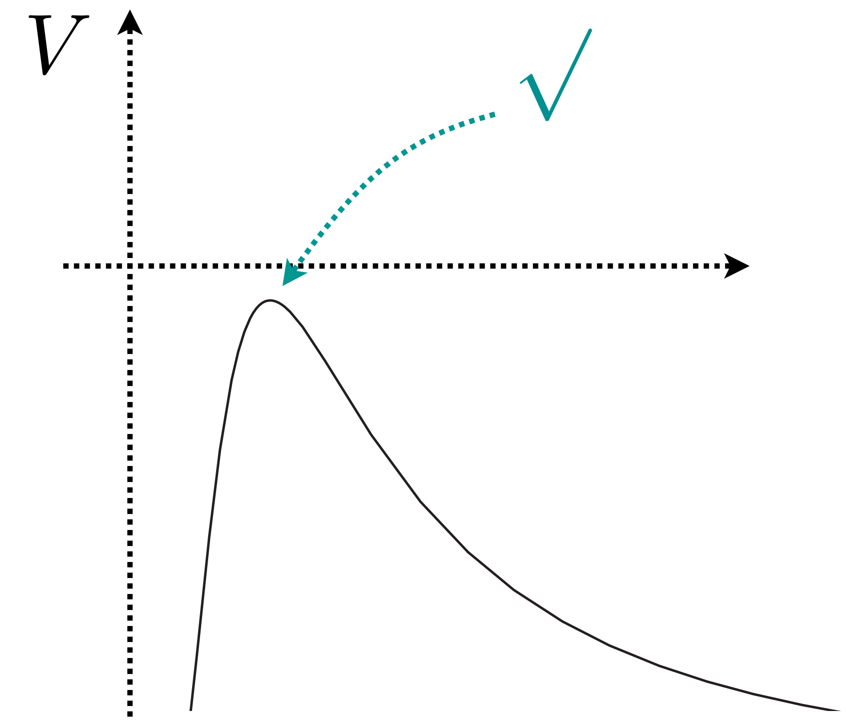
using
- classical ingredients
- O6

Step 2. Not easy (no-go, in some sense)

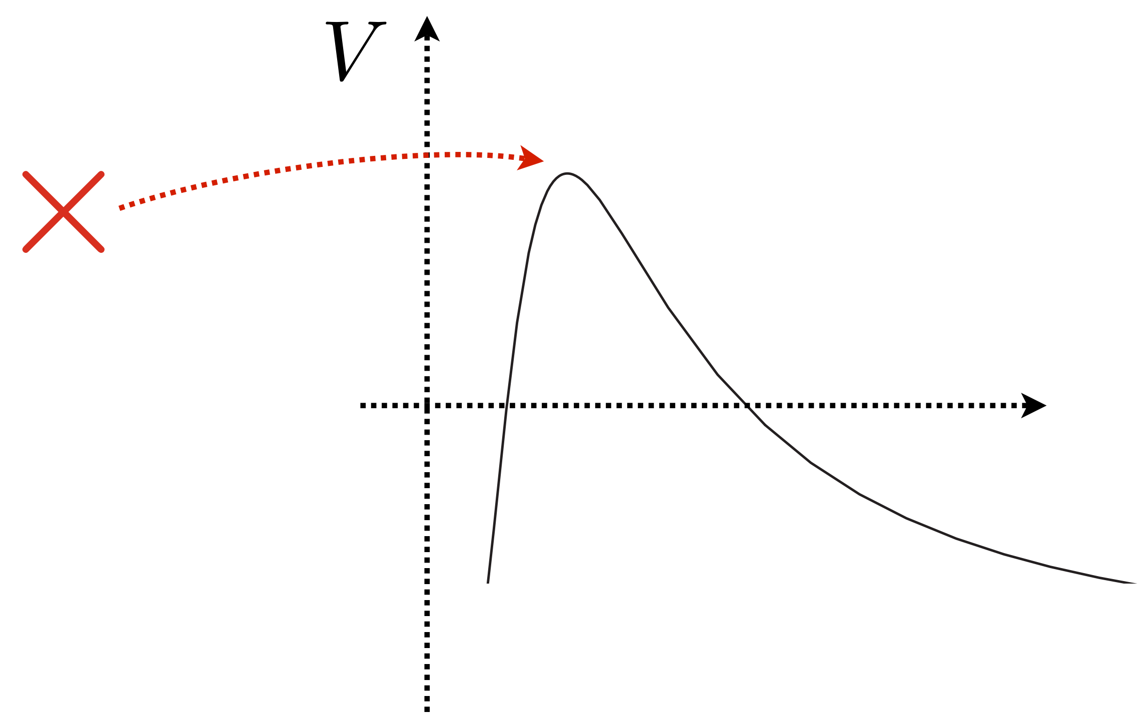
{Hertzberg, Kachru, Taylor, Tegmark'07}

General caveat about step 2:

from the point of view of the 4d effective action



At Step 1. a tachyon might be acceptable
(if above Breitenlohner-Freedman bound)



But it will not be so
after lifting (at Step 2.)

In this talk:

[only Step I. will be considered]

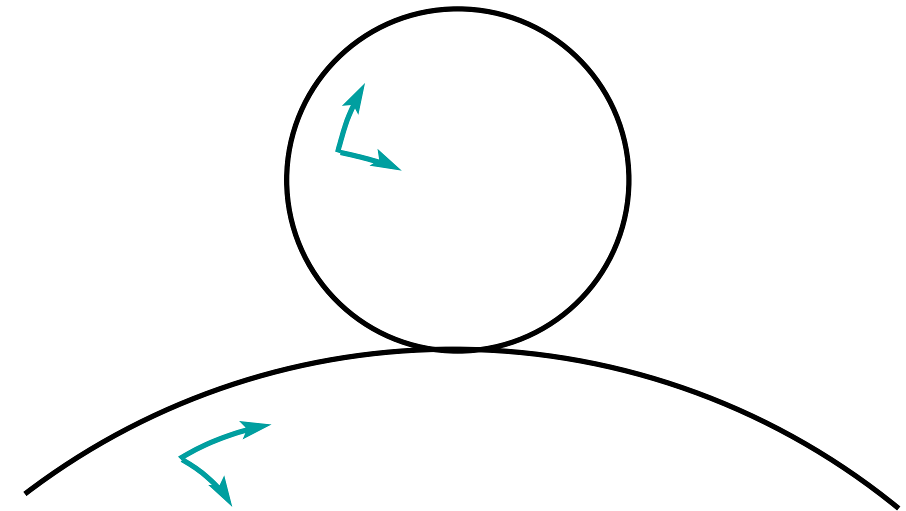
- $\mathrm{AdS}_4 \times \mathbb{CP}^3$ vacua in IIA

- no orientifolds, no brane instantons

- infinitely many

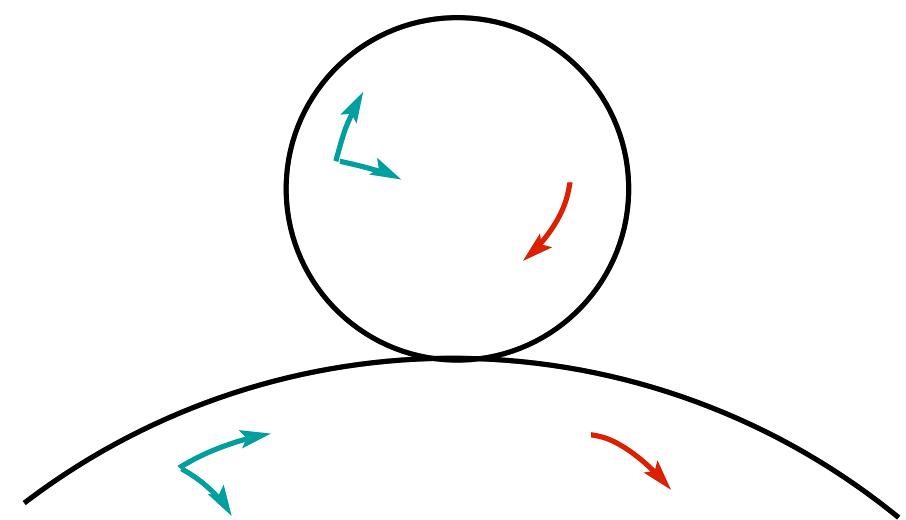
- all moduli stabilized: few to begin with

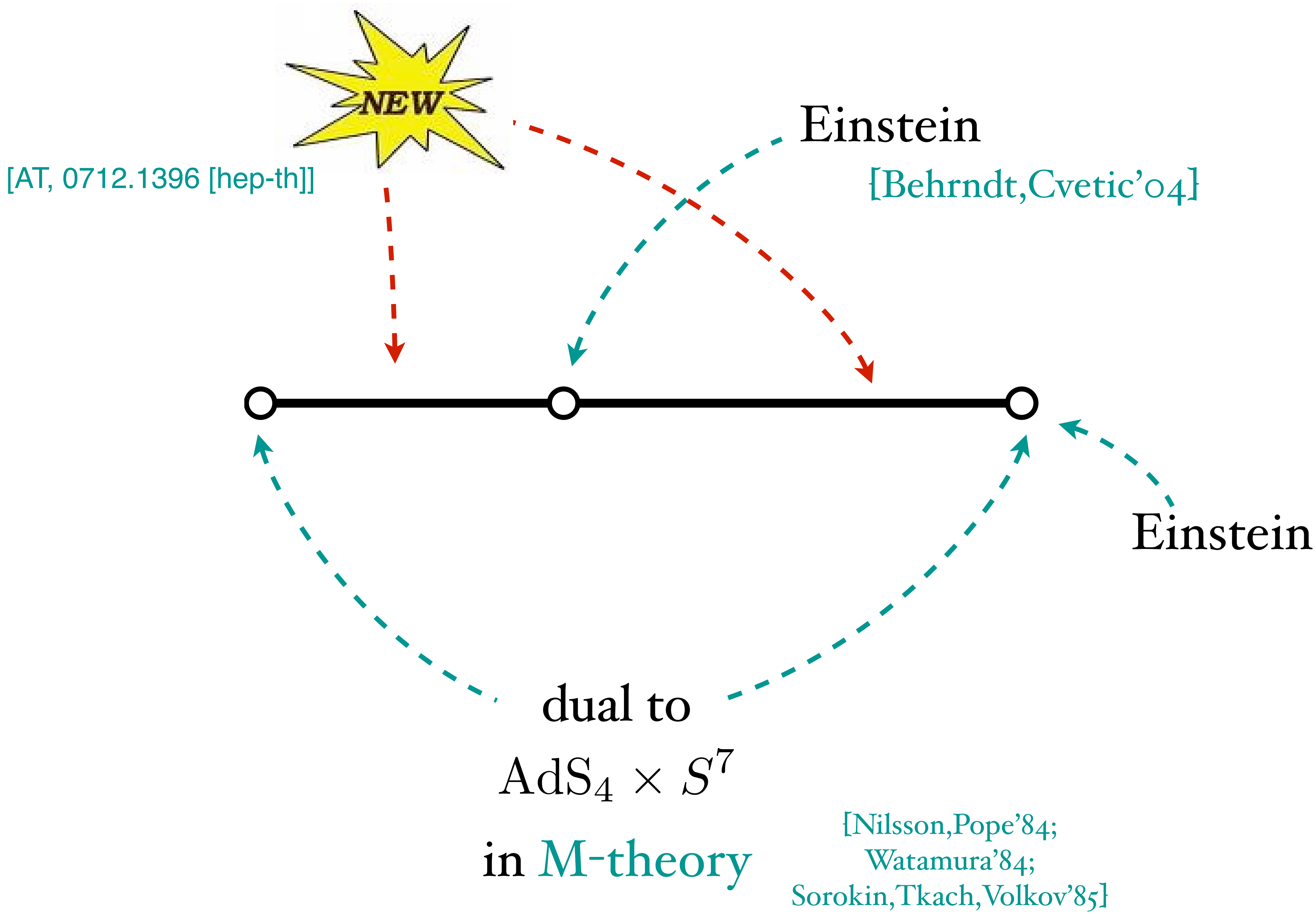
unlike usual
Freund-Rubin construction



(fluxes proportional to volume forms)

all fluxes will be on





NEW
[AT, 0712.1396 [hep-th]]
Einstein
[Behrndt,Cvetic'04]
Einstein
dual to
$\mathrm{AdS}_4 \times S^7$
in M-theory
[Nilsson,Pope'84;
Watamura'84;
Sorokin,Tkach,Volkov'85]

Plan

- General considerations about supersymmetry
- Some geometry of $\mathbb{CP}^3$
- Finding the new vacua

Supersymmetry

$\text{Minkowski}_4 \times M_6$ would be more interesting geometrically:

has to be 'generalized Calabi-Yau'

[Graña, Minasian, Petrini, AT'05]

'calibrations'

$$\text{susy} \quad \Longleftrightarrow \quad \begin{aligned} d\Phi_1 &= 0 \\ d\Phi_2 &= i * F \end{aligned}$$

RR fluxes

(includes all the particular cases you know already: [Klebanov,Strassler'00;Maldacena,Nuñez'00;...])

$\mathrm{AdS}_4 \times M_6$ slightly less so

has to be 'generalized half-flat'

[Graña, Minasian, Petrini, AT'06]

However, AdS vacua are easier to find.

Important subclass: "SU(3) vacua"

fluxes* are determined by two real numbers...

... and by a two-form

$$g_s F_0 = 5m \qquad g_s F_2 = \frac{1}{3}\tilde{m} J - W_2$$

$$g_s F_4 = \frac{3}{2} m J^2 \qquad g_s F_6 = -\frac{1}{2}\tilde{m} J^3$$

$$H = 2m \mathrm{Re}\Omega$$

metric determined by 2-form J and 3-form Ω

such that

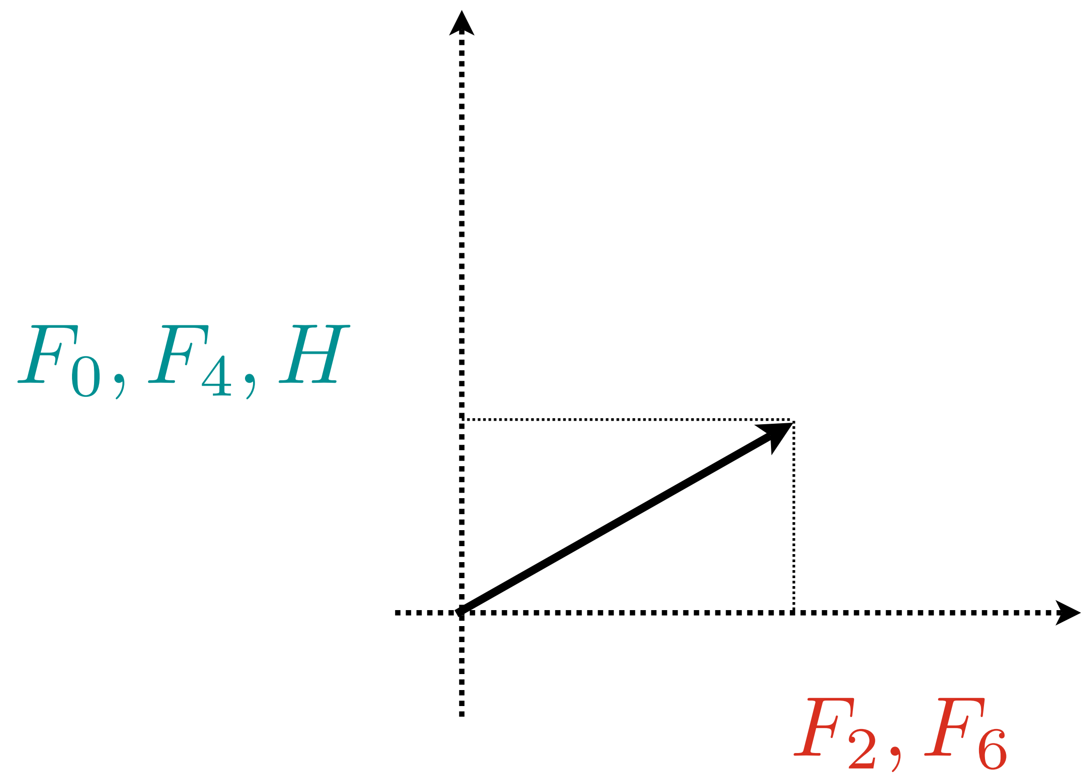
$$dJ = 2\tilde{m}\mathrm{Re}\Omega$$

$$d\mathrm{Im}\Omega = \frac{8}{3}\tilde{m} J^2 + W_2 \wedge J$$

Bianchi becomes

$$dW_2^- = \frac{2}{3}\left(\tilde{m}^2 - 15 m^2\right)\mathrm{Re}\Omega$$

*these are internal; external determined by duality



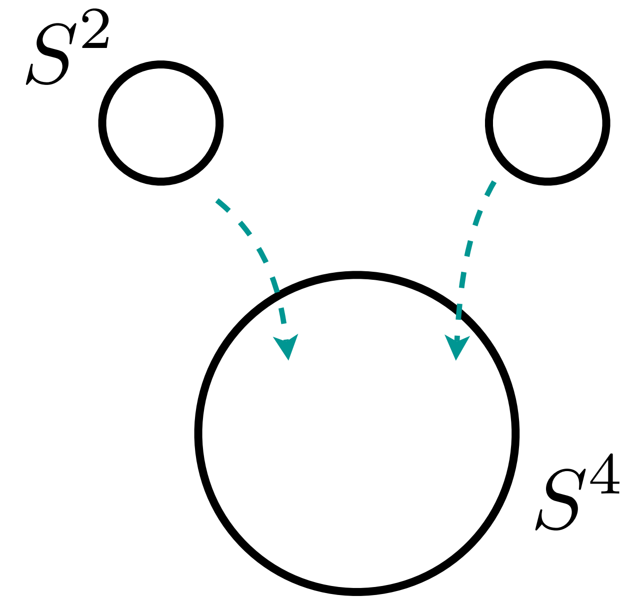
$$\Lambda = -3(m^2 + \tilde{m}^2)$$

Some geometry of $\mathbb{CP}^3$

- topology

$$S^2 \hookrightarrow \mathbb{CP}^3$$
$$\downarrow$$
$$S^4$$

'**twistor fibration**'



$$\text{fibre} = \frac{\text{spinors}}{\text{complex rescalings}} \equiv \text{twistors}$$

$$||$$

$$\mathbb{CP}^1 = S^2$$

cohomology:

h^0	h^1	h^2	h^3	h^4	h^5	h^6
1	0	1	0	1	0	1

- complex structure and metric

Usually on $\mathbb{CP}^3$: Fubini-Study (Kähler, Einstein) $I_{\rm FS}\,,\ g_{\rm FS}$

It can't be useful for us:

- susy ⇨ $dJ = 2\tilde{m}\,{\rm Re}\Omega$ ⇨ I not integrable

  dJ: $(1,1)$; ${\rm Re}\Omega$: $(3,0)+(0,3)$

- $I_{\rm FS}$ has no globally defined (3,0)-form: $\nexists\ \Omega_{\rm FS}$

We need a different (almost) complex structure

← not integrable

not

$$I_{\mathrm{FS}} = \begin{pmatrix} I_2 & \\ & I_4 \end{pmatrix}$$

but

$$I_{\mathrm{susy}} = \begin{pmatrix} -I_2 & \\ & I_4 \end{pmatrix}$$

- integrable — not integrable
- $c_1 = 4$ — $c_1 = 0$

$\Downarrow$ — $\Downarrow$

$\nexists\ \Omega_{\mathrm{FS}}$ — $\exists\ \Omega_{\mathrm{susy}}$

- We will also change the metric

rather than $g_{\rm FS} = g_2 + 2 g_4$

'squashing parameter'

$$g_{\rm susy} = R^2 (g_2 + \sigma g_4)$$

overall scale

notice that we have few parameters to begin with:

'stabilizing moduli' will be easy

Finding the new vacua

$g_{\rm susy}$

$I_{\rm susy}$

$$J_{\rm susy} \equiv g_{\rm susy} I_{\rm susy}$$

can solve (susy)

$$\begin{cases} dJ = 2\tilde{m}\,\mathrm{Re}\Omega \\ d\mathrm{Im}\Omega = \frac{8}{3}\tilde{m}J^2 + W_2^- \\ dW_2^- = \frac{2}{3}\left(\tilde{m}^2 - 15m^2\right)\mathrm{Re}\Omega \end{cases}$$

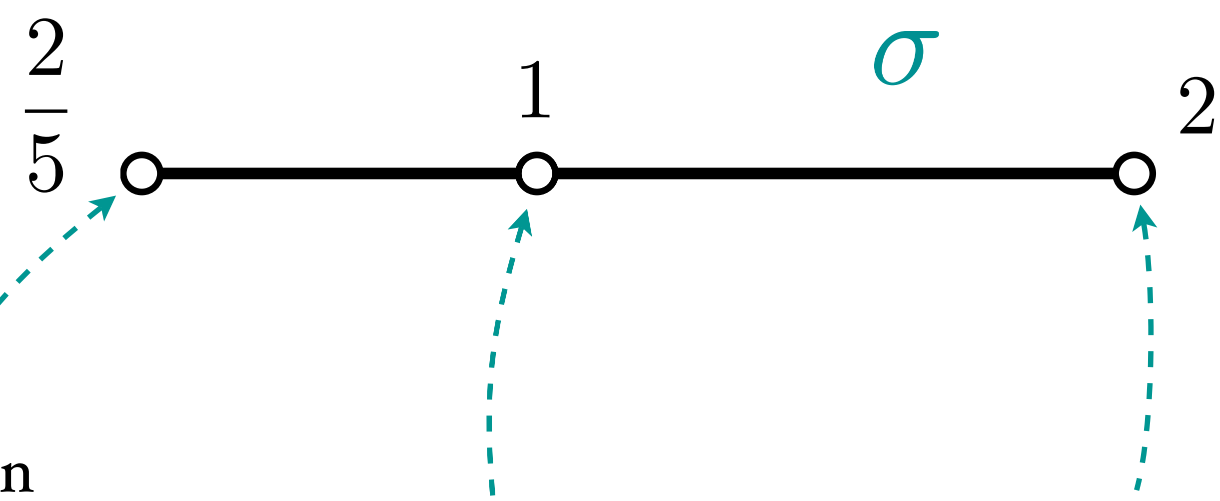
with

$$m = \frac{1}{2R}\sqrt{\left(\sigma - \frac{2}{5}\right)(2-\sigma)}$$

$$\tilde{m} = -\frac{1}{2R}(\sigma + 2)$$

$$(\sigma, R) \longrightarrow (m, \tilde{m}) \longrightarrow \text{fluxes}$$

$$m = \frac{1}{2R}\sqrt{\left(\sigma - \frac{2}{5}\right)(2-\sigma)} \quad \Rightarrow \quad \boxed{\frac{2}{5} \le \sigma \le 2}$$



not Einstein

can be lifted to another Einstein metric on S^7

$$W_2 = 0$$

also Einstein;
can be generalized to 'nearly Kähler'

[Behrndt,Cvetic'04]

$$g = g_{\rm FS}$$ Einstein;
can be lifted to S^7

[Nilsson,Pope'84; Watamura'84; Sorokin,Tkach,Volkov'85]

$$I \neq I_{\rm FS}$$

(would have been Kähler)

Flux quantization

- H is exact

$$H = 2m\mathrm{Re}\Omega$$

$$dJ = 2\tilde{m}\mathrm{Re}\Omega$$

$$\Rightarrow \quad H = d\left(\frac{m}{\tilde{m}}J\right)$$

- F_k are not closed

$$dF_k = H \wedge F_{k-2} \quad \Rightarrow \quad d(e^{-B\wedge}F)_k = 0$$

$$|||$$

$$d\tilde{F}_k = 0$$

using

$$m = \frac{1}{2R}\sqrt{\left(\sigma - \frac{2}{5}\right)(2-\sigma)}$$

$$\tilde{m} = -\frac{1}{2R}(\sigma + 2)$$

$$r \equiv \frac{R}{2\pi l_s}$$

$$\int \tilde{F}_k = f_k(\sigma) r^{k-1}$$

$$||$$

$$n_k$$

flux quantization

four equations for g_s, R, σ

- not all n_k are allowed
- all three are fixed
- for each vacuum, $\exists$ infinitely many others
- it can be arranged: $r \gg 1\,,\ g_s \ll 1$

Some lessons

- complexity of vacua not unique to Calabi-Yau

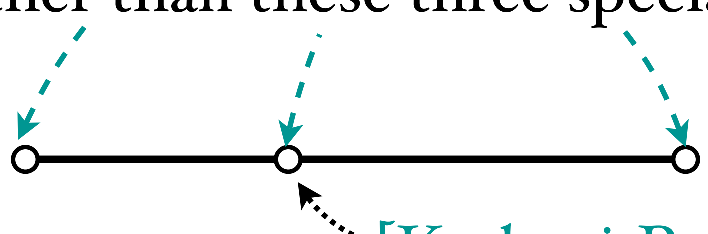
- "vacua first, then effective field theories"

KK reduction on general M_6 is hard!

it can be done on M_6
- Calabi-Yau
- "twisted tori" [Scherk-Schwarz]

For the vacua we found, 4d theory not known

other than these three special cases



[Kashani-Poor'07]

$\mathcal{N} = 2$; one vector multiplet, one hypermultiplet

Conclusions

- “Landscape” of vacua not exclusive to Calabi-Yau’s
- Even just classical physics goes a long way
- KK more difficult than finding vacua