DO WE KNOW IF OUR UNIVERSE IS STABLE?

Rutgers Seminar
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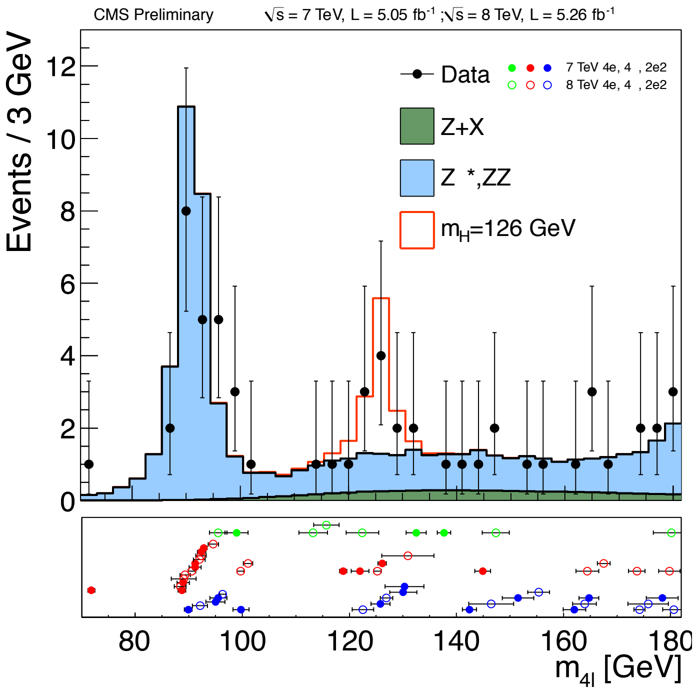
Based on arXiv:1408.0287 (PRD **91**)
and arXiv:1408.0292 (PRL **113**)
with Anders Andreassen and William Frost

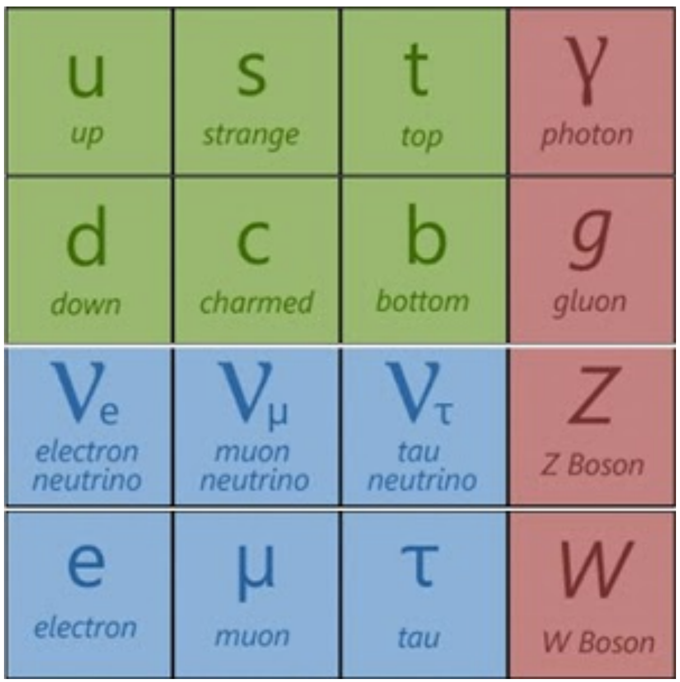
July 4, 2012: Higgs boson discovered!



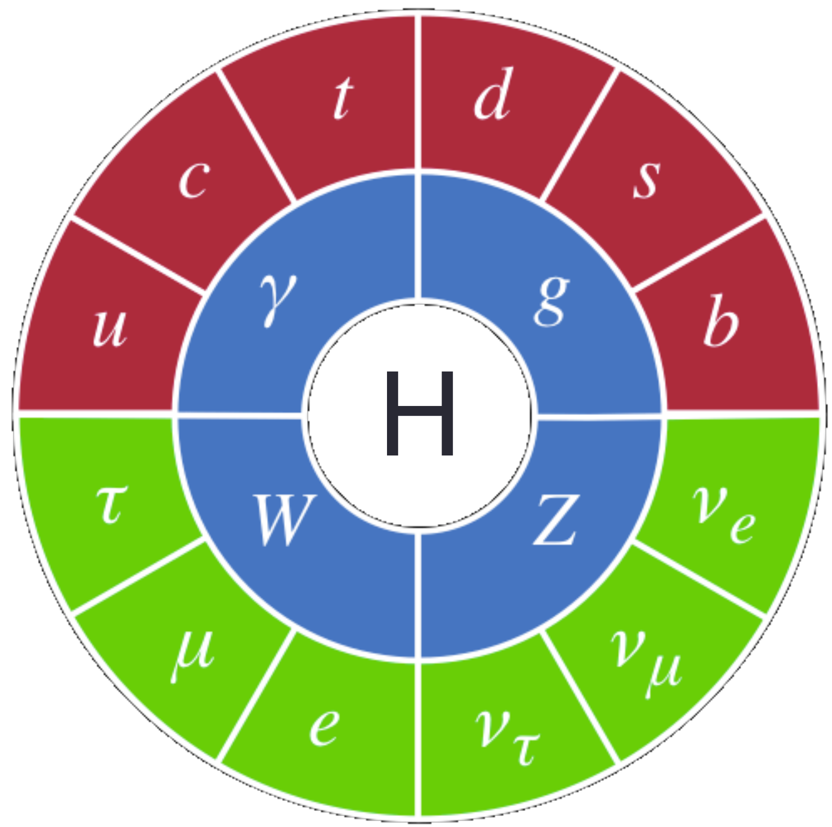
What did we learn?

The Standard Model

1980-2012

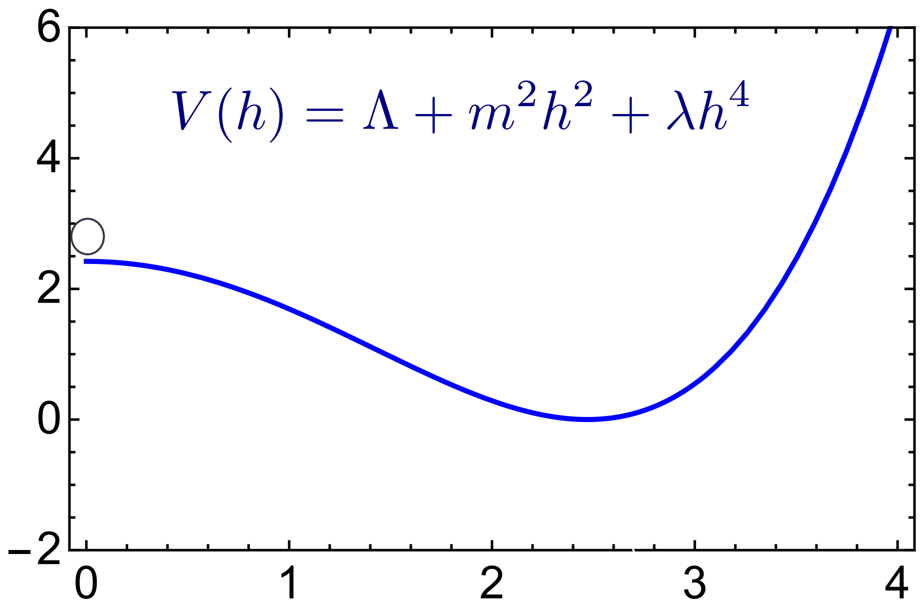


2012 -- ??



What is the Higgs field?

- The Higgs field h(x) pervades all space
- The Higgs field h(x) has charge under the weak force
  - If <h> = 0 space is not empty – it has weak charge too
- The Higgs field h(x) has a potential



- Lowest energy state has <h> = v
- This Higgs field value surrounds us all

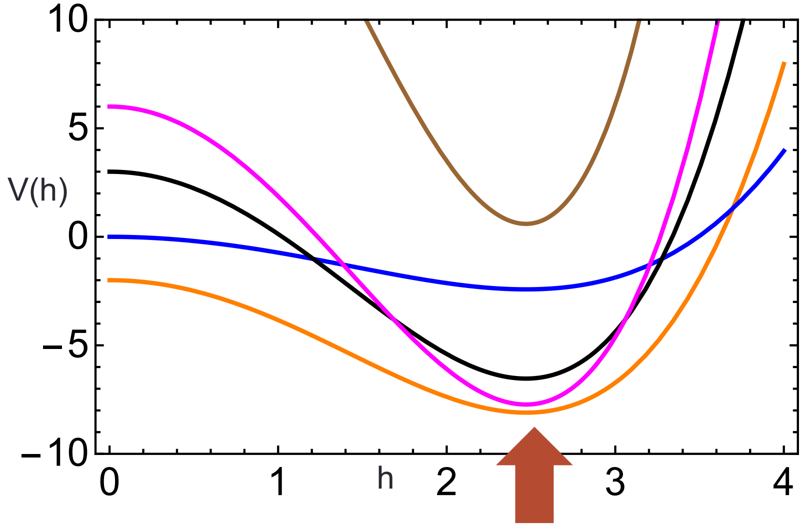
What do we know about this potential?

Classical potential: $V(h) = \Lambda + m^2 h^2 + \lambda h^4$

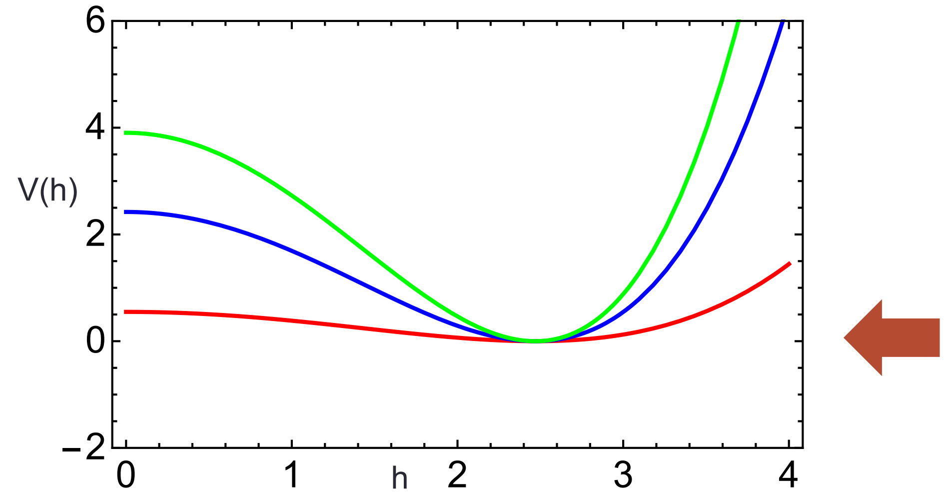
- 3 free parameters (Λ, m λ)
  - Must be measured from data

Higgs potential

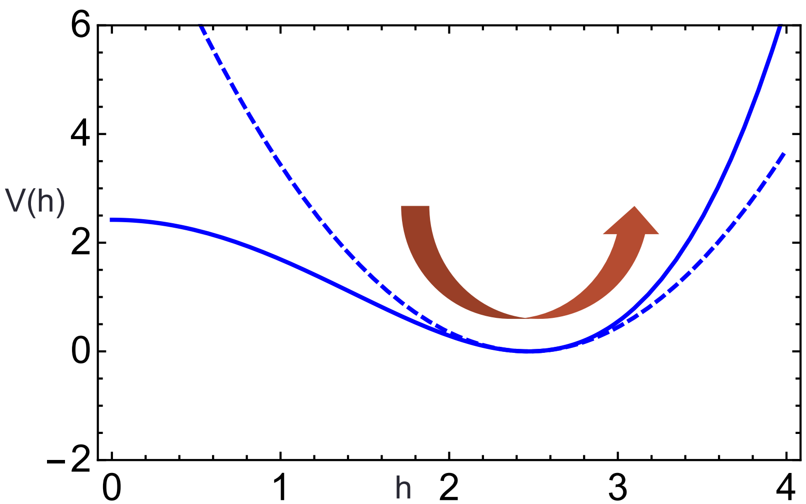
$$V(h) = \Lambda + m^2 h^2 + \lambda h^4$$



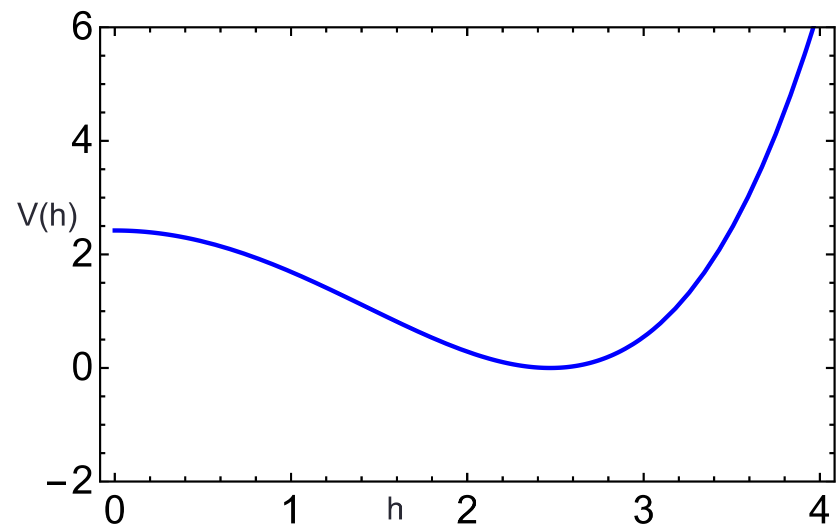
1933: Rate for beta decay (G_F=<v>$^{-2}$) gives vacuum expectation value



1998: acceleration of universe gives vacuum energy density V(v)= $(10^{-3}$ eV$)^4$

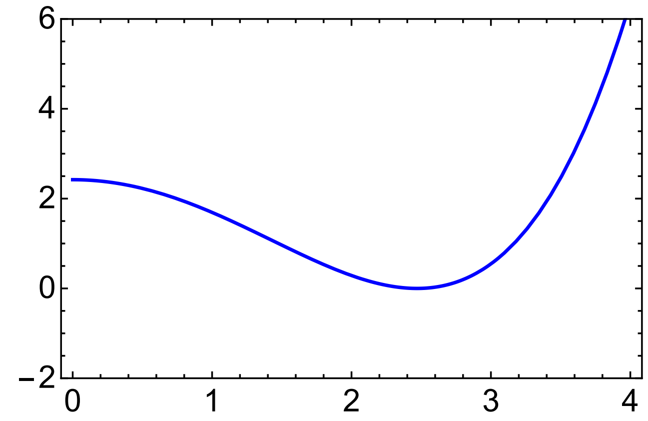


2012: Higgs boson mass V''(v)=(126 GeV$)^2$ gives curvature at minimum



Classical potential: $V(h) = \Lambda + m^2 h^2 + \lambda h^4$

- 3 free parameters (Λ, m, λ)
  - Must be measured from data ✓

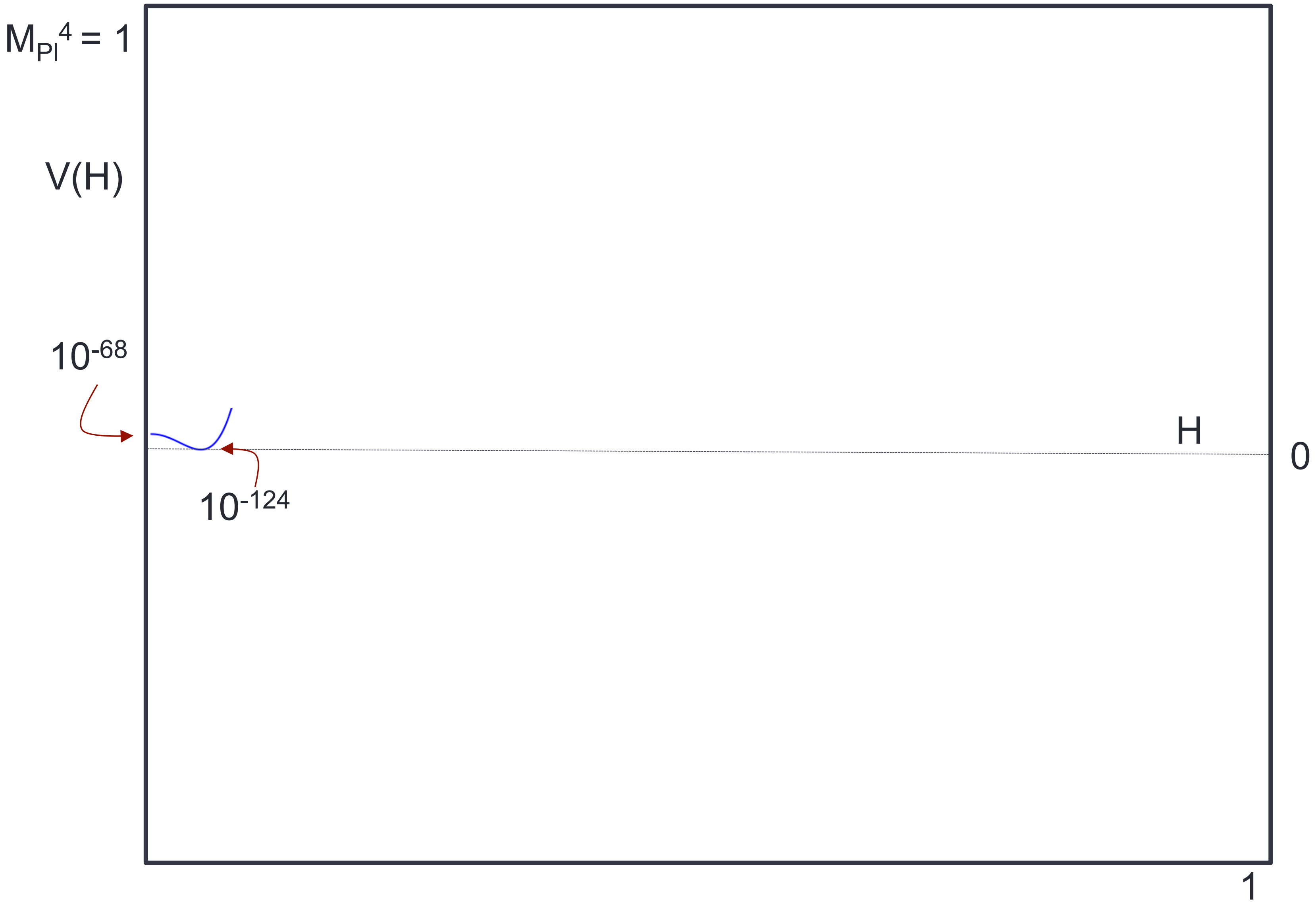


Why are the values of Λ, m, λ in nature **interesting**?

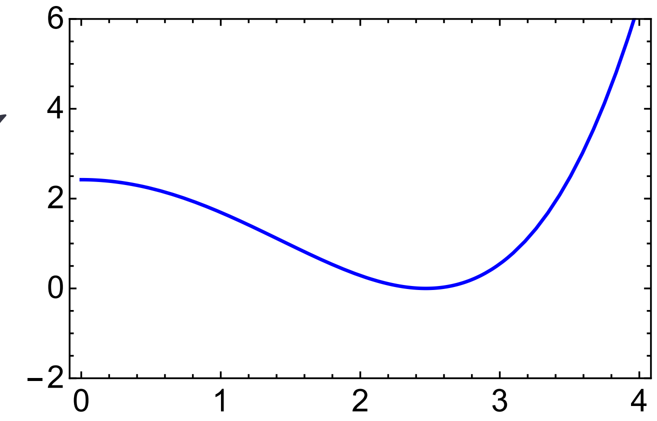
1. Fine tuning

2. Vacuum stability

1. Fine tuning

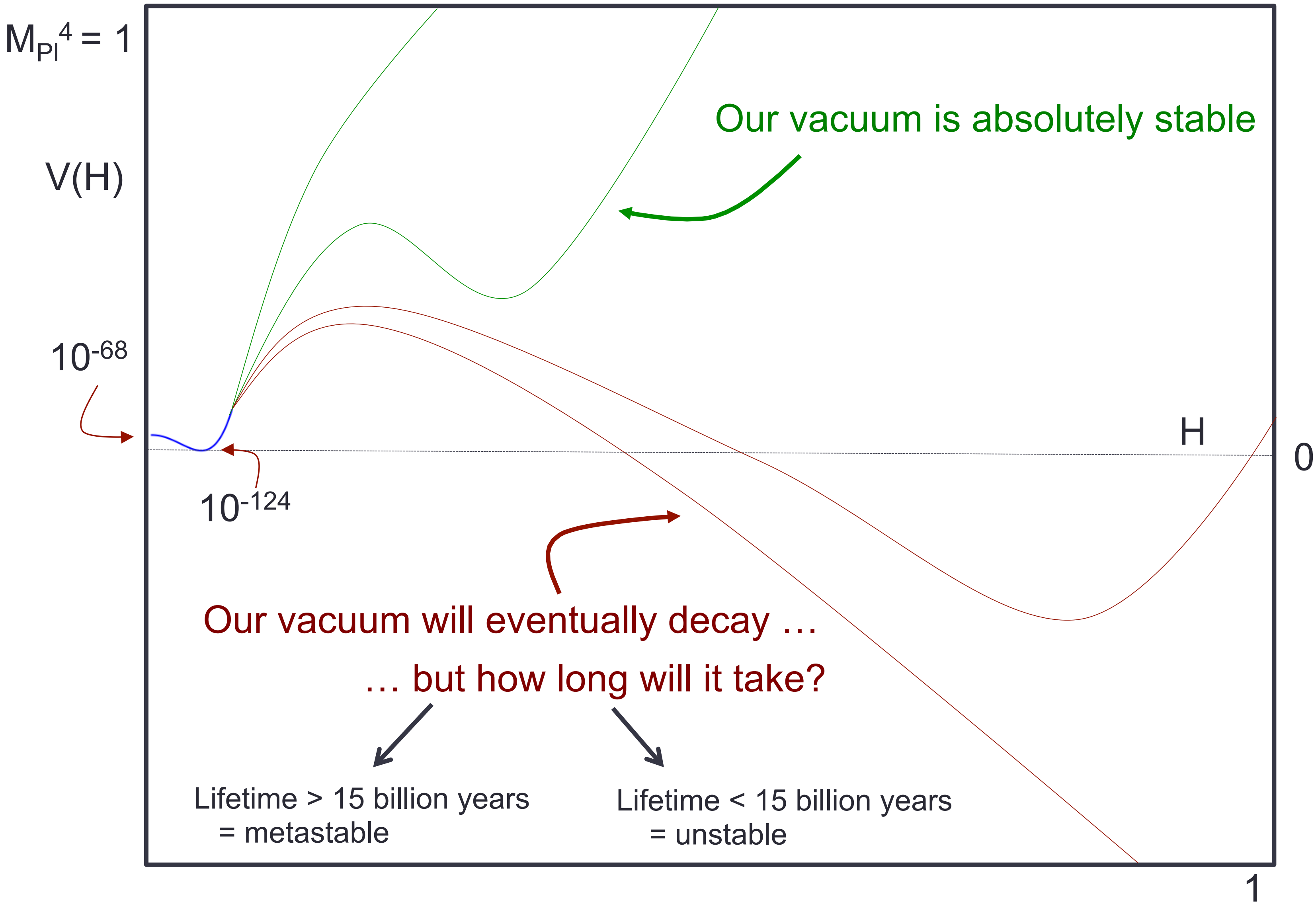


Classical potential: $V(h) = \Lambda + m^2 h^2 + \lambda h^4$

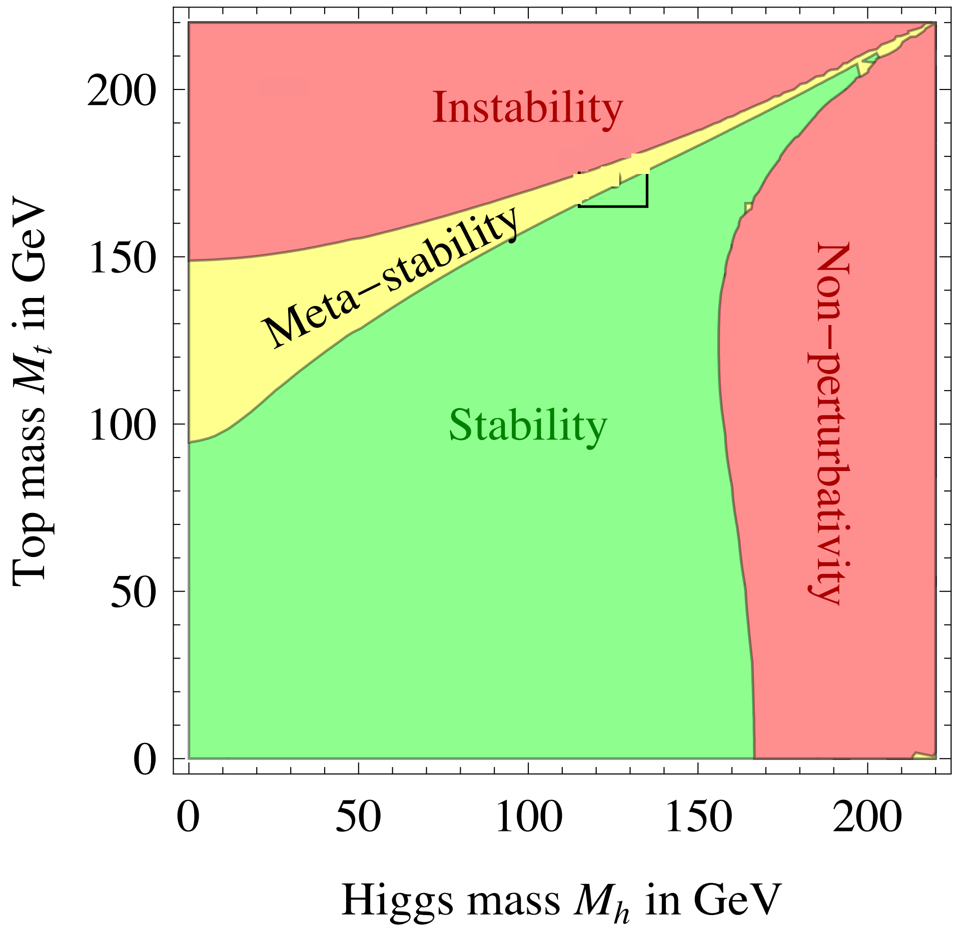


- 3 free parameters (Λ, m λ)
  - Must be measured from data ✓
- **Only** 3 free parameters
  - Quantum Field Theory
    determines V(h) for arbitrarily large h
  - Called the quantum-corrected or Effective Potential

Fine tuning Stability

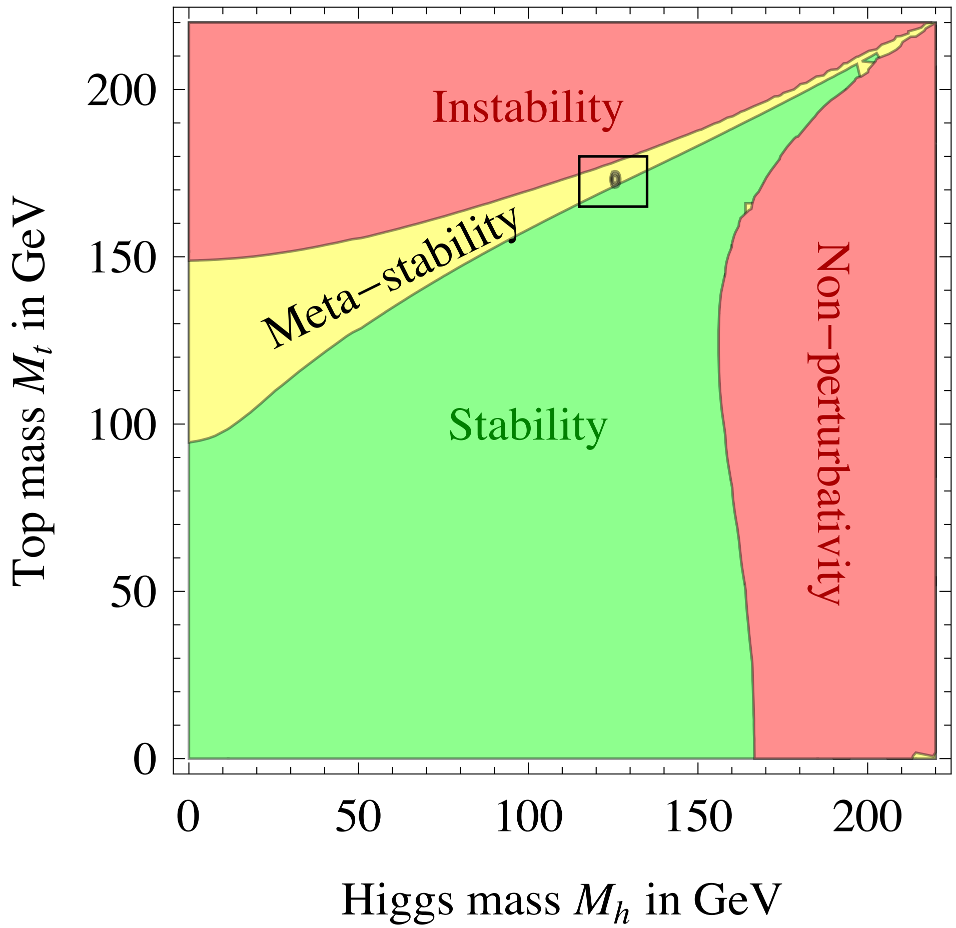


Absolute stability or metastability depends on Higgs and top masses



From Degrassi et al (arXiv:1205.6497)

Absolute stability or metastability depends on Higgs and Top masses

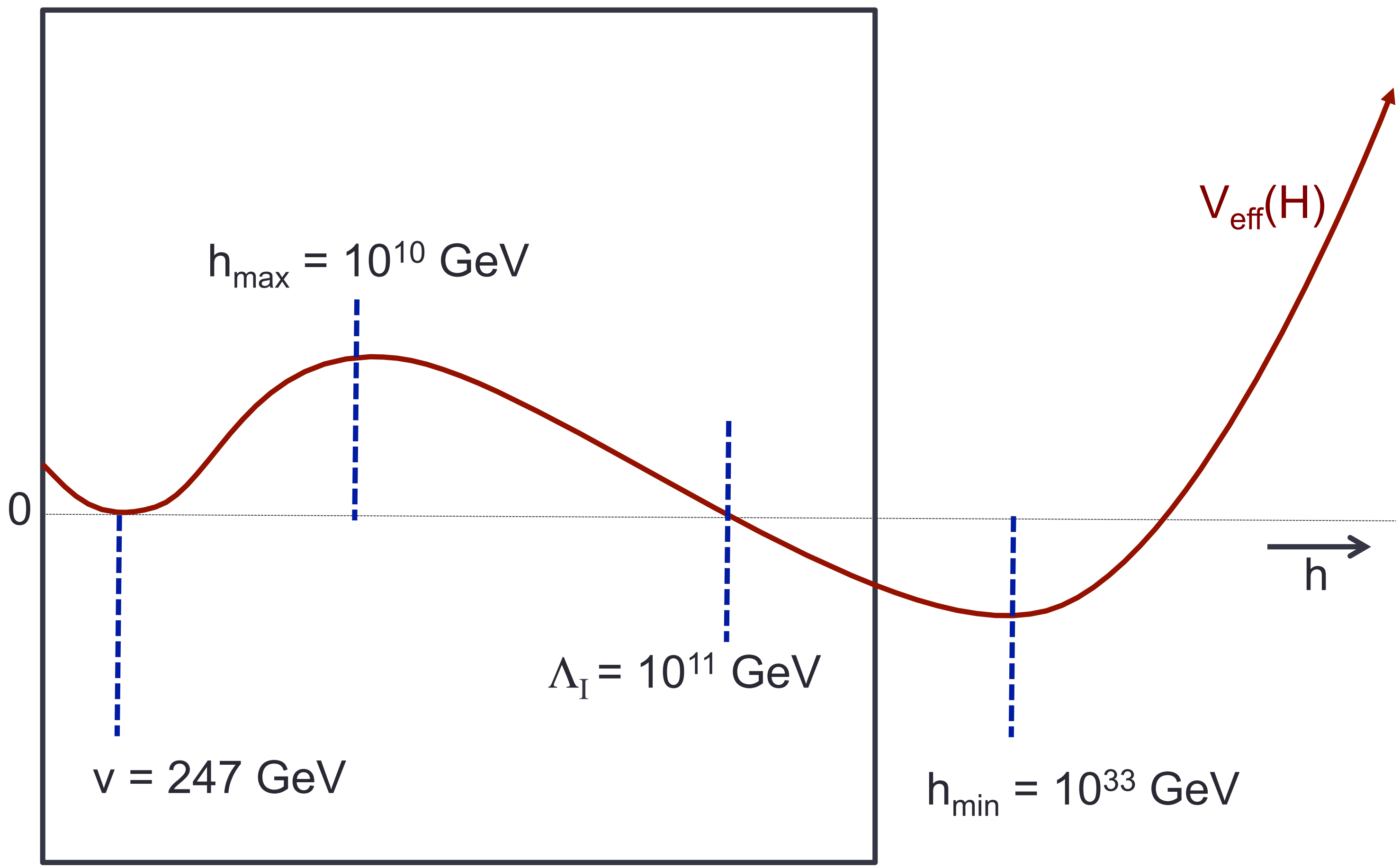


From Degrassi et al (arXiv:1205.6497)

Our standard model in metastability funnel region

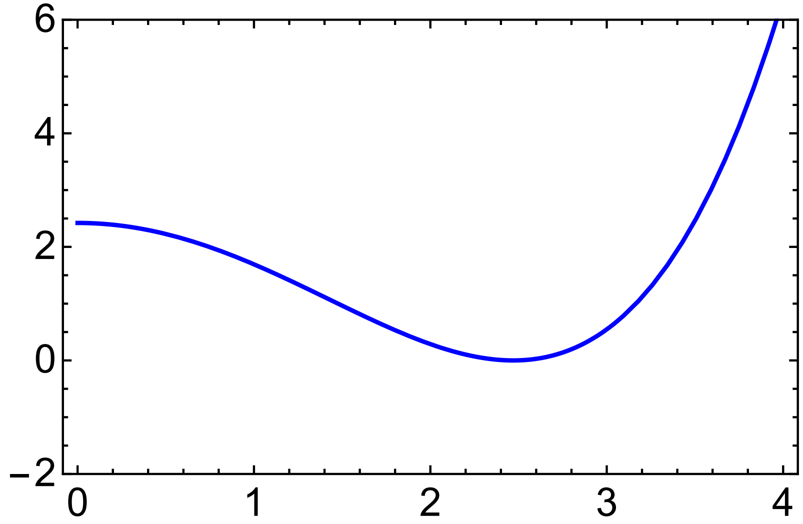
This is now precision Standard Model physics.
Is it correct?

Standard Model Effective Potential



Are these scales physical?

How do we compute V_{eff}?



Classical potential: $V(h) = \Lambda + m^2h^2 + \lambda h^4$

- Renormalizable
- Three parameters (Λ, m^2 and λ), measured from data

How can the quantum-corrected potential be computed?

How do we compute V_{eff}?

Method 1:

$$\int \mathcal{D}H e^{i\Gamma} = \int \mathcal{D}H \underbrace{\mathcal{D}\psi \cdots \mathcal{D}A e^{iS}}$$

Classical action

Effective Action

Integrate out everything but H

$$\Gamma = \int d^4x \Big\{ -Z[H]H\Box H - V_{\text{eff}}(H) + \cdots \Big\}$$

Problems:

- Generally non-local (has nasty things like $\ln \frac{1+\Box/m_t^2}{H^2}$ in it)
- Nearly impossible to compute

OK if $H \approx \langle H \rangle$ (applies to the two points above)

- Can't include loops of H itself this way

If we integrate over everything, effective action is just a number

$$e^{i\Gamma} = \int \mathcal{D}H \cdots \mathcal{D}A e^{iS}$$

Method 2: Legendre transform

Classical action

$$\frac{\delta S}{\delta H}\Big|_{H=v} = 0$$

Classical minimum

We want an effective action

$$\frac{\delta \Gamma}{\delta H}\Big|_{H=H_q} = 0$$

True quantum minimum

1. Compute W[J] $$e^{W[J]} \equiv \int \mathcal{D}H \cdots \mathcal{D}A e^{i\int d^4x\{\mathcal{L}+JH\}}$$

Current introduced by hand
So that Γ depends on something

2. Solve $H = \dfrac{\partial W}{\partial J}$ for J[H]

3. Compute $$\Gamma[H] = W[J[H]] - \int d^4x H J[H]$$

Has the property that $\dfrac{\delta \Gamma}{\delta H} = J[H]$ so that $\dfrac{\delta \Gamma}{\delta H} = 0$ when J=0 (i.e. in original theory)

- Agrees with method 1 in perturbation theory

What do you get?

Tree-level (classical)

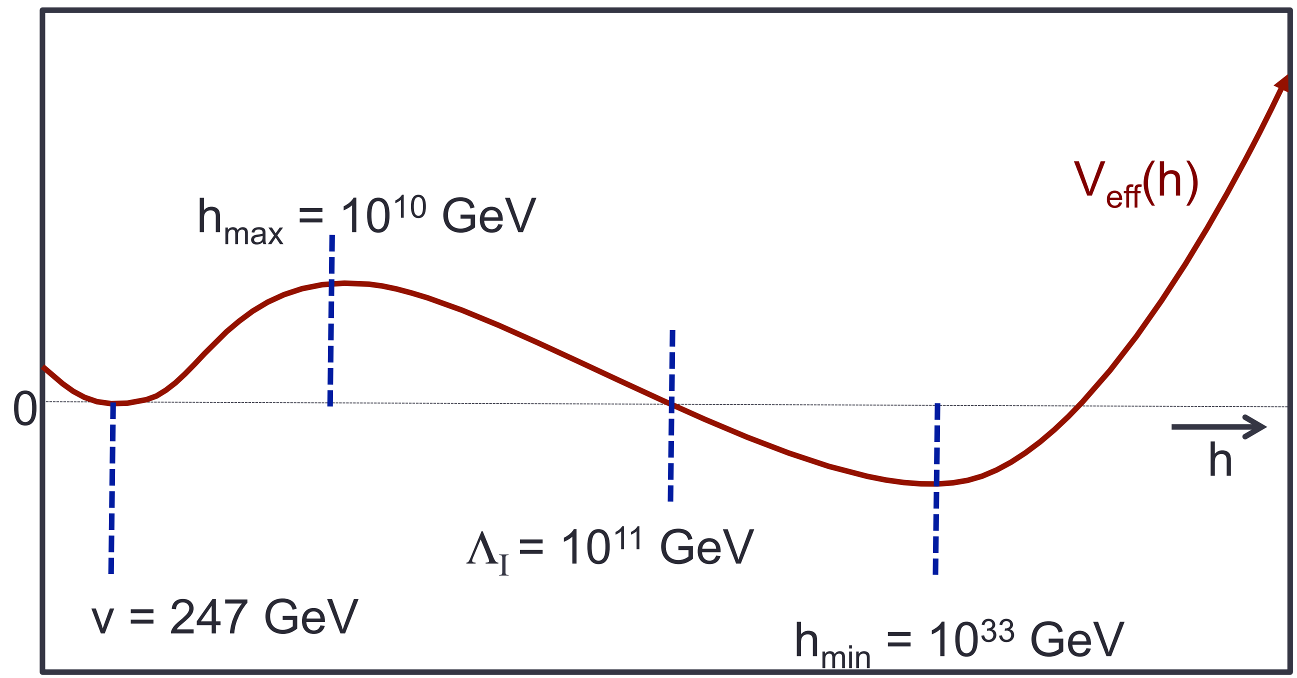
$$V_{\rm eff} = \frac{1}{4}\lambda h^4 - m^2 h^2$$

$$+ h^4 \frac{1}{2048\pi^2}\Big[-5g_1^4 + 6(g_1^2+g_2^2)^2 \ln\frac{h^2(g_1^2+g_2^2)}{4\mu^2} - 10 g_1^2 g_2^2 - 15 g_2^4 + 12 g_2^4 \ln\frac{g_2^2 h^2}{4\mu^2} + 144 y_t^4 - 96 y_t^4 \ln\frac{y_t^2 h^2}{2\mu^2}\Big]$$

$$\frac{-1}{256\pi^2}\left[\xi_B g_1^2\left(\ln\frac{\lambda h^4(\xi_B g_1^2+\xi_W g_2^2)}{4\mu^4} - 3\right) + \xi_W g_2^2\left(\ln\frac{\lambda^3 h^{12}\xi_W^2 g_2^4(\xi_B g_1^2+\xi_W g_2^2)}{64\mu^{12}} - 9\right)\right]\lambda h^4$$

$$+\cdots$$

one-loop



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$$\frac{-1}{256\pi^2}\left[\xi_B g_1^2\left(\ln\frac{\lambda h^4(\xi_B g_1^2+\xi_W g_2^2)}{4\mu^4} - 3\right) + \xi_W g_2^2\left(\ln\frac{\lambda^3 h^{12}\xi_W^2 g_2^4(\xi_B g_1^2+\xi_W g_2^2)}{64\mu^{12}} - 9\right)\right]\lambda h^4$$

$$+\cdots$$

Two curious features

1. Not gauge-invariant
2. Large logarithms

1. Gauge-dependence

Method 1 to compute Γ **is** gauge-invariant:

$$\int \mathcal{D}H e^{i\Gamma} \equiv \int \mathcal{D}H \underbrace{\mathcal{D}\psi \cdots \mathcal{D}A e^{iS}}$$

Completely integrate over gauge-orbits

Action/energy at minimum also gauge-invariant: $e^{i\Gamma} = \int \mathcal{D}H \cdots \mathcal{D}A e^{iS}$

Method 2 to compute Γ introduces a charged source J

$$e^{W[J]} \equiv \int \mathcal{D}H \cdots \mathcal{D}A e^{i\int d^4x\{\mathcal{L}+JH\}}$$

$$\Gamma = W - HJ$$

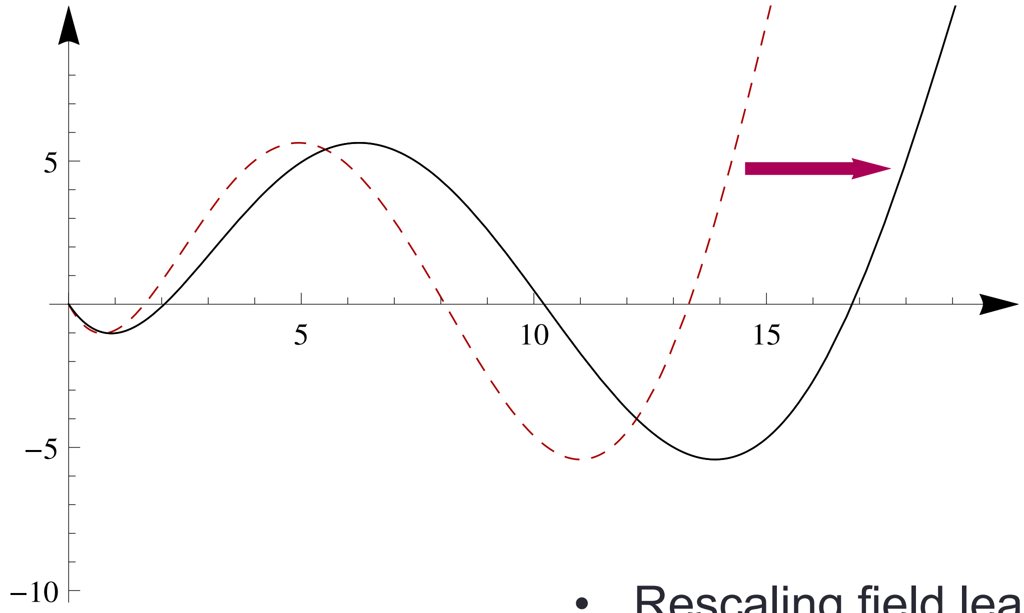
$$\frac{\delta\Gamma}{\delta H} = J$$

- Action away from minimum has current present
- Action **at minimum** has **no current**, should be gauge-invariant

Encoded in Nielsen identity

$$\left[\xi\frac{\partial}{\partial\xi} + C(h,\xi)\frac{\partial}{\partial h}\right]V_{\text{eff}}(h,\xi) = 0$$

Potential at minimum indep. of rescaling

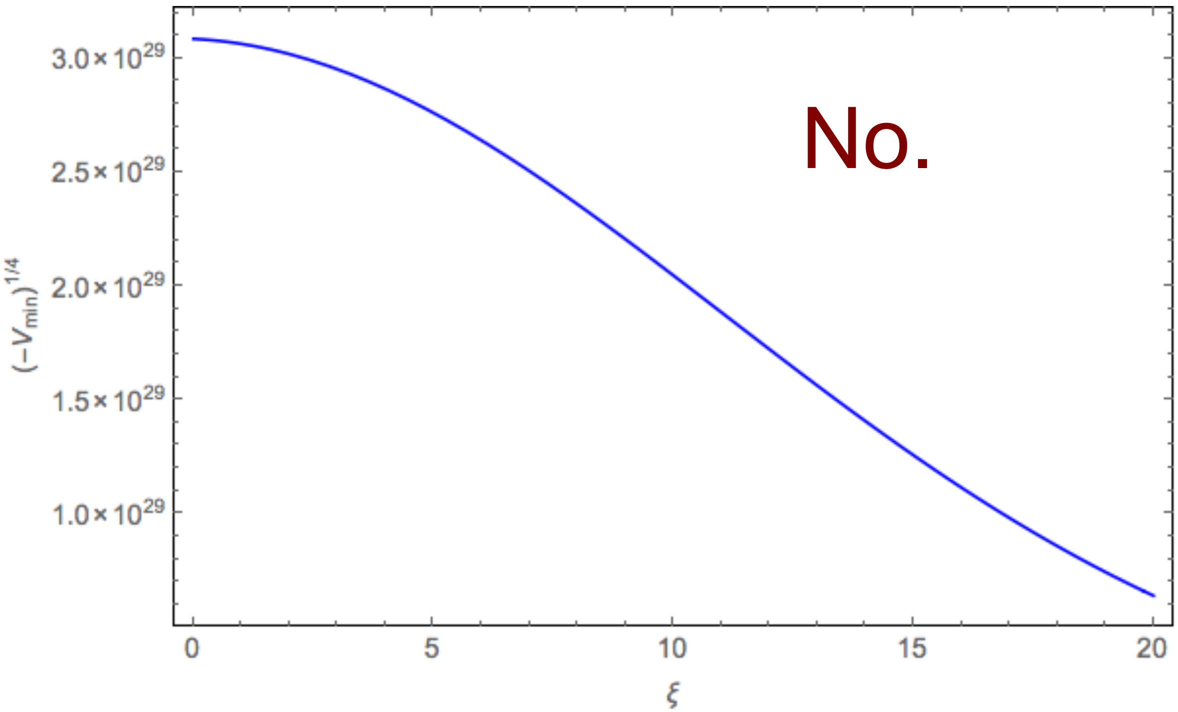


- Rescaling field leaves V_{min} unchanged

Nielsen identity

$$\left[\xi \frac{\partial}{\partial \xi} + C(h,\xi)\frac{\partial}{\partial h}\right] V_{\text{eff}}(h,\xi) = 0$$

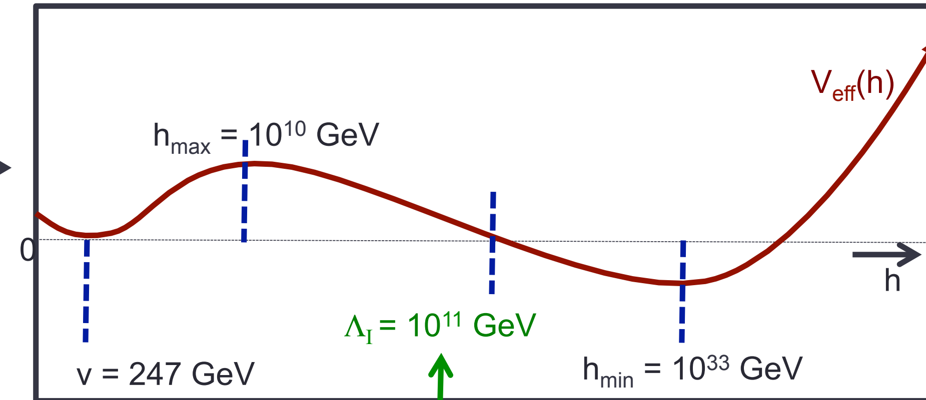
But is it?



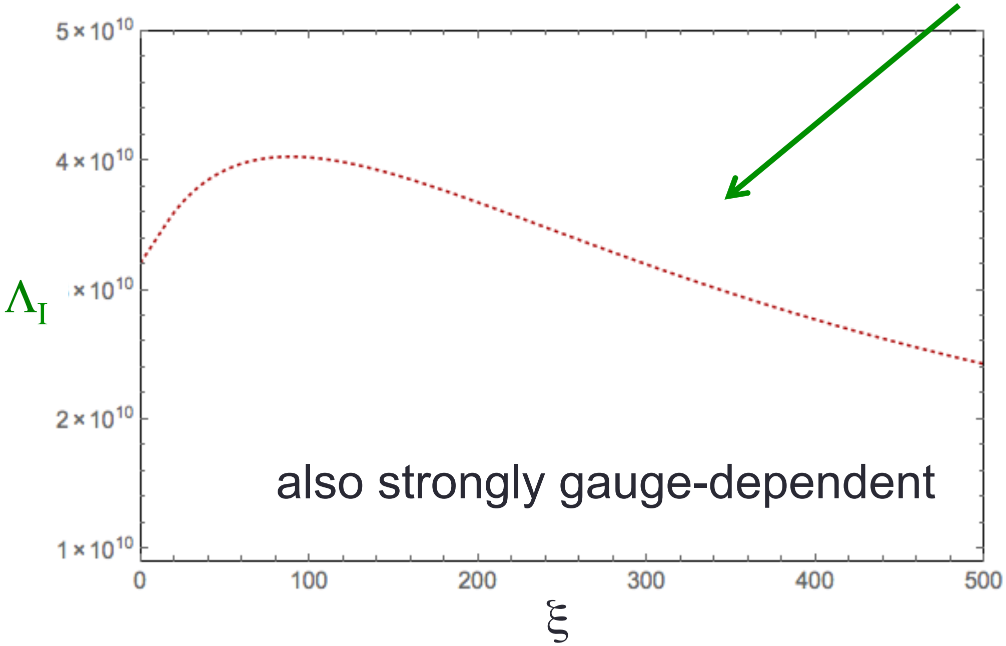
$(-V_{\min})^{1/4}$ appears linearly-dependent on gauge parameter ξ

What about field values?

Landau gauge (ξ=0) →



Instability scale Λ_I = value of h where V(h) = 0



- h_{min} also gauge dependent
- h_{max} also gauge dependent
- …

2. Large Logarithms

Can be resummed with RGE:

Explicit μ dependence

$$\left(\mu\frac{\partial}{\partial\mu} + \beta_i\frac{\partial}{\partial g_i} - \gamma h\frac{\partial}{\partial h}\right) V_{\text{eff}} = 0$$

compensated for by rescaling couplings and fields

- Same RGE as 1PI Green's functions or off-shell matrix elements
- Observables/S-matrix elements satisfy simpler RGE:

$$\left(\mu\frac{\partial}{\mu} + \beta_i\frac{\partial}{\partial g_i}\right)\sigma = 0$$

- Field-rescaling term canceled by LSZ wavefunction Z-factors

⇒ Effective potential depends on the normalization of fields??!!

Resum logarithms

1. Compute V_{eff} to fixed order (say 2-loops) at scale (say) μ_0 ~ 100 GeV

2. Solve RGE $\left(\mu\frac{\partial}{\partial\mu}+\beta_i\frac{\partial}{\partial g_i}-\gamma h\frac{\partial}{\partial h}\right)V_{\text{eff}}=0$

$$V_{\text{eff}}(h,g_i,\mu)\to V_{\text{eff}}(e^{\Gamma(\mu_0,\mu)}h,g_i(\mu),\mu)$$

$$\Gamma(\mu_0,\mu)\equiv\int_{\mu_0}^{\mu}\gamma(\mu')d\ln\mu'$$

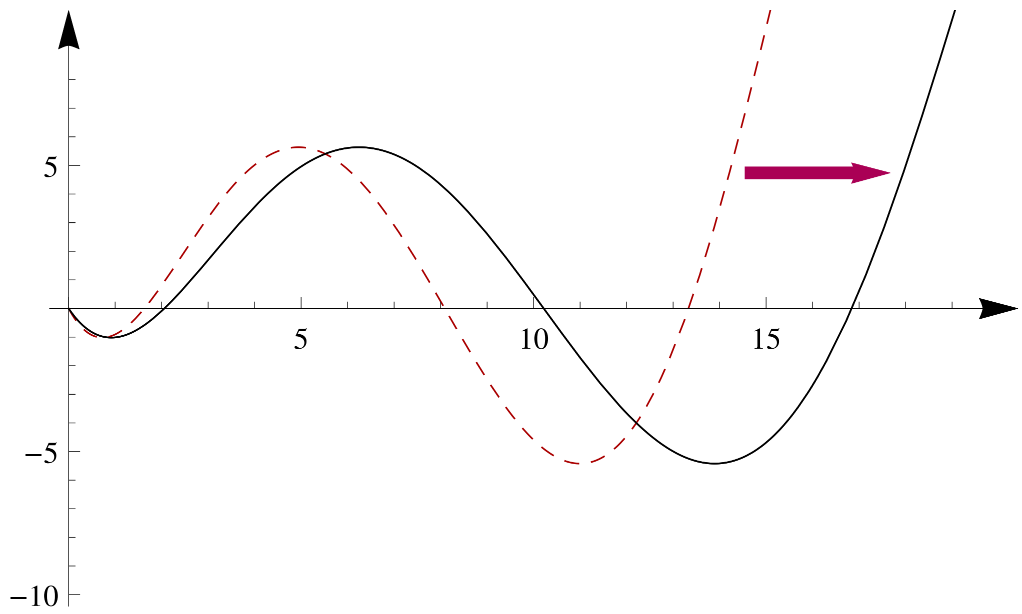
3. Set μ ~ h

$$V_{\text{eff}}(h,\mu_0)=V_{\text{eff}}(e^{\Gamma(\mu_0,h)}h,g_i(h),h)$$

Potential depends on scale μ_0 where it is calculated??!!

$$\Rightarrow\quad\left(\frac{\partial}{\partial\mu_0}-\gamma h\frac{\partial}{\partial h}\right)V(h,\mu_0)=0$$

Potential at minimum



Nielsen identity (gauge invariance)

$$\left[\xi\frac{\partial}{\partial\xi}+C(h,\xi)\frac{\partial}{\partial h}\right]V_{\text{eff}}(h,\xi)=0$$

Calculation-scale invariance

$$\left(\frac{\partial}{\partial\mu_0}-\gamma h\frac{\partial}{\partial h}\right)V(h,\mu_0)=0$$

V_{min} should be gauge invariant and independent of how it is calculated

Even gauge-invariant Γ is unphysical

Even if we source a gauge-invariant field $e^{W[J]} \equiv \int \mathcal{D}H \cdots \mathcal{D}A e^{i\int d^4x\{\mathcal{L}+JH\}}$

$$e^{W[J]} \equiv \int \mathcal{D}H \cdots \mathcal{D}A e^{i\int d^4x\{\mathcal{L}+JH^\dagger H\}}$$
$$e^{W[J]} \equiv \int \mathcal{D}H \cdots \mathcal{D}A e^{i\int d^4x\{\mathcal{L}+J|H|\}}$$

Γ(h) is now gauge-invariant

Effective potential still depends on how it is calculated $\left(\frac{\partial}{\partial\mu_0} - \gamma h\frac{\partial}{\partial h}\right)V(h,\mu_0) = 0$

- This is OK.
- Off-shell quantities can be unphysical

- **Observables should be physical**
  - S-matrix elements
  - Vacuum energy (V_{min})
  - Tunnelling rates
  - Critical temperature

But are they?

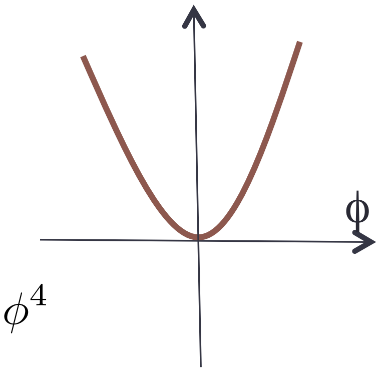
What about field values?

Instability scale?
Inflation scale?
Planck/new physics sensitivity?

Are these questions about observables?

SCALAR QED

Scalar QED



$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}|D_\mu\phi|^2 - V(\phi)$$

$$V_0(\phi) = \frac{\lambda}{24}\phi^4$$

- mass term gives small corrections, so we drop it

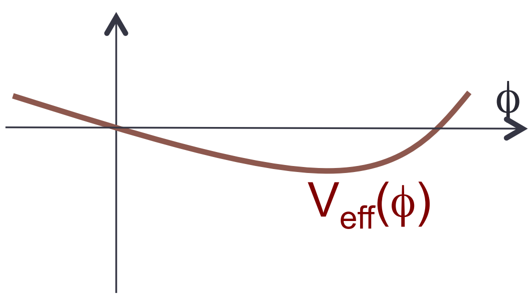
1-loop potential in R_ξ gauges:

$$V_1(\phi) = \phi^4 \frac{\hbar}{16\pi^2}\left[\frac{3}{4}e^4\left(\ln\frac{e^2\phi^2}{\mu^2} - \frac{5}{6}\right) + \frac{\lambda^2}{16}\left(\ln\frac{\lambda\phi^2}{2\mu^2} - \frac{3}{2}\right) + \left(\frac{\lambda^2}{144} - \frac{1}{12}e^2\lambda\xi\right)\left(\ln\frac{\phi^2}{\mu^2} - \frac{3}{2}\right) + \frac{1}{4}K_+^4 \ln K_+^2 + \frac{1}{4}K_-^4 \ln K_-^2\right]$$

$$K_\pm^2 = \frac{1}{12}\left(\lambda \pm \sqrt{\lambda^2 - 24\lambda e^2\xi}\right)$$

- Not gauge-invariant

- For most values of e and λ, there is no minimum
- When $\lambda \approx \frac{e^4}{16\pi^2}$ $\Rightarrow V_0 \approx V_1$
- And…. V_{min} depends on ξ



Spontaneous symmetry breaking

When is $\lambda \approx \frac{e^4}{16\pi^2}$?

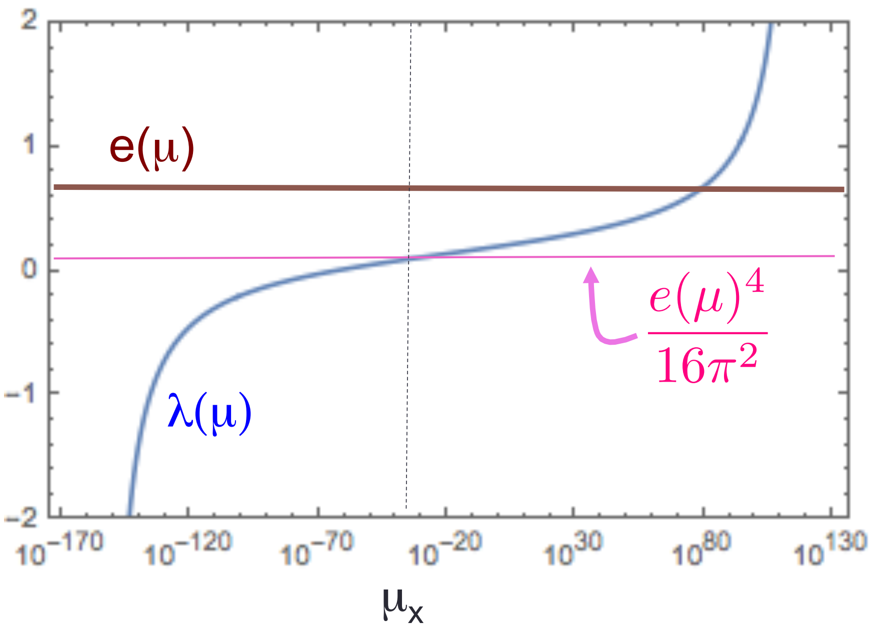
Solve RGEs:

$$\beta_e = \frac{\hbar}{16\pi^2}\left(\frac{e^3}{3}\right) + \cdots$$

$$\beta_\lambda = \frac{\hbar}{16\pi^2}\left(36e^4 - 12e^2\lambda + \frac{10\lambda^2}{3}\right)$$

$$e^2(\mu) = \frac{e^2(\mu_0)}{1 - \frac{e^2(\mu_0)}{24\pi^2}\ln\frac{\mu}{\mu_0}}$$

$$\lambda(\mu) = \frac{e^2(\mu)}{10}\left[19 + \sqrt{719}\tan\left(\frac{\sqrt{719}}{2}\ln\frac{e(\mu)^2}{C}\right)\right]$$



- e runs relatively slowly
- For any e, λ runs through all values
- There is always a scale μ_X where

$$\lambda(\mu_X) \approx \frac{e(\mu_X)^4}{16\pi^2}$$

- Near this scale, V_{eff} is pertubative

Proper loop expansion

$$V_0(\phi) = \frac{\lambda}{24}\phi^4$$

$$V_1(\phi) = \phi^4 \frac{\hbar}{16\pi^2}\left[\frac{3}{4}e^4\left(\ln\frac{e^2\phi^2}{\mu^2} - \frac{5}{6}\right) + \frac{\lambda^2}{16}\left(\ln\frac{\lambda\phi^2}{2\mu^2} - \frac{3}{2}\right) + \left(\frac{\lambda^2}{144} - \frac{1}{12}e^2\lambda\xi\right)\left(\ln\frac{\phi^2}{\mu^2} - \frac{3}{2}\right) + \frac{1}{4}K_+^4 \ln K_+^2 + \frac{1}{4}K_-^4 \ln K_-^2\right]$$

$$K_\pm^2 = \frac{1}{12}\left(\lambda \pm \sqrt{\lambda^2 - 24\lambda e^2 \xi}\right)$$

Comparable when

$$\lambda \approx \hbar \frac{e^4}{16\pi^2}$$

- Then V0 and V1 of order $\hbar$

These terms all have extra $\hbar$ suppression

Expanding in $\hbar$ with $\lambda \sim \hbar$

order $\hbar$: $V^{\text{LO}} = \frac{\lambda}{24}\phi^4 + \frac{\hbar e^4}{16\pi^2}\phi^4\left(-\frac{5}{8} + \frac{3}{2}\ln\frac{e\phi}{\mu}\right) \longrightarrow V_{\text{min}}^{\text{LO}} = -\frac{3}{128\pi^2}e^4\langle\phi\rangle^4$

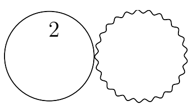
order $\hbar^2$: $V^{\text{NLO}} = \frac{\hbar e^2\lambda}{16\pi^2}\phi^4\left(\frac{\xi}{8} - \frac{\xi}{24}\ln\frac{e^2\lambda\xi\phi^4}{6\mu^4}\right) \longrightarrow V_{\text{min}}^{\text{NLO}} = \cdots$

Problem: higher-loop contributions also of order $\hbar^2$

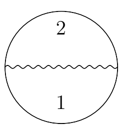
2-Loop potential in scalar QED

- Known in Landau gauge
- Some terms computed by Kang (1974), not in $\overline{\text{MS}}$
- Some terms at order $e^6\hbar^2$ unknown

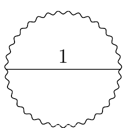
We computed all the relevant 2-loop graphs:



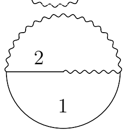
$$= \frac{\hbar^2\phi^4 e^6}{(16\pi^2)^2}\xi\left[-12\ln^2\frac{e\phi}{\mu} + \left(8 - 3\ln\frac{\lambda\xi}{6e^2}\right)\ln\frac{e\phi}{\mu} - \frac{5}{2} - \frac{\pi^2}{16} - \frac{3}{16}\ln^2\frac{\lambda\xi}{6e^2} + \ln\frac{\lambda\xi}{6e^2}\right]$$



$$= \frac{\hbar^2\phi^4 e^6}{(16\pi^2)^2}\left[(2+6\xi)\ln^2\frac{e\phi}{\mu} - (3+7\xi)\ln\frac{e\phi}{\mu} + \frac{7}{4} + \frac{\pi^2}{8} + \frac{15}{4}\xi + \frac{3\pi^2}{8}\xi\right]$$



$$= \frac{\hbar^2\phi^4 e^6}{(16\pi^2)^2}\left[(18+6\xi)\ln^2\frac{e\phi}{\mu} - (21+7\xi)\ln\frac{e\phi}{\mu} + \frac{47}{4} + \frac{7\pi^2}{24} + \frac{15}{4}\xi + \frac{3\pi^2}{8}\xi\right]$$



$$= \frac{\hbar^2\phi^4 e^6}{(16\pi^2)^2}\xi\left[-12\ln^2\frac{e\phi}{\mu} + 14\ln\frac{e\phi}{\mu} - \frac{15}{2} - \frac{3\pi^2}{4}\right]$$

Then the relevant part of the 2-loop potential is

$$V_2 = \left(\frac{\hbar}{16\pi^2}\right)^2 e^6\phi^4\left[(10-6\xi)\ln^2\frac{e\phi}{\mu} + \left(-\frac{62}{3} + 4\xi - \frac{3}{2}\xi\ln\frac{\lambda\xi}{6e^2}\right)\ln\frac{e\phi}{\mu}\right.$$
$$\left. + \xi\left(-\frac{1}{2} + \frac{1}{4}\ln\frac{\lambda\xi}{6e^2}\right) + \frac{71}{6}\right] + \cdots$$

terms of order $\hbar^3$

Potential at minimum

$$V^{\rm LO} = \frac{\lambda}{24}\phi^4 + \frac{\hbar e^4}{16\pi^2}\phi^4\left(-\frac{5}{8}+\frac{3}{2}\ln\frac{e\phi}{\mu}\right)$$

$$V^{\rm NLO} = \frac{\hbar e^2\lambda}{16\pi^2}\phi^4\left(\frac{\xi}{8}-\frac{\xi}{24}\ln\frac{e^2\lambda\xi\phi^4}{6\mu^4}\right)$$

$$+\frac{\hbar^2 e^6}{(16\pi^2)^2}\phi^4\left[(10-6\xi)\ln^2\frac{e\phi}{\mu}+\left(-\frac{62}{3}+4\xi-\frac{3}{2}\xi\ln\frac{\lambda\xi}{6e^2}\right)\ln\frac{e\phi}{\mu}+\xi\left(-\frac{1}{2}+\frac{1}{4}\ln\frac{\lambda\xi}{6e^2}\right)+\frac{71}{6}\right]$$

- Solve V'(ϕ=v) =0 for λ(v):

$$\lambda = \frac{\hbar e^4}{16\pi^2}\left(6-36\ln\frac{ev}{\mu}\right)+\frac{\hbar e^6}{(16\pi^2)^2}\left\{-160-24\xi+(376+90\xi)\ln\frac{ev}{\mu}-240\ln^2\frac{ev}{\mu}+9\xi\ln\left[\frac{\xi\hbar\mu^2}{16\pi^2v^2}\left(1-6\ln\frac{ev}{\mu}\right)\right]\right\}$$

- Plug in to V(v):

$$V_{\rm min} = v^4\frac{e^4\hbar}{16\pi^2}\left(-\frac{3}{8}\right)+v^4\frac{e^6\hbar^2}{(16\pi^2)^2}\frac{1}{12}\left\{62-9\xi+(-60+18\xi)\ln\frac{ev}{\mu}+\frac{9}{2}\xi\ln\left[\frac{e^2\xi\hbar}{16\pi^2}\left(1-6\ln\frac{ev}{\mu}\right)\right]\right\}$$

Still gauge-dependent!

Problem : $v = \langle\phi\rangle$ is gauge-dependent

Express $V_{\rm min}$ in terms of only other dimensionful scale: μ

In terms of μ_X

Define μ_X by

$$\lambda(\mu_X) \equiv \frac{\hbar}{16\pi^2} e^4(\mu_X) \Big[6 - 36 \ln[e(\mu_X)]\Big]$$

- Tree-level vev is $v=\mu_X$
- Exact (non-perturbative) definition of μ_X

Then, vev is:

$$v = \mu_X + \mu_X \frac{\hbar e^2}{16\pi^2} \left\{ -\frac{40}{9} + \frac{94}{9} \ln e - \frac{20}{3} \ln^2 e - \frac{\xi}{2} + \frac{3}{2} \xi \ln e + \frac{\xi}{4} \ln \left[\frac{\xi \hbar}{16\pi^2} (1 - 6 \ln e) \right] - \frac{1}{6} \xi + \xi \ln e \right\}.$$

- gauge-dependent vev is OK – not physical

Potential at minimum is:

$$V_{\min} = \frac{e^4 \hbar}{16\pi^2} \mu_X^4 \left(-\frac{3}{8} \right) + \frac{e^6 \hbar}{(16\pi^2)^2} \mu_X^4 \left(\frac{71}{6} - \frac{62}{3} + 10 \ln^2 e \right) + \frac{e^6 \hbar}{(16\pi^2)^2} \mu_X^4 \left(\frac{\xi}{4} - \frac{3}{2} \xi \ln e \right)$$

- gauge-dependent vacuum energy is **not OK**

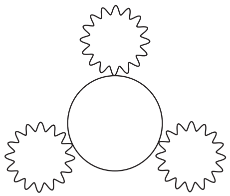
Still gauge-dependent!

What's missing?

More diagrams!

Daisy resummation

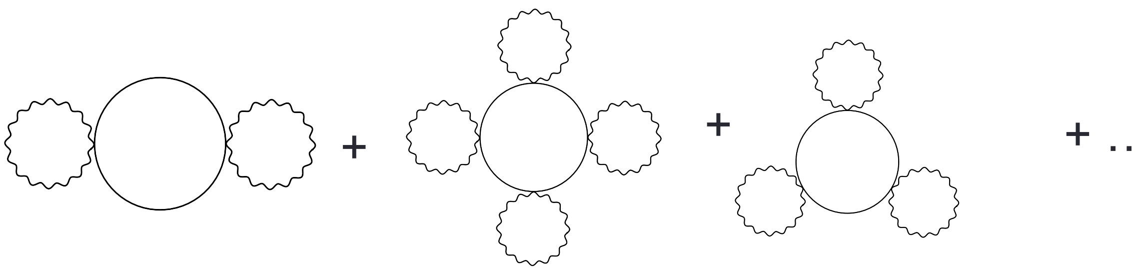
Higher order graphs can scale like inverse powers of λ:



$$\propto (e^2)^3(e^2\phi^2)^3 \int \frac{d^4k}{2\pi^4}\left(\frac{i}{k^2-\frac{\lambda}{2}\phi^2}\right)^3 \propto \phi^4\frac{e^{12}}{\lambda}$$

Effective masses depend on λ

Only one series of graphs contribute at order $\sim \hbar^2$



"daisy diagrams"

We can sum the series:

$$V^{e^6\text{daisies}} = \phi^4\frac{\hbar}{16\pi^2}\left(-\frac{e^2\lambda\xi}{24}\right)\left[\frac{\widehat{\lambda}(\phi)}{\lambda}+\left(1-\frac{\widehat{\lambda}(\phi)}{\lambda}\right)\ln\left(1-\frac{\widehat{\lambda}(\phi)}{\lambda}\right)\right]$$

$$\widehat{\lambda}(\phi) \equiv \frac{\hbar e^4}{16\pi^2}\left(6-36\ln\frac{e\phi}{\mu}\right)$$

Full potential at NLO:

$$V^{\rm NLO} = \frac{\hbar e^2\lambda}{16\pi^2}\phi^4\left(\frac{\xi}{8} - \frac{\xi}{24}\ln\frac{e^2\lambda\xi\phi^4}{6\mu^4}\right)$$

$$+\frac{\hbar^2 e^6}{(16\pi^2)^2}\phi^4\left[(10-6\xi)\ln^2\frac{e\phi}{\mu} + \left(-\frac{62}{3}+4\xi-\frac{3}{2}\xi\ln\frac{\lambda\xi}{6e^2}\right)\ln\frac{e\phi}{\mu} + \xi\left(-\frac{1}{2}+\frac{1}{4}\ln\frac{\lambda\xi}{6e^2}\right)+\frac{71}{6}\right]$$

$$+\phi^4\frac{\hbar e^2\lambda}{16\pi^2}\left(-\frac{\xi}{24}\right)\left[\frac{\widehat{\lambda}(\phi)}{\lambda} + \left(1-\frac{\widehat{\lambda}(\phi)}{\lambda}\right)\ln\left(1-\frac{\widehat{\lambda}(\phi)}{\lambda}\right)\right]$$

Now… vacuum energy is gauge-invariant!

$$V_{\rm min} = -\frac{3\hbar e^4}{128\pi^2}\mu_X^4 + \frac{e^6\hbar^2}{(16\pi^2)^2}\mu_X^4\left(\frac{71}{6}-\frac{62}{3}\ln e + 10\ln^2 e\right)$$

Field values are still gauge-dependent:

$$v = \mu_X + \mu_X\frac{\hbar e^2}{16\pi^2}\left\{-\frac{40}{9}+\frac{94}{9}\ln e - \frac{20}{3}\ln^2 e - \frac{\xi}{2} + \frac{3}{2}\xi\ln e + \frac{\xi}{4}\ln\left[\frac{\xi\hbar}{16\pi^2}(1-6\ln e)\right] - \frac{1}{6}\xi + \xi\ln e\right\}.$$

$$\Lambda_I = \mu_I + \mu_I\frac{\hbar e^2}{16\pi^2}\left\{-\frac{77}{9}+\frac{124}{9}\ln e - \frac{20}{3}\ln^2 e - \frac{\xi}{2} + \frac{3}{2}\xi\ln e + \frac{\xi}{4}\ln\left[\frac{\xi\hbar}{16\pi^2}(1-6\ln e)\right] - \frac{5}{12}\xi + \xi\ln e\right\}.$$

STANDARD MODEL

Lessons from scalar QED

1. Gauge invariance requires consistent expansion in $\hbar$

To N^{n}LO order

Drop some n-loop contributions

Include contributions from > n loops

2. Don't resum logs by solving RGE for V_{eff}

$$\left(\mu\frac{\partial}{\partial\mu}+\beta_i\frac{\partial}{\partial g_i}-\gamma h\frac{\partial}{\partial h}\right)V_{\text{eff}}=0$$

- Mixes up orders in $\hbar$ in an uncontrolled way

3. Do resum logs by using couplings at some scale μ_X

- Natural condition for μ_X is that $V_{LO}'(\phi=\mu_X) = 0$

4. Don't express V_{min} in terms of $v = \langle\phi\rangle$

- Express V_{min} in terms of μ_X instead

Standard Model

$$V^{(\mathrm{LO})}(h) = \frac{1}{4}\lambda h^4 + h^4 \frac{1}{2048\pi^2}\Big[-5g_1^4 + 6(g_1^2+g_2^2)^2 \ln\frac{h^2(g_1^2+g_2^2)}{4\mu^2} - 10g_1^2g_2^2 - 15g_2^4 + 12g_2^4 \ln\frac{g_2^2h^2}{4\mu^2} + 144y_t^4 - 96y_t^4 \ln\frac{y_t^2h^2}{2\mu^2}\Big]$$

Tree-level

Part of 1-loop $\lambda \sim \mathcal{O}(\hbar)$

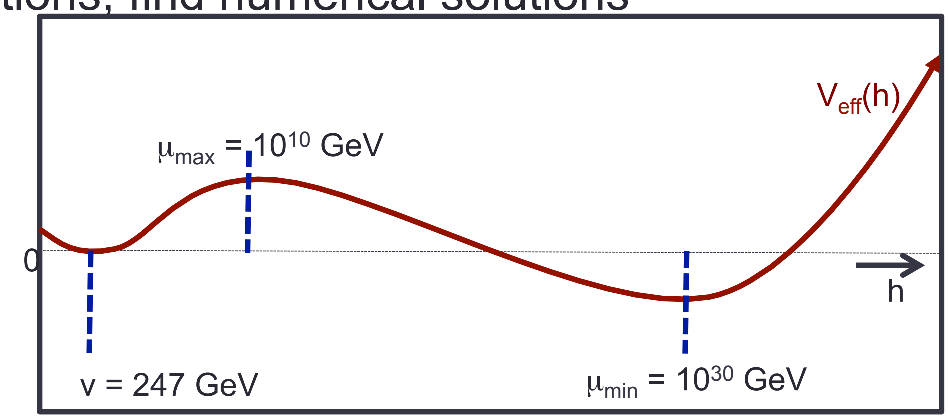
- Scale h=μ_X where $\frac{d}{dh}V^{(\mathrm{LO})}(h) = 0$ is

$$\lambda = \frac{1}{256\pi^2}\Big[g_1^4 + 2g_1^2g_2^2 + 3g_2^4 - 48y_t^4 - 3\left(g_1^2+g_2^2\right)^2 \ln\frac{g_1^2+g_2^2}{4} - 6g_2^4 \ln\frac{g_2^2}{4} + 48y_t^4 \ln\frac{y_t^2}{2}\Big]$$

- Run couplings with 3-loop β-functions, find numerical solutions

$$\mu_X^{\max} = 2.46\times 10^{10}\ \mathrm{GeV}$$

$$\mu_X^{\min} = 3.43\times 10^{30}\ \mathrm{GeV}$$



Standard Model at NLO

- We know the 1-loop contribution to V_{NLO}

$$V^{(1,\mathrm{NLO})}(h) = \frac{-1}{256\pi^2}\left[\xi_B g_1^2\left(\ln\frac{\lambda h^4(\xi_B g_1^2+\xi_W g_2^2)}{4\mu^4}-3\right)+\xi_W g_2^2\left(\ln\frac{\lambda^3 h^{12}\xi_W^2 g_2^4(\xi_B g_1^2+\xi_W g_2^2)}{64\mu^{12}}-9\right)\right]\lambda h^4$$

- We know the 2-loop contribution to V_{NLO} in Landau gauge

$$\begin{aligned}\lambda_{\mathrm{eff}}^{(2)} = \frac{1}{(4\pi)^4}\Big[& 8g_3^2y_t^4\left(3r_t^2-8r_t+9\right)+\frac{1}{2}y_t^6\left(-6r_tr_W-3r_t^2+48r_t-6r_{tW}-69-\pi^2\right)+\\ & +\frac{3y_t^2g_2^4}{16}\left(8r_W+4r_Z-3r_t^2-6r_tr_Z-12r_t+12r_{tW}+15+2\pi^2\right)+\\ & +\frac{y_t^2g_Y^4}{48}\left(27r_t^2-54r_tr_Z-68r_t-28r_Z+189\right)+\frac{y_t^2g_2^2g_Y^2}{8}(9r_t^2-18r_tr_Z+4r_t+44r_Z-57)+\\ & +\frac{g_2^6}{192}(36r_tr_Z+54r_t^2-414r_Wr_Z+69r_W^2+1264r_W+156r_Z^2+632r_Z-144r_{tW}-2067+90\pi^2)+\\ & +\frac{g_2^4g_Y^2}{192}(12r_tr_Z-6r_t^2-6r_W(53r_Z+50)+213r_W^2+4r_Z(57r_Z-91)+817+46\pi^2)+\\ & +\frac{g_2^2g_Y^4}{576}(132r_tr_Z-66r_t^2+306r_Wr_Z-153r_W^2-36r_W+924r_Z^2-4080r_Z+4359+\\ & +\frac{g_Y^6}{576}(6r_Z(34r_t+3r_W-470)-102r_t^2-9r_W^2+708r_Z^2+2883+206\pi^2)+\\ & +\frac{y_t^4}{6}\left(4g_Y^2(3r_t^2-8r_t+9)-9g_2^2\left(r_t-r_W+1\right)\right)+\frac{3}{4}\left(g_2^6-3g_2^4y_t^2+4y_t^6\right)\mathrm{Li}_2\frac{g_2^2}{2y_t^2}+\\ & +\frac{y_t^2}{48}\xi\left(\frac{g_2^2+g_Y^2}{2y_t^2}\right)\left(9g_2^4-6g_2^2g_Y^2+17g_Y^4+2y_t^2\left(7g_Y^2-73g_2^2+\frac{64g_2^4}{g_Y^2+g_2^2}\right)\right)+\\ & +\frac{g_2^2}{64}\xi\left(\frac{g_2^2+g_Y^2}{g_2^2}\right)\left(18g_2^2g_Y^2+g_Y^4-51g_2^4-\frac{48g_2^6}{g_Y^2+g_2^2}\right)\Big].\end{aligned}$$

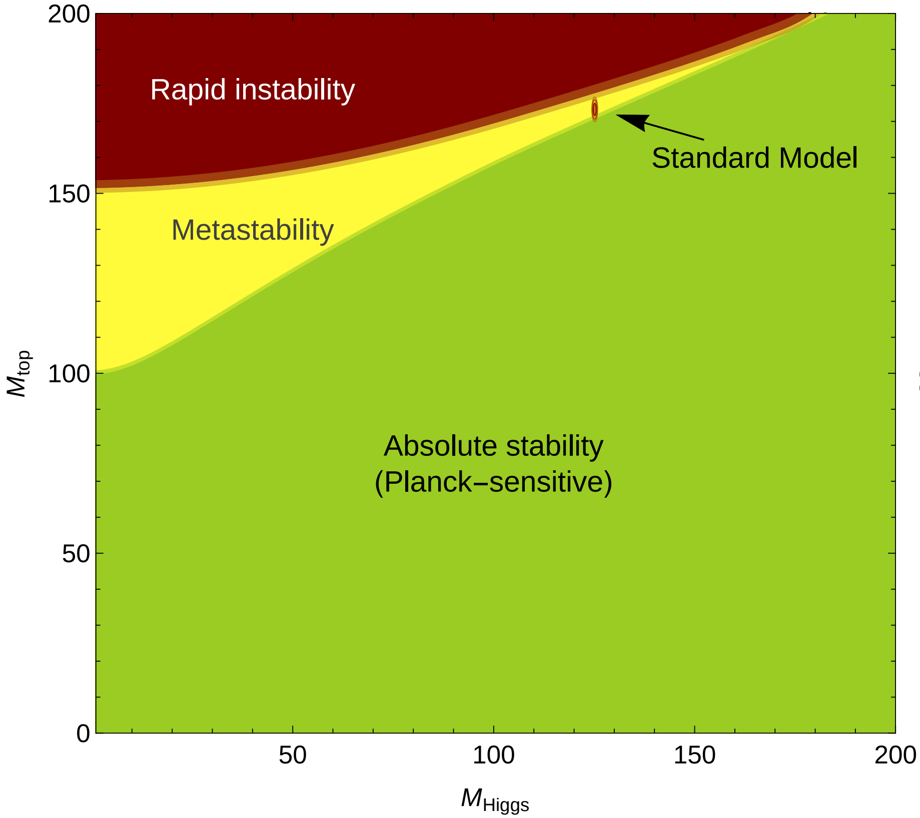
- We don’t know the Daisy contribution. But we do know if vanishes in Landau gauge at NLO

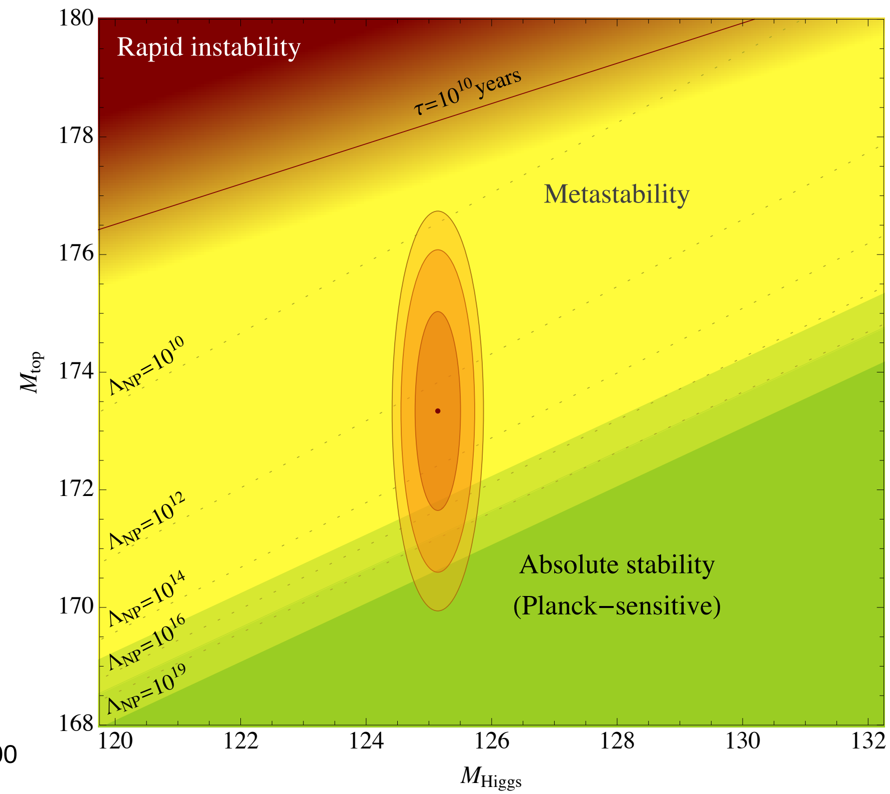
$$V^{e^6\mathrm{daisies}} = \phi^4\frac{\hbar}{16\pi^2}\left(-\frac{e^2\lambda\xi}{24}\right)\left[\frac{\widehat{\lambda}(\phi)}{\lambda}+\left(1-\frac{\widehat{\lambda}(\phi)}{\lambda}\right)\ln\left(1-\frac{\widehat{\lambda}(\phi)}{\lambda}\right)\right]$$

- Assuming everything works like in scalar QED, we have everything we need for NLO

Results

Absolute stability: for what values of the Higgs and top masses is is $V_{min} = 0$?





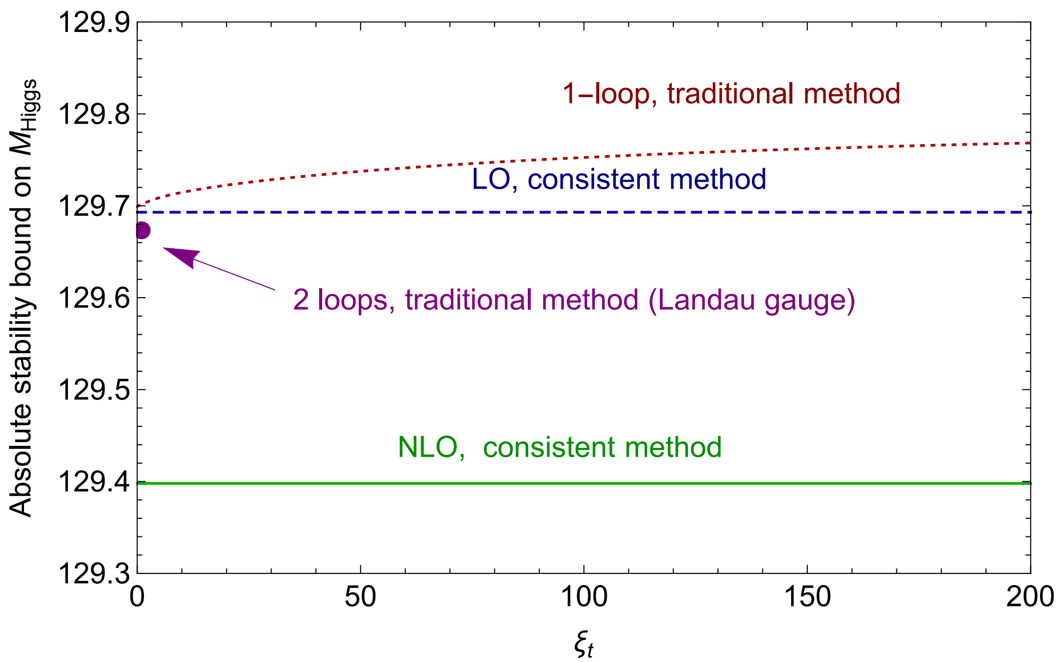
$$m_h^{\rm pole} = (125.14 \pm 0.23)\ {\rm GeV}$$

$$m_t^{\rm pole} = (173.34 \pm 1.12)\ {\rm GeV}$$

Results

Absolute stability: for what values of the Higgs mass is V_{min} = 0 at fixed top mass?

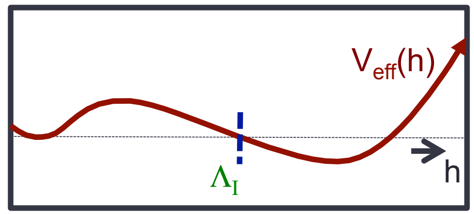
Holding top mass fixed



- Absolute stability bound lowered by 300 MeV
- Larger shift that including the 2-loop V_{eff}

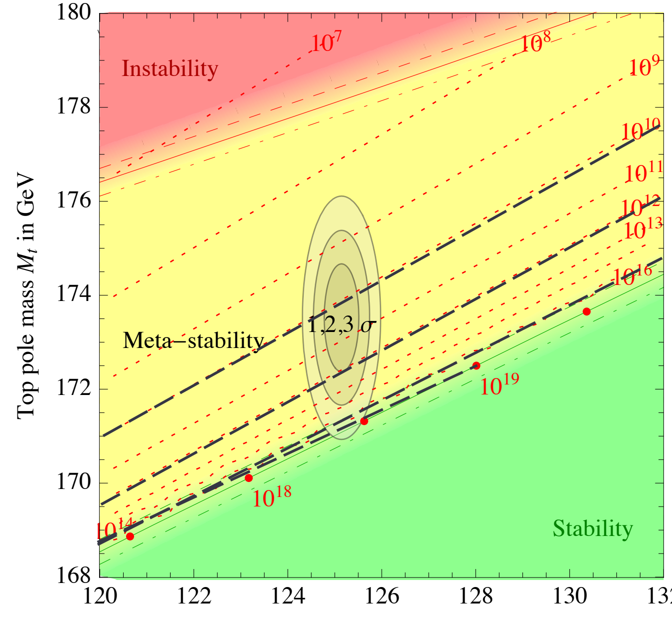
Sensitivity to new physics

Old way:
when is $\Lambda_I = \Lambda_{NP}$?



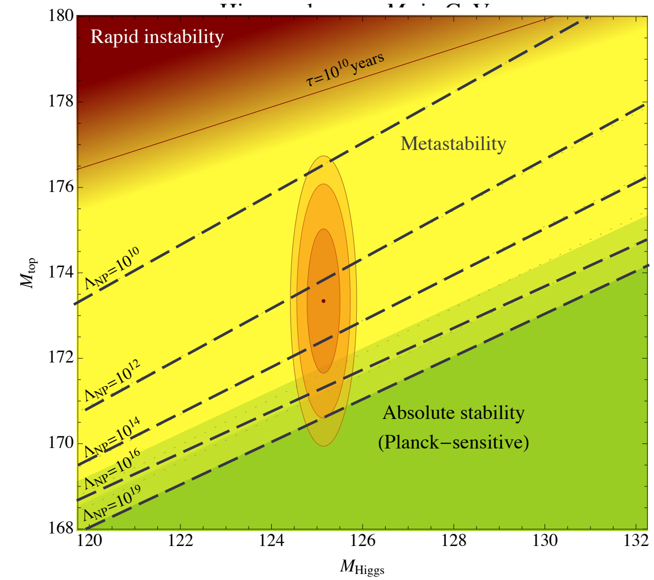
- gauge dependent, since Λ_I is gauge-dependent

From Buttazzo et al (arXiv:1307.3536)



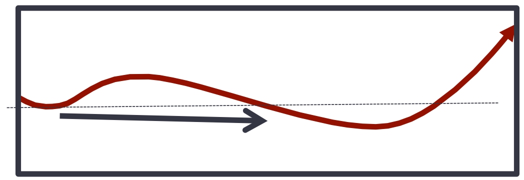
New gauge-invariant way

- Add $\mathcal{O}_6 = \frac{1}{\Lambda_{\rm NP}^2}|H|^2$ to the SM Lagrangian
- See how big Λ_{NP} must be so that V_{min} =0



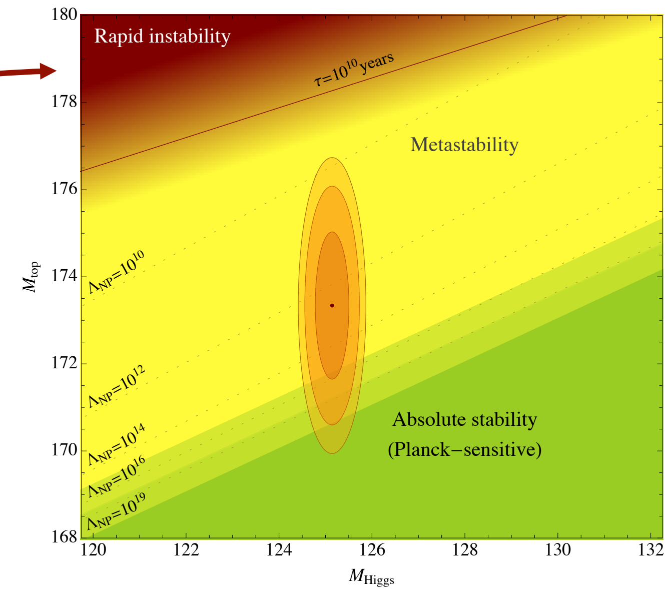
Open questions…

1. Lifetime of our universe



- Tunneling rate

$$\Gamma \sim e^{-\Gamma_{\text{eff}}(\phi_b)}$$



- Action on bounce solution formally gauge invariant
- Resummation/truncation to fixed order breaks gauge-invariance
- Is there a similar consistent perturbative calculation scheme?
- Is the rate Planck sensitive?
  - Guidice, Strumia et al (arXiv:1307.3536): minimum below Mpl, so **no**.

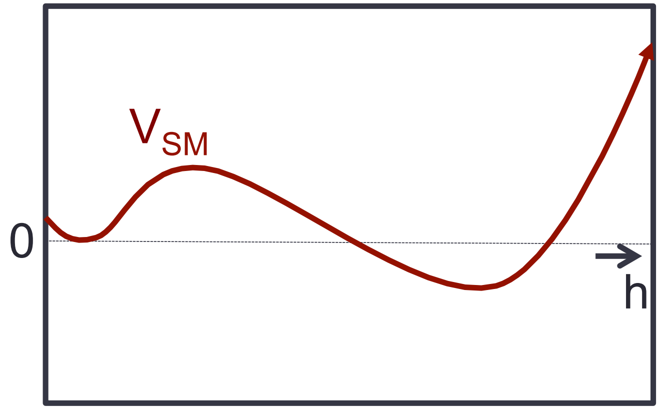
    β_λ =0 at μ = 10^{17} GeV < M_{Pl}

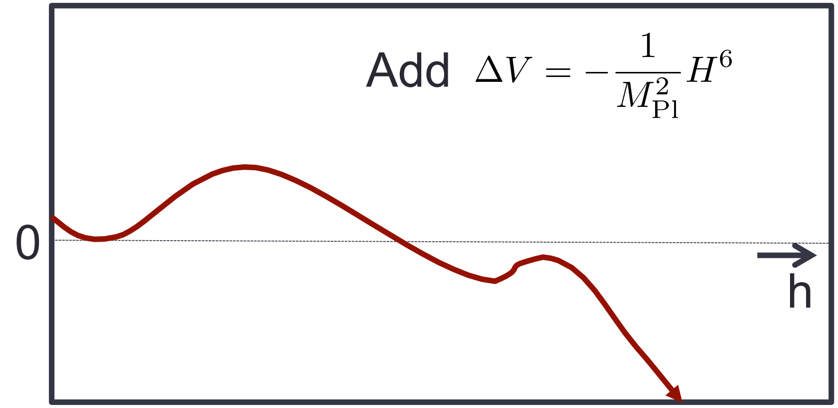
  - Sher, Brandina et al (arXiv:1408.5302):

    field at center of bubble greater than Mpl, so **yes**

$$\phi_B(r=0) = 10^{19}\text{GeV} \sim M_{\text{Pl}}$$

Metastability (work in progress)

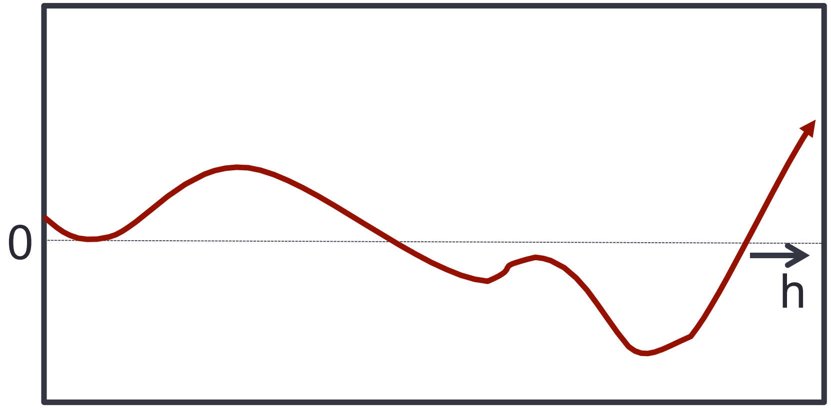




Standard Model potential

Liftetime = 10^{600} years

- **Lifetime = 0 sec**
- Arbitrarily small bubbles form and grow

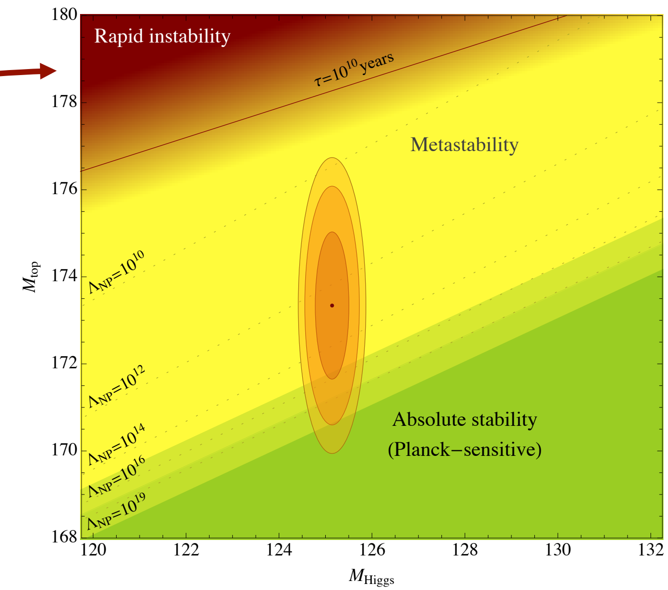


Add $\Delta V = -\alpha \frac{1}{M_{\rm Pl}^2} H^6 + \beta \frac{1}{M_{\rm Pl}^2} H^8$

- **Lifetime can be anything**!

- Planck sensitivity not due to coincidence that $\beta_\lambda = 0$ at $\mu \sim M_{Pl}$
- Tunnelling is **non-perturbative** and ***always*** UV sensitive.

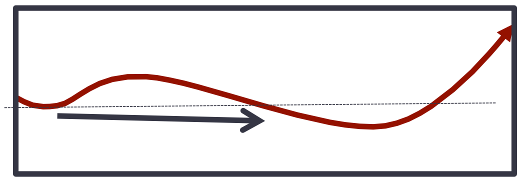
Open questions…



1. Lifetime of our universe

- Tunneling rate

$$\Gamma \sim e^{-\Gamma_{\text{eff}}(\phi_b)}$$



- Action on bounce solution formally gauge invariant
- Resummation/truncation to fixed order breaks gauge-invariance
- Is there a similar consistent perturbative calculation scheme?
- Is the rate Planck sensitive?

2. Temperature dependent potential

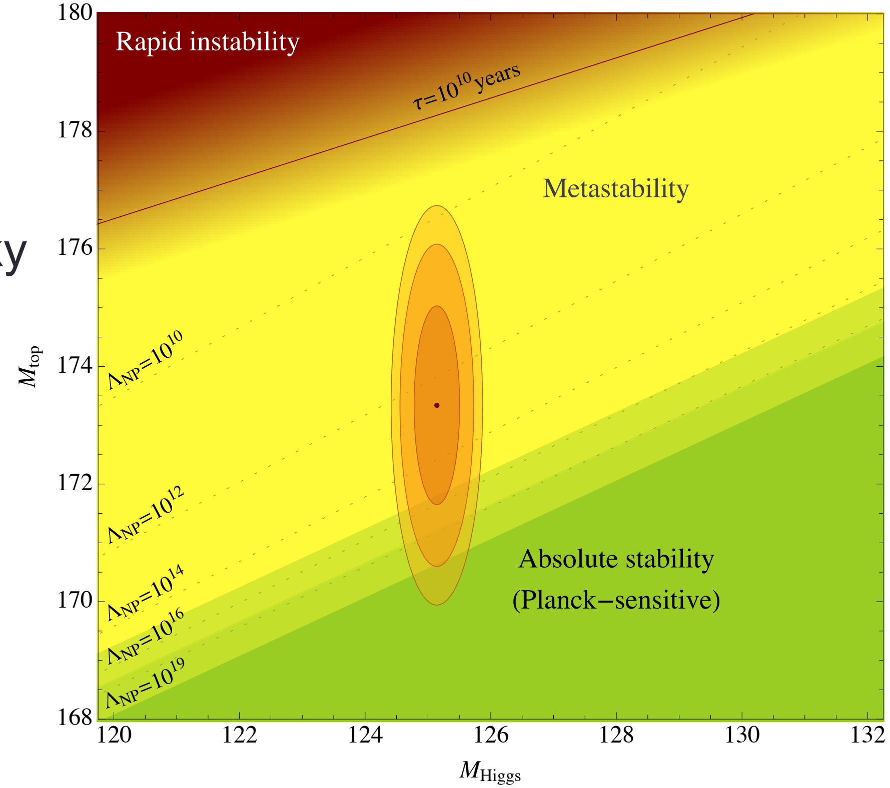
- Physical quantities also formally gauge invariant
  - Critical T: T_C
  - Transition rates
  - Gravity wave spectrum
  - …

3. Inflation

- Field values are unphysical
- What is the right way to construct short-distance models of inflation?

Conclusions

- Using effective actions consistently is tricky
- Field values ϕ are unphysical
  - Don't compare ϕ to some fixed scale
- Consistent use of perturbation theory is important



Do we know if the universe is stable?

- Our universe will probably decay, eventually.
- We don't know how long it will last