Dynamics of 3D gauge theories with antisymmetric matter

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  - Important
  - Interesting
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  - Important
  - Interesting
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- 4d SUSY theories offer a perfect laboratory for studying non-perturbative dynamics
  - Gaugino condensation, instantons, confinement with and without chiral symmetry breaking, duality...
  - Still hard!
- 3d SUSY theories: a simpler lab for strong QFT dynamics
  - Many 4d phenomena in 3d setting
  - Instanton-monopoles, Chern-Simons terms. real masses...
  - Lab for study of condensed matter systems?
  - Many results over the years, significant progress in the last year

Affleck, Harvey, Witten (82); Seiberg, Witten (96); Intriligator, Seiberg (96)

de Boer, Hori, Oz (97); Aharony, Hanany, Intriligator, Seiberg, Strassler (97)

Aharony, Razamat, Seiberg, Willet (13); Aharony, Seiberg, Tachikawa (13)

Motivation

SUSY in 3d

- Instanton-monopoles
- Fermion zero modes
- Global coordinates on the Coulomb branch

s-confinment in 3d

- $SU(4)$ with two antisymmetrics
- Matching 4d to 3d
- Consistency checks
- $SU(4)$ with a single antisymmetric

Summary and outlook

SUSY in 3 dimensions

- $\mathcal{N}=2$ 3d theory from $\mathcal{N}=1$ 4d theory:
  - Real scalar from dimensional reduction $A_\mu \to A_i^{(3)}, \sigma$
  - 3d photon dual is dual to a scalar $\partial^i \gamma = \epsilon^{ijk} F_{jk}$
  - Holomorphic modulus $\Phi = \sigma + i\gamma$
- r_G dimensional Coulomb branch parameterized by

$$\sigma = \mathrm{diag}(\sigma_1, \sigma_2, \ldots, \sigma_N)\,, \quad \mathrm{Tr}\,(\sigma) = 0$$

- Work in Weyl chamber to remove gauge redundancy:

$$\sigma_1 > \sigma_2 > \ldots > \sigma_N$$

- Convenient parameterization

$$Y_i \sim \exp(\vec{\Phi} \cdot \vec{\alpha}_i / g_3^2)\,, \quad i = 1, \ldots, r_G$$

- Classical moduli space is a cylinder

3d duality from 4d duality

- Straightforward reduction to 3d is problematic:
  - 4d scales in terms of 3d coupling:

$$g_4^2 = 2\pi r g_3^2, \quad \Lambda_e^b \sim \exp\left(-4\pi/rg_3^2\right)$$

  - $r \to 0$ limit with fixed g_3^2 implies $\Lambda_e \to 0, \quad \Lambda_m \to \infty$ and does not commute with $E \ll \Lambda_e, \Lambda_m$
  - Low energy limit keeping r, Λ_e, Λ_m fixed still works
- Duality on $\mathbb{R}^4$ versus $\mathbb{R}^3 \times S^1$:

	$SU(N)$, F flavors	$SU(F-N)$, F flavors
$4d$	$W = 0$	$W = Mq\bar{q}$
$3d$	$W = \eta Y$	$W = Mq\bar{q} + \tilde{\eta}\tilde{Y}$

- Must decouple ηY to obtain true 3d duality:
  $F + 1$ flavors in 4d $\longrightarrow$ F flavors in 3d

Instanton-monopoles

- In 3d instantons can only exist on Coulomb branches of non-abelian theories.
- Instanton in $SU(2) \to U(1)$:
  - Symmetry breaking pattern $\mathrm{diag}(\sigma, -\sigma)$
  - Start with 4d theory on $\mathbb{R}^3 \times S^1$ and wrap monopole around compact direction: 3 dimensional instanton-monopole
  - Gauginos have 2 zero mode in the instanton-monopole background and aquire a mass

$$W = \frac{1}{Y}, \quad Y \sim \exp(2\sigma)$$

- Dynamics of $SU(N)$
  - $N-1$ dimensional Coulomb branch: $Y_i \sim \exp(\sigma_i - \sigma_{i+1})$
  - $N-1$ linearly independent $SU(2)$ factors lead to $N-1$ fundamental instanton-monopoles
  - Each fundamental monopole has 2 gaugino zero modes:

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KK monopoles

- Coulomb branch of $\mathbb{R}^3 \times S^1$ theory is periodic:

$$\sigma_i \to \sigma_i + \frac{1}{r}$$

- An unbroken $SU(2)$ when $\sigma_1 = \sigma_N + 1/r$
- KK monopole winding around compact dimension

$$W_{KK} = \frac{1}{\exp(\frac{\sigma_N + 1/r - \sigma_1}{g_3^2})} = \eta Y, \quad \eta = e^{-1/rg_3^2} = e^{-1/g_4^2} = \Lambda^b$$

- In pure SYM KK monopole generated superpotential corresponds to gaugino condensate:

$$\eta Y = S = \Lambda^3 = \eta^{1/N_c}$$

- Full $\mathbb{R}^3 \times S^1$ SYM superpotential

$$W = \sum_i^{N-1} \frac{1}{Y_i} + \eta Y$$

Matter fields in 3d

- SUSY in 3d allows real mass terms. E.g. gauge baryon number and give vev to $A_3 = \sigma_b = m_R$:

$$K = Q^\dagger e^V Q + \bar{Q}^\dagger e^{-V}\bar{Q} \supset Q^\dagger e^{\sigma_b \theta\bar{\theta}} Q + \bar{Q}^\dagger e^{-\sigma_b \theta\bar{\theta}}\bar{Q}$$

- Q and $\bar{Q}$ have real masses $m_{\mathbb{R}}$ and $-m_R$ respecively
- Additional contributions on the Coulomb branch

$$\left| \langle \sigma^a T^a \rangle^\alpha{}_\beta Q^\beta{}_f \right|^2 \quad \text{no sum over } \beta$$

- The fermion has zero modes in i^{th} monopole background if effective real mass is

$$\sigma_i > m_R > \sigma_{i+1}$$

Matter fields in 3d: examples

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- Massless fundamental on the Coulomb branch of $SU(4)$

$$\begin{pmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \sigma_3 & \\ & & & \sigma_4 \end{pmatrix}, \quad \sigma_4 = -\sum_i \sigma_i$$

  - Zero mode in the 3^{rd} instanton-monopole
  - No AHW suportotential for Y_3: $W = 1/Y_1 + 1/Y_2$

Matter fields in 3d: examples

- Doublet of $SU(2)$ has one zero mode if $\sigma > m_R > -\sigma$
- Massless fundamental on the Coulomb branch of $SU(4)$

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  - Zero mode in the 2^{nd} and 3^{rd} instanton-monopoles
  - No AHW superpotential for Y_2 and Y_3: $W = 1/Y_1$

Matter fields in 3d: examples

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  - Zero mode in all instanton-monopoles
  - No superpotential
  - One dimensional Coulomb branch: $Y = \prod Y_i$

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  - Fundamental has no zero modes in KK monopole
  - The superpotential on a circle is $W = \eta Y$

Matter fields in 3d: moduli charges

$$\begin{pmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \sigma_3 & \\ & & & \sigma_4 \end{pmatrix}$$

- Region I: $\sigma_1 > 0 > \sigma_2 > \sigma_3 > \sigma_4$

$$W = \frac{1}{Y_2} + \frac{1}{Y_3}\,, \quad R(Y_2) = R(Y_3) = -2,\ R(Y_1) = F - 2$$

- Region II: $\sigma_1 > \sigma_2 > 0 > \sigma_3 > \sigma_4$

$$W = \frac{1}{Y_1'} + \frac{1}{Y_3}\,, \quad R(Y_1') = R(Y_3) = -2,\ R(Y_2') = F - 2$$

- Region III: $\sigma_1 > \sigma_2 > \sigma_3 > 0 > \sigma_4$

$$W = \frac{1}{Y_1'} + \frac{1}{Y_2''}\,, \quad R(Y_1') = R(Y_2'') = -2,\ R(Y_3') = F - 2$$

- $Y = \prod Y_i$ is globally defined

Matter fields in 3d: antisymmetric of $SU(4)$

- On the Coulomb branch A_{ij} has a real mass $\sigma_i + \sigma_j$

$$\begin{pmatrix} \sigma_1 & & \text{x} & \text{x} \\ & \sigma_2 & \text{x} & \text{x} \\ & & \sigma_3 & \\ & & & \sigma_4 \end{pmatrix}$$

- Instanton in the first $SU(2)$:
  Zero modes if $\sigma_1 + \sigma_{3,4} \geq 0$, $\sigma_2 + \sigma_{3,4} \leq 0$
- Instanton in the third $SU(2)$:
  Zero modes if $\sigma_3 + \sigma_{1,2} \geq 0$, $\sigma_4 + \sigma_{1,2} \leq 0$
- Instanton in the second $SU(2)$:
  Doublets have zero modes if $\sigma_2 \to \sigma_1$ and $\sigma_3 \to \sigma_4$
- One dimensional Coulomb branch $\widetilde{Y} = \sqrt{Y_1 Y_2^2 Y_3}$

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$$\begin{pmatrix} \sigma & & & \\ & \sigma & & \\ & & -\sigma & \\ & & & -\sigma \end{pmatrix}$$

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s-confinement in $SU(4)$

- Two antisymmetrics and two fundamental flavors

	$SU(4)$	$SU(2)$	$SU(2)_L$	$SU(2)_R$	$U(1)_1$	$U(1)_2$	$U(1)'$	$U(1)_R$
A	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$	$\Box$	1	1	0	0	-3	0
Q	$\Box$	1	$\Box$	1	1	0	2	$\frac{1}{3}$
$\overline{Q}$	$\overline{\Box}$	1	1	$\overline{\Box}$	0	-1	2	$\frac{1}{3}$

- Expect s-confinement
- Potentially 2-dimensional Coulomb branch, Y and $\widetilde{Y}$
- Symmetries and quantum numbers

	$U(1)_1$	$U(1)_2$	$U(1)'$	$U(1)_R$
$Y = \prod_i Y_i$	-2	2	4	$\frac{2}{3}$
$\tilde{Y} = \sqrt{Y\tilde{Y}_2}$	-2	2	-2	$\frac{2}{3}$

From 4d to 3d: s-confinement in $\mathbb{R}^4$

- The 4d model s-confining model

	$SU(4)$	$SU(2)$	$SU(3)_L$	$SU(3)_R$	$U(1)$	$U(1)'$	$U(1)_R$
A	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\square$	1	1	0	-3	0
Q	$\square$	1	$\square$	1	1	2	$\frac{1}{3}$
$\overline{Q}$	$\overline{\square}$	1	1	$\overline{\square}$	-1	2	$\frac{1}{3}$

- Charges of the composites

	$SU(4)$	$SU(2)$	$SU(3)_L$	$SU(3)_R$	$U(1)$	$U(1)'$	$U(1)_R$
$M_0 = Q\bar{Q}$	1	1	$\square$	$\overline{\square}$	0	4	$\frac{2}{3}$
$M_2 = QA^2\bar{Q}$	1	1	$\square$	$\overline{\square}$	0	-2	$\frac{2}{3}$
$H = AQ^2$	1	$\square$	$\overline{\square}$	1	2	1	$\frac{2}{3}$
$\bar{H} = A\bar{Q}^2$	1	$\square$	1	$\square$	-2	1	$\frac{2}{3}$
$T = A^2$	1	$\square\square$	1	1	0	-6	0

- Exact non-perturbative superpotential

$$W_{\text{dyn}} = \frac{1}{\Lambda^7}\left(T^2 M_0^3 - 12 T H \bar{H} M_0 - 24 M_0 M_2^2 - 24 H \bar{H} M_2\right)$$

From 4d to 3d: s-confinement on $\mathbb{R}^3 \times S^1$

- KK instanton generates the superpotential

$$W = W_{\text{dyn}} + \eta Y$$

- Need to decouple KK instanton contribution:
  - Gauge a diagonal $U(1)$ in $SU(3)_L \times SU(3)_R \times U(1)$
  - Decouple the third (massive) flavor and ηY
  - Unbroken global symmetries the same as in 3d model
  - Composites without third flavor quark remain masslees
  - Composites with only one third flavor quark are heavy
  - M_0^{33} and M_2^{33} are neutral under $U(1)$: massless
  - M_0^{33} and M_2^{33} have the same charges as Y and $\widetilde{Y}$
- Claim: 3d dual desription:

$$W_{\text{dyn}} = Y\Big(3T^2 \det M_0 - 12Th\overline{h} - 24 \det M_2\Big) + \widetilde{Y}\Big(2M_0M_2 + h\overline{h}\Big)$$

Tests

- Back to $\mathbb{R}^3 \times S^1$:
  - 4d quantum modified moduli space:

$$W = \lambda \left(3T^2 \det M_0 - 12Th\bar{h} - 24 \det M_2 - \Lambda^8\right) + \mu \left(2M_0 M_2 + h\bar{h}\right) + \eta Y$$

  - Identify λ and μ with Y and $\widetilde{Y}$ ($\eta \sim \Lambda^8$)
- Coulomb branch: large $\tilde{Y}$:
  - Semiclassical symmetry breaking pattern:

$$SU(4) \to SO(4) \times U(1)$$

  - All fundamentals obtain large real mass
  - Two light $SO(4)$ vectors survive from antisymmetric
  - Low energy 3d physics is known to s-confine with one Coulomb modulus $Y_{SO} \sim Y^2/\widetilde{Y}^2$

s-confinement with a single antisymmetric

- $SU(4)$ with and antisymmetric and three flavors
- How many Coulomb branch moduli?
- Y and $\widetilde{Y}$ are not lifted by instanton-monopoles
- No candidate for $\tilde{Y}$ in $\mathbb{R}^3 \times S^1$ model
  - There is no $M_2 \sim QA^2\bar{Q}$ composite
- Expect new dynamical effects to lift $\tilde{Y}$
- The low energy dynamics

$$W = Y\left(T\det\, M + HM\bar{H}\right)$$

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- Compare models with one and two antisymmetrics:
  - Holomorphic mass for the fundamental flavor in theory II
  - Holomorphic mass for antisymmetric in theory I
  - $\widetilde{Y}$ decouples when integrating out antisymmetric
  - Low energy descriptions agree
- Coulomb branch again: large $\tilde{Y}$
  - $SU(4) \to SO(4) \times U(1)$
  - A single $SO(4)$ vector
  - ADS-like superpotential

Aharony, Shamir

$$W = \frac{1}{Y_{SO}^4 T}$$

  - Carefully matching moduli: $\widetilde{Y}$ lifted (preliminary).

To be continued: generalizations

$SU(N)$ group with A, $\bar{A}$ and two flavors

$$
\begin{array}{llllll}
SU(4): & \tilde{Y} & \to & \sqrt{YY_2} & \to & \mathrm{diag}(\sigma,\sigma,-\sigma,-\sigma) \\
SU(5): & \tilde{Y}_5 & \to & \sqrt{YY_2Y_3} & \to & \mathrm{diag}(\sigma,\sigma,0,-\sigma,-\sigma) \\
SU(6): & \tilde{Y}_6 & \to & \sqrt{YY_2Y_3Y_4} & \to & \mathrm{diag}(\sigma,\sigma,0,0,-\sigma,-\sigma) \\
 & \hat{Y}_6 & \to & (\tilde{Y}_1^2Y_3)^{\frac{1}{3}} & \to & \mathrm{diag}(\sigma,\sigma,\sigma,-\sigma,-\sigma,-\sigma)
\end{array}
$$

- Antisymmetric of $SU(5)$ has zero modes under KK monopole in some regions of moduli space
- Total number of matter zero modes (including KK monopole contributions) adds up to number of 4d instanton zero modes predicted by Atiah-Singer index
- Expectations for the number of unlifted Coulomb branch moduli are based on 4d s-confining theories

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