Effective String Theory in a Tank

Riccardo Penco
Columbia University

in collaboration with
Alberto Nicolis + Bart Horn
arXiv: 1507.05635

Rutgers University
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Superfluids in a Nutshell

- **Superfluid** = spontaneously broken U(1) at finite density
- Ground state $|\mu\rangle$ minimizes $H' = H - \mu Q$
- **Pattern:**

$$\left.\begin{matrix} Q \\ H \\ K_i \end{matrix}\right\} = \text{broken} \qquad \left.\begin{matrix} H' = H - \mu Q \\ P_i \\ J_i \end{matrix}\right\} = \text{unbroken}$$

Superfluids in a Nutshell

2 low-energy, model-independent features:

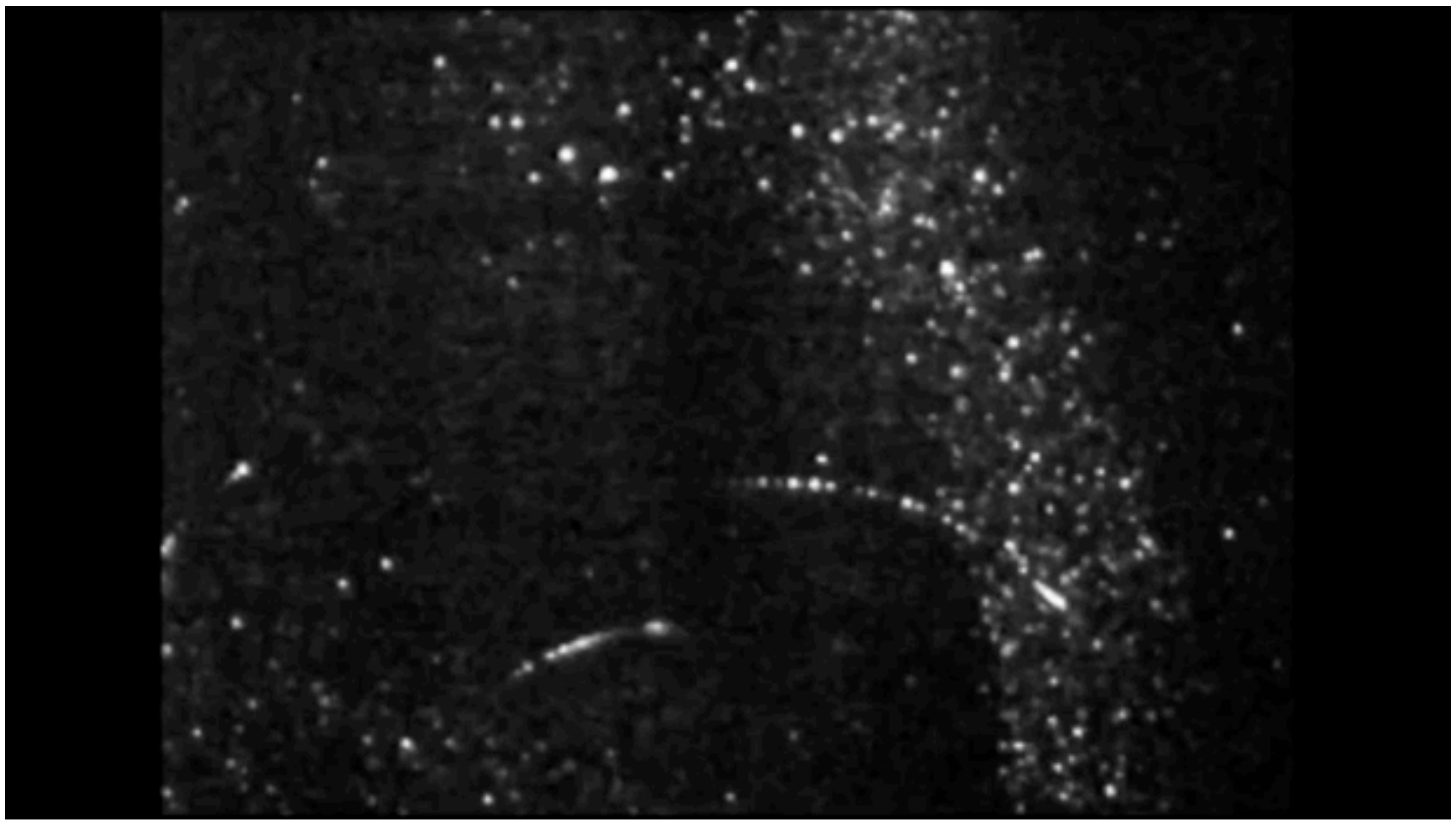
- **Goldstone boson:** phonon

$$\langle \Phi \rangle = \Phi_0 \, e^{-i\mu t}$$

- **Topological defect:** vortex line

$$\Phi = f(r) \, e^{-i\mu t + in\theta}$$

What we want to describe:



Despite appearances, vortices don’t behave like usual strings.

[Video credit: E. Fonda, D. P. Meichle, N. T. Ouellette, S. Hormoz, K. R. Sreenivasan, D. P. Lathrop]

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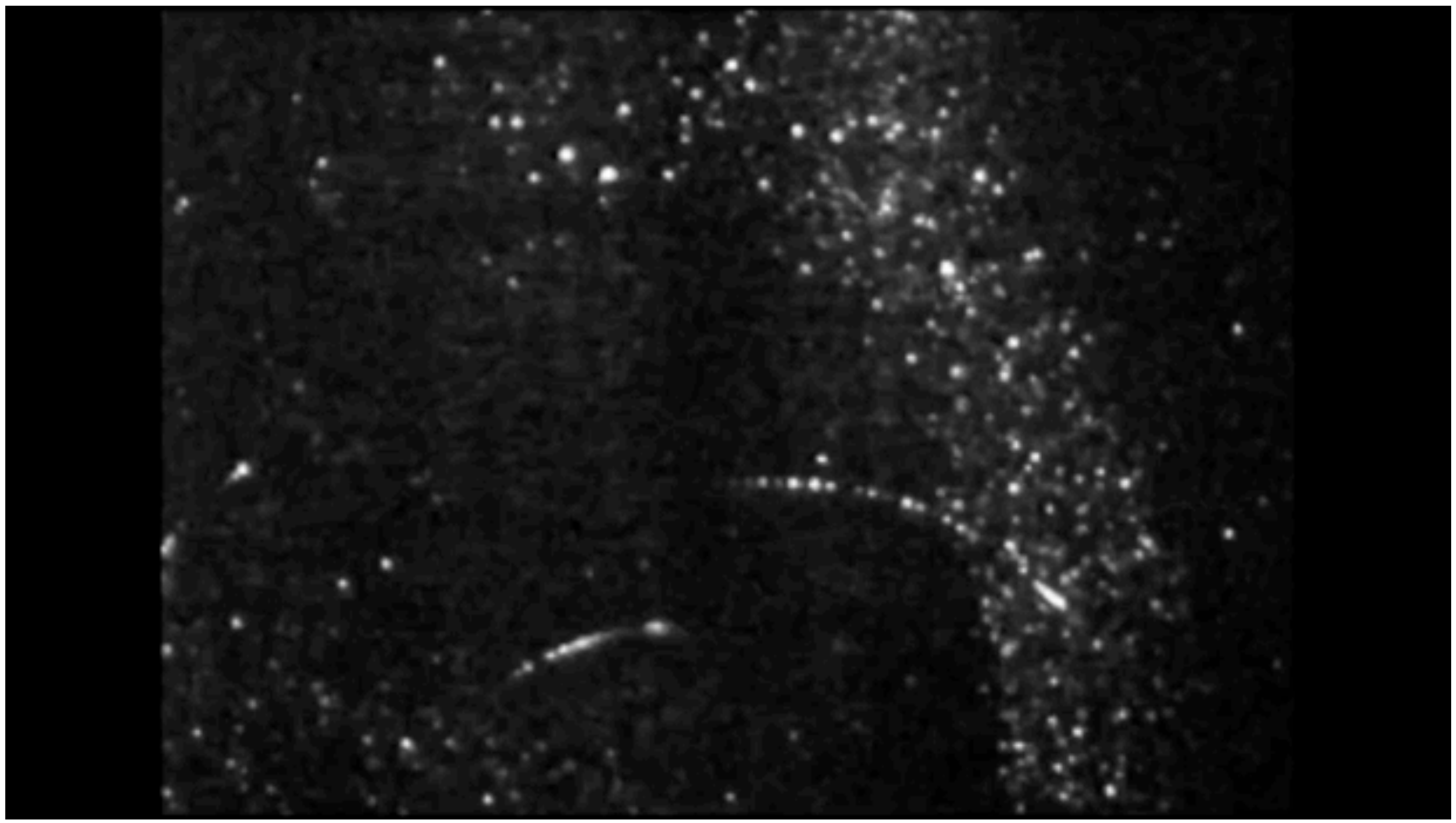

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Outline

1. Phonons

2. Vortex lines

3. Formal aspects

4. Applications

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1. **Phonons**

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EFT of Goldstone

$$\Phi = \Phi_0 \, e^{-i\phi}, \qquad \phi = \mu t + \pi$$

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- **Effective Lagrangian:**

$$\mathcal{L} = P(X), \qquad X = -\partial_\mu \phi \, \partial^\mu \phi, \qquad \phi = \mu t + \pi$$

[Son 02]

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- **Macroscopic quantities:**

$$U_\mu = -\frac{\partial_\mu \phi}{\sqrt{X}}, \qquad \rho = 2XP'(X) - P(X), \qquad p = P(X)$$

[Son 02]

Goldstone = Phonon

Expanded Lagrangian:

$$\mathcal{L} = N\left\{\dot{\pi}^2 - c_s^2(\nabla\pi)^2 + \cdots\right\}$$

with $c_s^2 = \frac{P'}{P' + 2XP''} = \frac{\partial p}{\partial \rho}$

π describes compression waves, i.e. phonons

Dual Description

$$\partial^\mu \phi \sim \epsilon^{\mu\nu\lambda\rho} \partial_\nu A_{\lambda\rho} \equiv F^\mu$$

$$\begin{cases} A_{0i} = n A_i \\ A_{ij} = n\, \epsilon_{ijk} (-\tfrac{1}{3} x^k + B^k) \end{cases}$$

[Lund + Regge 76, Zee 94]

Dual Description

- **Dual 2-form field:**

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$$U_\mu = -\frac{F_\mu}{\sqrt{Y}}, \qquad p = G(Y) - 2YG'(Y), \qquad \rho = -G(Y)$$

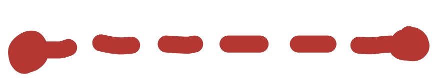
Dual Description

- Gauge invariance: $A_{\mu\nu} \to A_{\mu\nu} + \partial_{[\mu}\xi_{\nu]}$

- Coulomb gauge: $\partial_i A^{i\mu} = 0 \quad \to \quad \vec{\nabla} \cdot \vec{A} = 0, \quad \vec{\nabla} \times \vec{B} = 0$

- Propagators:

$\vec{B}$:  $= \dfrac{i}{\omega^2 - c_s^2 k^2} \hat{k}^i \hat{k}^j$ Phonon

$\vec{A}$:  $= \dfrac{i}{k^2} (\delta^{ij} - \hat{k}^i \hat{k}^j)$ Hydrophoton

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1. Phonons

2. **Vortex lines**

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EFT of Vortex Lines

EFT of Vortex Lines

- Total action: $S = S_{\rm bulk} + S_{\rm worldsheet}$
- Bulk: $S_{\rm bulk} = \int d^4x\, G(Y) + S_{\rm g.f.}\,,\qquad Y = -F^\mu F_\mu$
- Worldsheet:

symmetries: Lorentz + gauge + reparametrizations

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Nambu-Goto, right? No!

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Bi-metric Theory

on the worldsheet

- Usual induced metric: $g_{\alpha\beta} = g_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu$

- 2nd induced metric: $h_{\alpha\beta} = U_\mu U_\nu \partial_\alpha X^\mu \partial_\beta X^\nu$

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0-derivative term in bigravity: $\int d^D x \sqrt{-g}\, f(g^{\alpha\beta} h_{\beta\gamma})$

$$S_{\text{worldsheet}} = \int d\tau d\sigma \, \{\lambda \, A_{\mu\nu} \partial_\tau X^\mu \partial_\sigma X^\nu - \sqrt{-g}\, \mathcal{T}(g^{\alpha\beta} h_{\alpha\beta}, Y)\}$$

Expanded Action

Bulk

$$\mathcal{L}_{\text{bulk}} \to (\vec{\nabla} \times \vec{A})^2 + \dot{\vec{B}}^{\,2} - c_s^2(\vec{\nabla} \cdot \vec{B})^2 + (1 - c_s^2)\vec{\nabla} \cdot \vec{B}(\vec{\nabla} \times \vec{A})^2 + \cdots$$

Worldsheet $(\tau = X^0)$

$$\mathcal{L}_{\text{worldsheet}} \to n\lambda \left[-\tfrac{1}{3}\vec{X} \cdot \dot{\vec{X}} \times \vec{X}' + \vec{A} \cdot \vec{X}' + \vec{B} \cdot \dot{\vec{X}} \times \vec{X}' \right]$$

$$+|\vec{X}'| \left[-T + T_{(10)}(\dot{\vec{B}} - \vec{\nabla} \times \vec{A})_\perp \cdot \dot{\vec{X}}_\perp + T_{(01)}\vec{\nabla} \cdot \vec{B} + \cdots \right]$$

Expansion Parameters

in superfluids

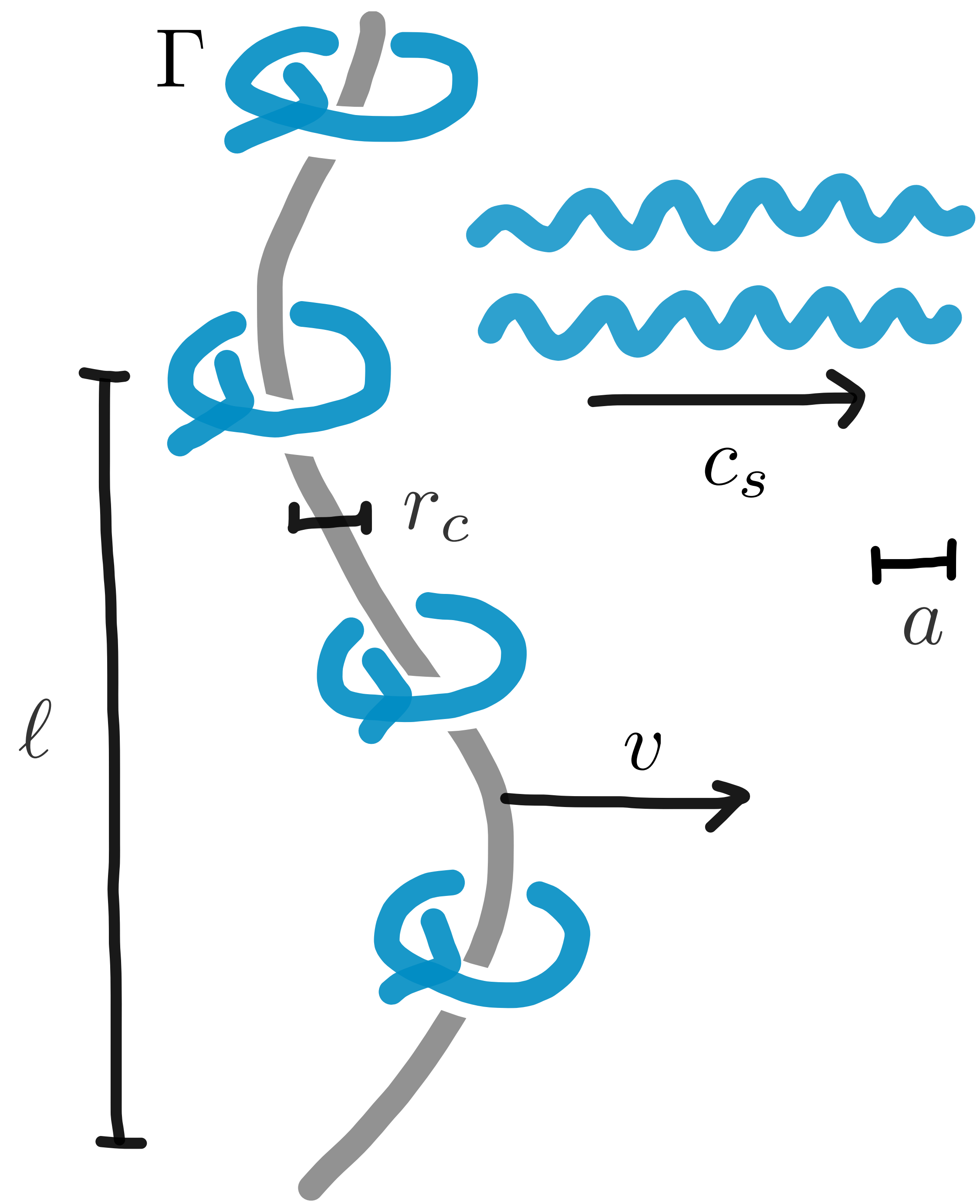
Lengths: $a, \ r_c, \ \ell$

Velocities: $v, \ c_s, \ c$

$$v \sim \frac{\Gamma}{\ell} \sim \frac{c_s a}{\ell},$$

$$r_c \sim a$$

→ $\boxed{\dfrac{v}{c_s}, \ \dfrac{c_s}{c}}$



Small-speed Approximations

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- Non-relativistic limit: $c \gg c_s, v$

$$S^{\rm nr}_{\rm bulk} \to \int d^4x \left[\frac{m\, n_{\rm nr} \vec{u}^2}{2} - U(n_{\rm nr})\right]$$

$$\begin{cases} n_{\rm nr} = n(1 - \nabla \cdot B) \\ \vec{u} = \dfrac{\dot{\vec{B}} - \vec{\nabla} \times \vec{A}}{1 - \vec{\nabla} \cdot \vec{B}} \end{cases}$$

$$S^{\rm nr}_{\rm worldsheet} \to \int dt d\sigma \left[\lambda\, A_{\mu\nu} \dot{X}^\mu X^{\nu\prime} - |\vec{X}'| \mathcal{T}(|\dot{\vec{X}}_\perp - \vec{u}_\perp|, n_{\rm nr})\right]$$

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- Incompressible limit: $c, c_s \gg v$

$$S \to \int d^4x \tfrac{1}{2} (\nabla \times A)^2 + \int dt d\sigma \left[-\tfrac{n\lambda}{3} \vec{X} \cdot \dot{\vec{X}} \times \vec{X}' + n\lambda \vec{A} \cdot \vec{X}' - T|\vec{X}'| \right]$$

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circulation $\Gamma = n\lambda$

$(\nabla \times v = \Gamma\delta)$

$$S \to \int dt d\sigma \left\{ -\frac{n\lambda}{3} \vec{X} \cdot \dot{\vec{X}} \times \partial_\sigma \vec{X} - T|\partial_\sigma \vec{X}| - \frac{n^2\lambda^2}{8\pi} \int d\sigma' \frac{\partial_\sigma \vec{X} \cdot \partial_{\sigma'} \vec{X}'}{|\vec{X} - \vec{X}'|} \right\}$$

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Expanded Action

Bulk

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Self-energy

- Integrate out bulk modes:

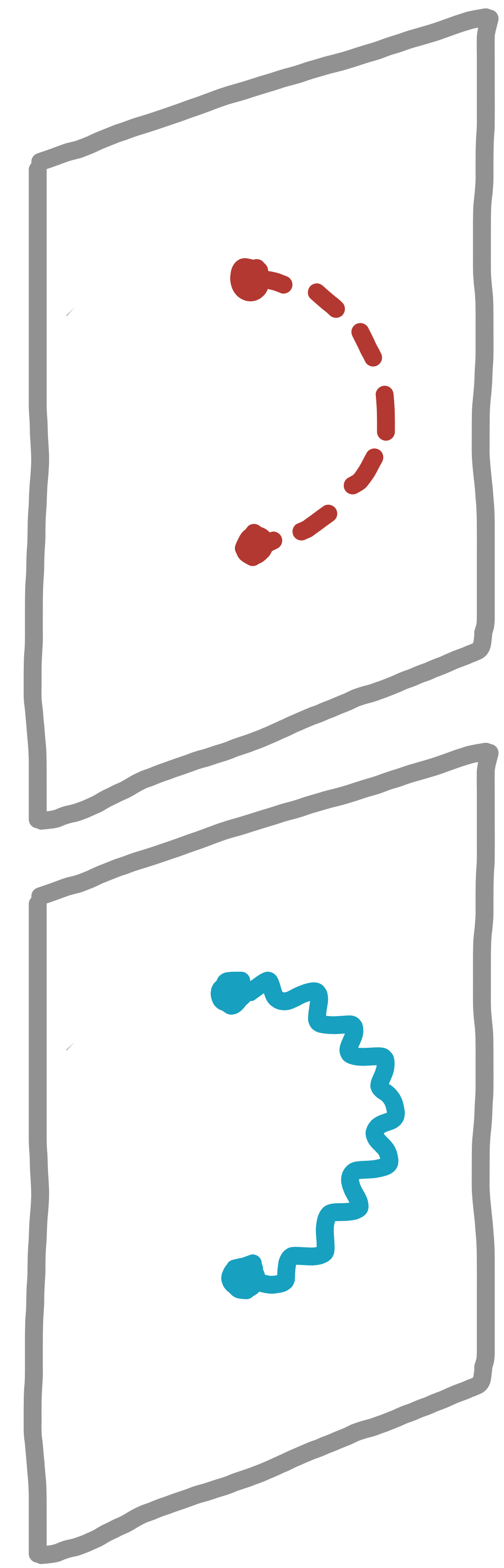
$$e^{iS_{\rm eff}[X]} = \int \mathcal{D}A\,\mathcal{D}B\, e^{iS[X,A,B]}$$

- Self-energy of static straight vortex:

$$S_{\rm eff}[X] = -\int dt d\sigma\, \frac{dE}{d\sigma}$$

- Energy / length:

$$\frac{dE}{d\sigma} = T + \frac{n^2\lambda^2}{2}\int \frac{d^2p_\perp}{(2\pi)^2\, p_\perp^2} - \frac{T_{(01)}^2}{2c_s^2}\int \frac{d^2p_\perp}{(2\pi)^2}$$



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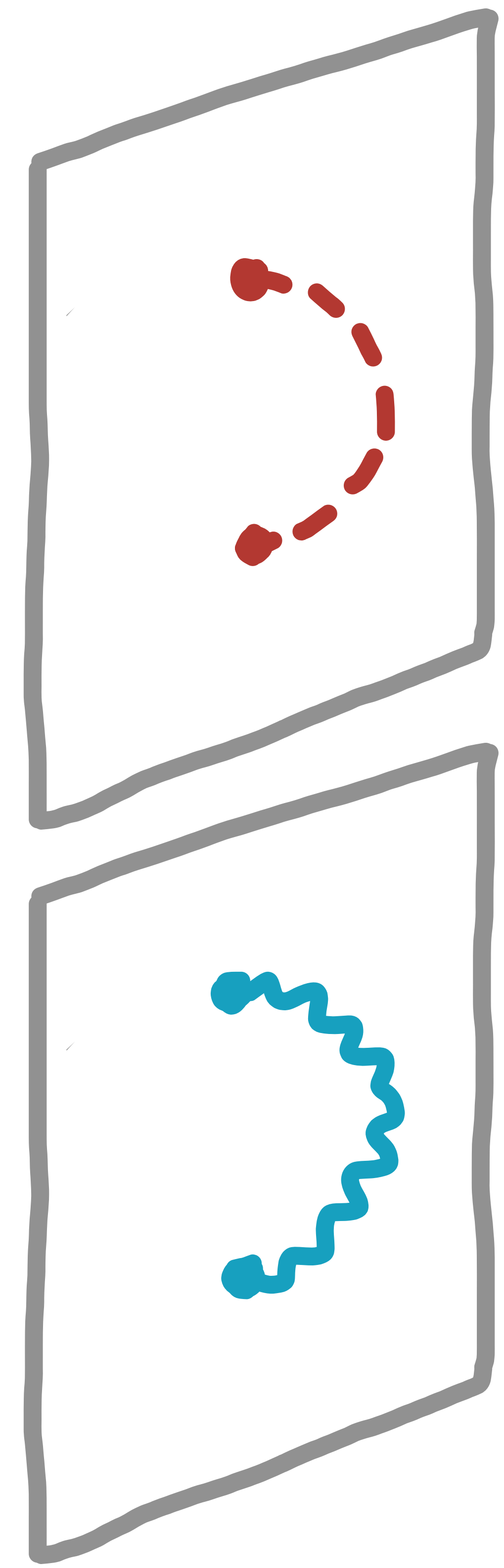
$$\frac{dE}{d\sigma} = T - \frac{n^2\lambda^2}{8\pi}\left[\frac{2}{\varepsilon} + \gamma_E - \log 4\pi - 2\log(\mu L)\right]$$



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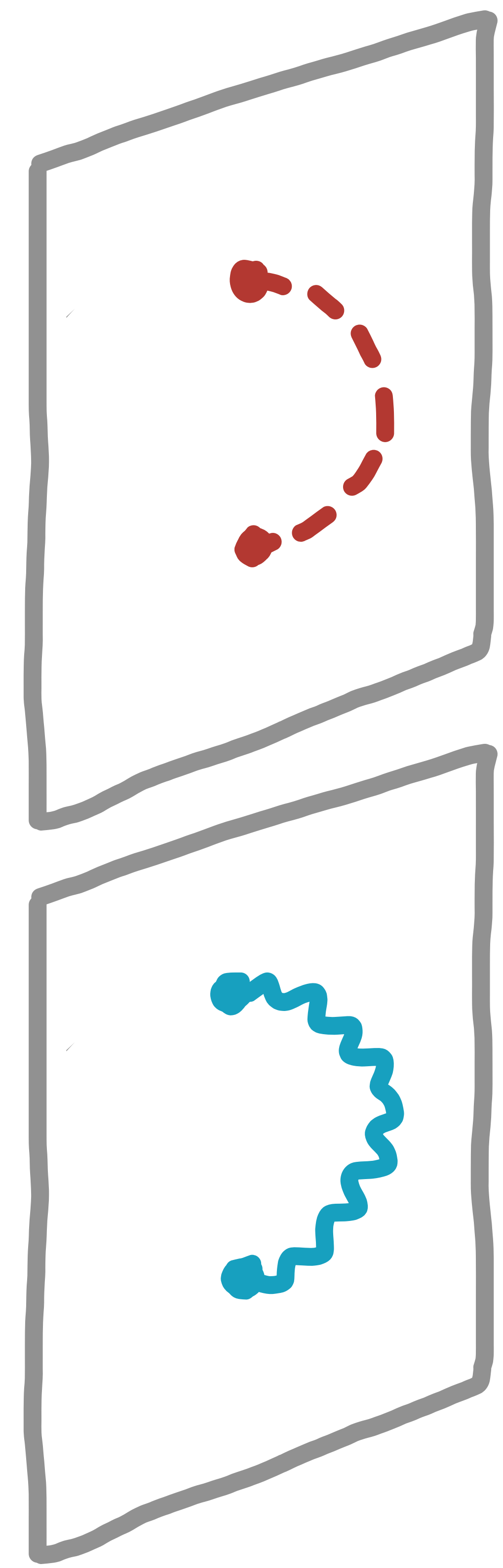

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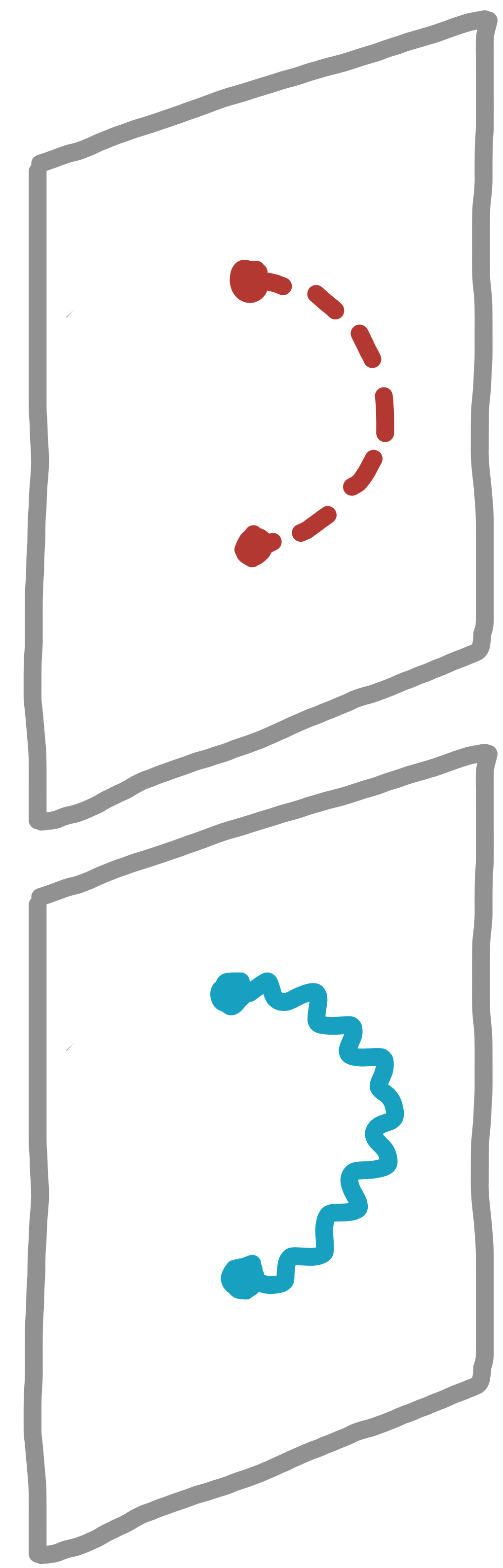
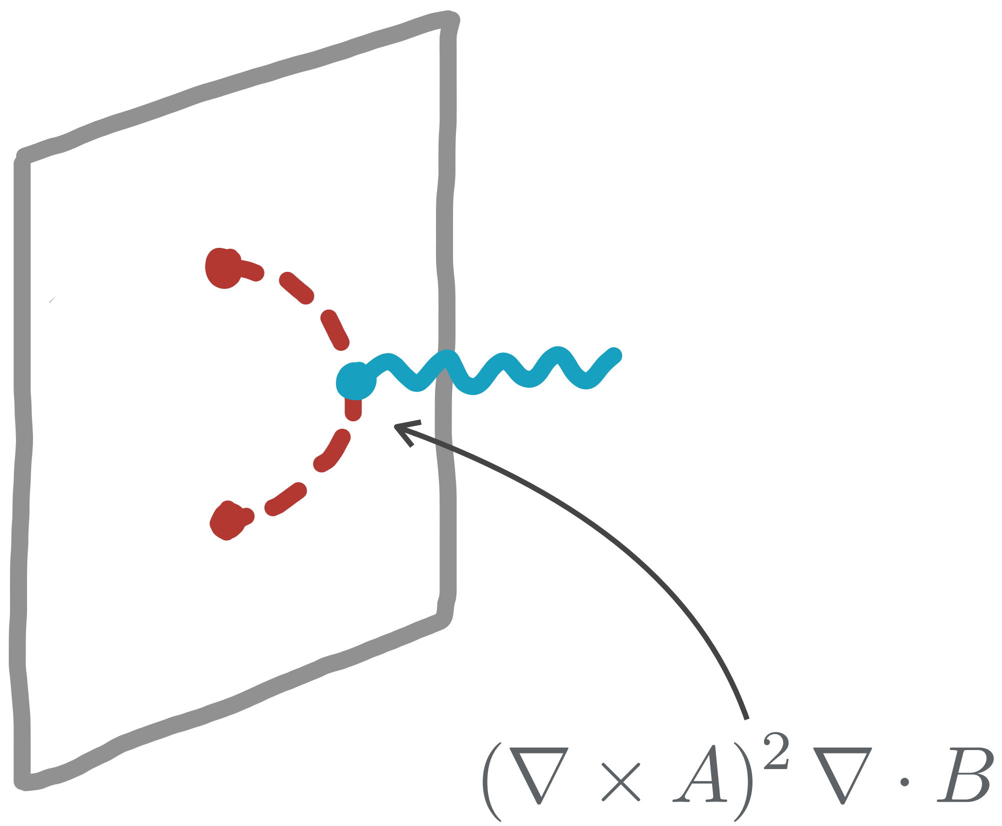
- Energy / length:

$$\frac{dE}{d\sigma} = T(1/L)$$



Coupling to Phonons



$$S_{\text{eff}} \supset \int dt d\sigma \, |\vec{X}'| \Big\{ T_{(01)} - \frac{n^2 \lambda^2}{8\pi} (1 - c_s^2) \Big[\frac{2}{\varepsilon} + \gamma_E - \log 4\pi - 2 \log(\mu / q_\perp) \Big] \Big\} \vec{\nabla} \cdot \vec{B}$$

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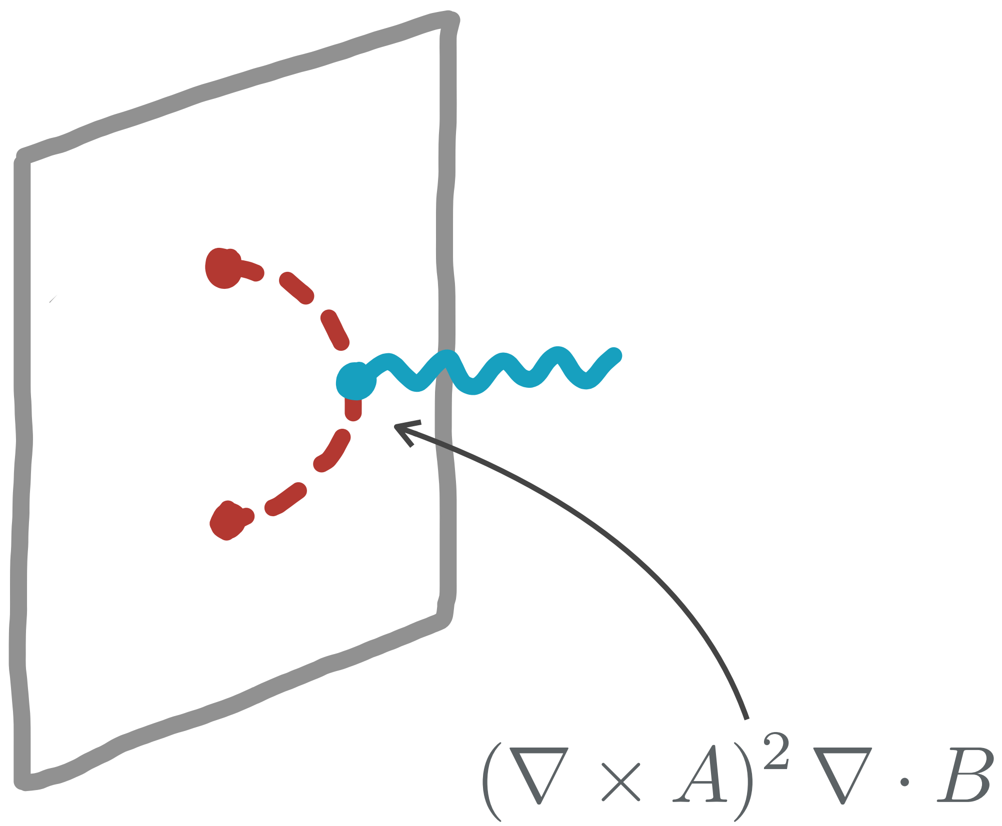

$$S_{\rm eff} \supset \int dt d\sigma \, |\vec{X}'| \Big\{ T_{(01)}(\mu) + \frac{n^2 \lambda^2}{8\pi} (1 - c_s^2) \log(\mu / q_\perp) \Big] \Big\} \, \vec{\nabla} \cdot \vec{B}$$

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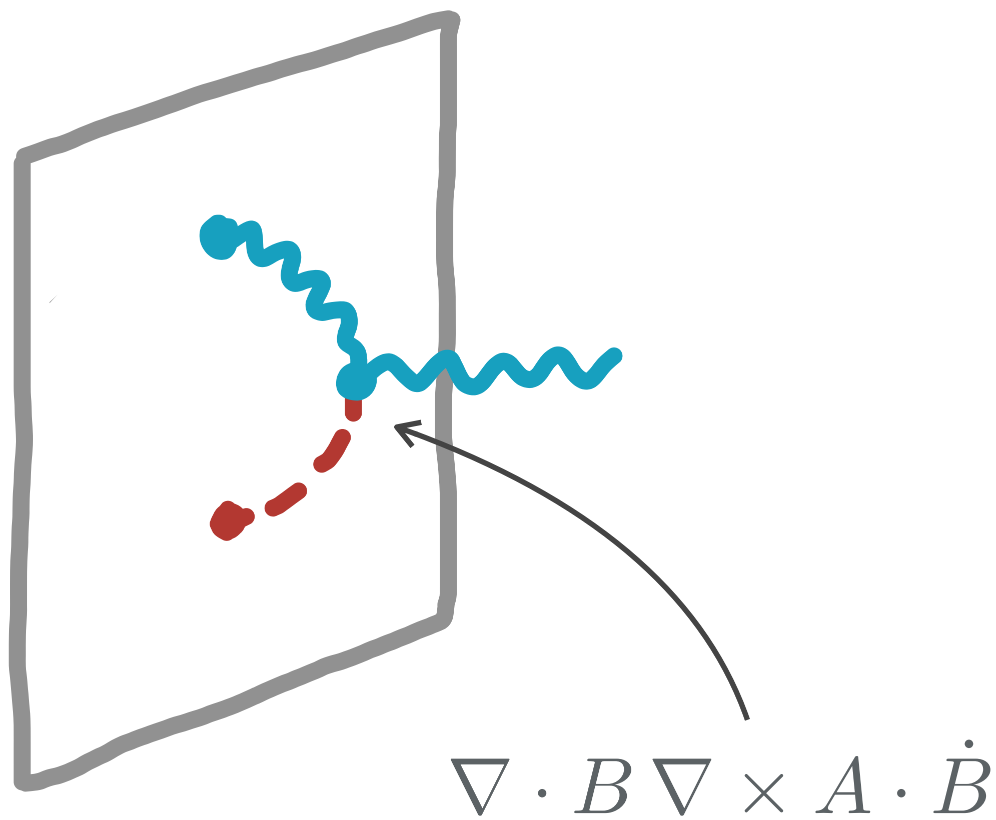
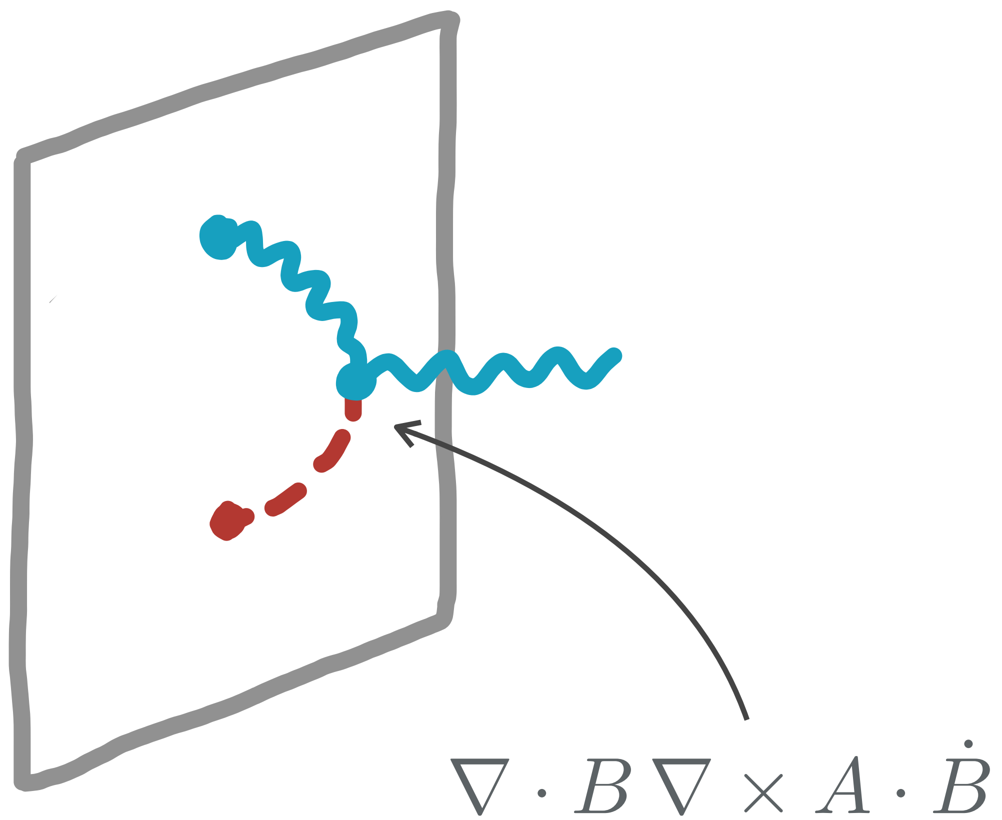


$$S_{\rm eff} \supset \int dt d\sigma \, |\vec{X}'| \Big\{ T_{(10)} + \frac{n^2 \lambda^2}{8\pi c_s^2}(1 - c_s^2)\Big[\frac{2}{\varepsilon} + \gamma_E - \log 4\pi - 2\log(\mu/q_\perp)\Big]\Big\}(\dot{\vec{B}}_\perp \cdot \dot{\vec{X}}_\perp)$$

Coupling to Phonons



$$S_{\text{eff}} \supset \int dt d\sigma \, |\vec{X}'| \Big\{ T_{(10)}(\mu) - \frac{n^2 \lambda^2}{4\pi c_s^2}(1 - c_s^2) \log(\mu/q_\perp) \Big\} (\dot{\vec{B}}_\perp \cdot \dot{\vec{X}}_\perp)$$

Non-renormalization Theorem

λ

$$S \supset \int d\tau d\sigma \lambda \, A_{\mu\nu} \partial_\tau A^\mu \partial_\sigma A^\nu$$

Non-renormalization Theorem

λ is the only coupling on the worldsheet that does not run.

In fact, it does not get renormalized at all.

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Hint: from a UV perspective, we know that vortex lines have a quantized circulation, which implies that $\lambda = \Gamma/n$ is quantized in units of 2π.

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Hint: from a UV perspective, we know that vortex lines have a quantized circulation, which implies that $\lambda = \Gamma/n$ is quantized in units of 2π.

Proof: consider perturbations around straight line: $\vec{X} = (\vec{\pi}, z)$

$$\mathcal{L}_\lambda = \frac{n}{2} \epsilon_{ab} \pi^b \partial_t \pi^a + \tilde{A}_z + \tilde{A}_a \partial_z \pi^a + \epsilon_{ba} \tilde{B}^b \partial_t \pi^a + \epsilon_{ab} \tilde{B}^z \partial_t \pi^a \partial_z \pi^b + \cdots$$

Main Message

We can describe the classical interaction of vortex lines
with sound using just a local worldsheet action
in which all but one coupling constants are running.

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2. Vortex lines

3. Formal aspects

4. **Applications**

Kelvin Waves

$$\vec{X} = (\vec{\pi}, z) \quad \rightarrow \quad \phi = \tfrac{1}{\sqrt{2}}(\pi_x + i\pi_y)$$

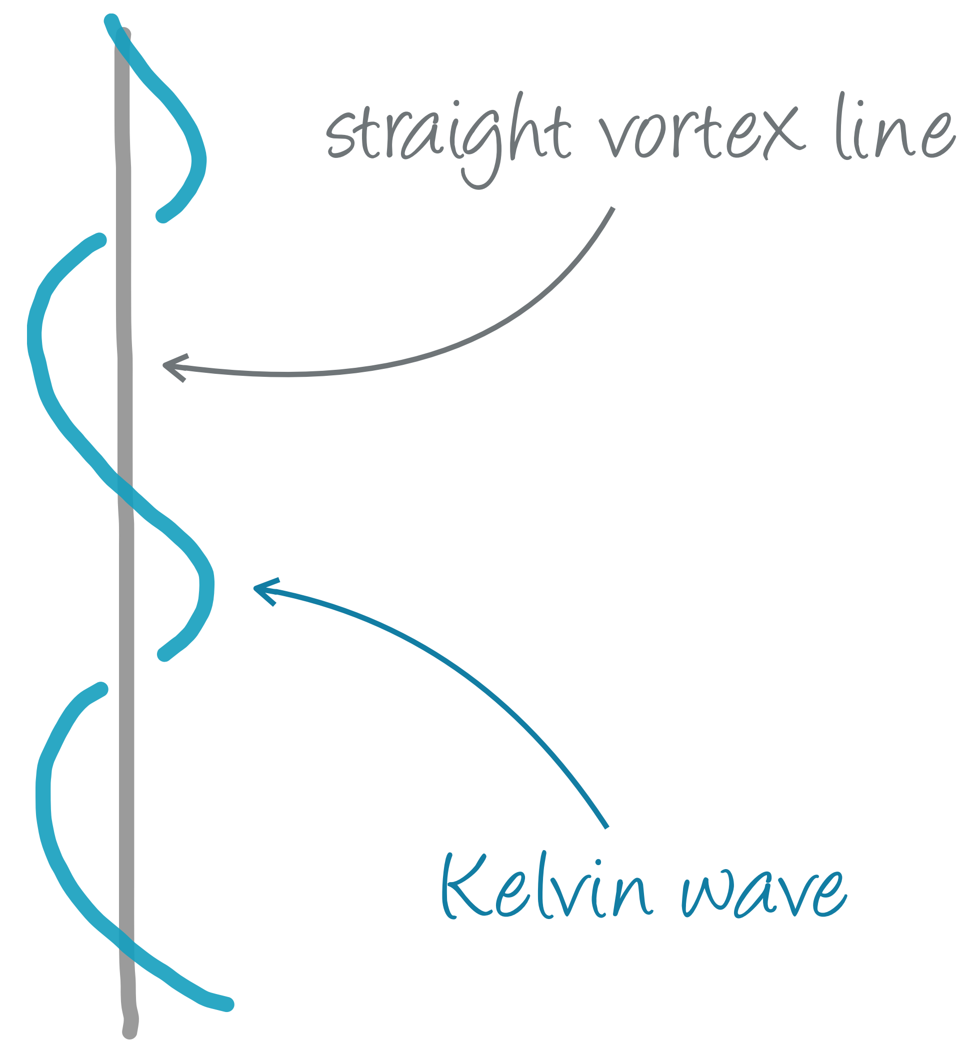
Effective Lagrangian:

$$\mathcal{L}_{\text{eff}} = i\, n\lambda\, \phi^* \partial_t \phi - T(k)\sqrt{1 + 2|\partial_z \phi|^2}$$

with $T(k) = -\dfrac{n^2\lambda^2}{4\pi} \log(k/\mu_0)$

Look for wave solutions:

$$\phi(t, z) = R\, e^{-i\omega t + ikz}$$

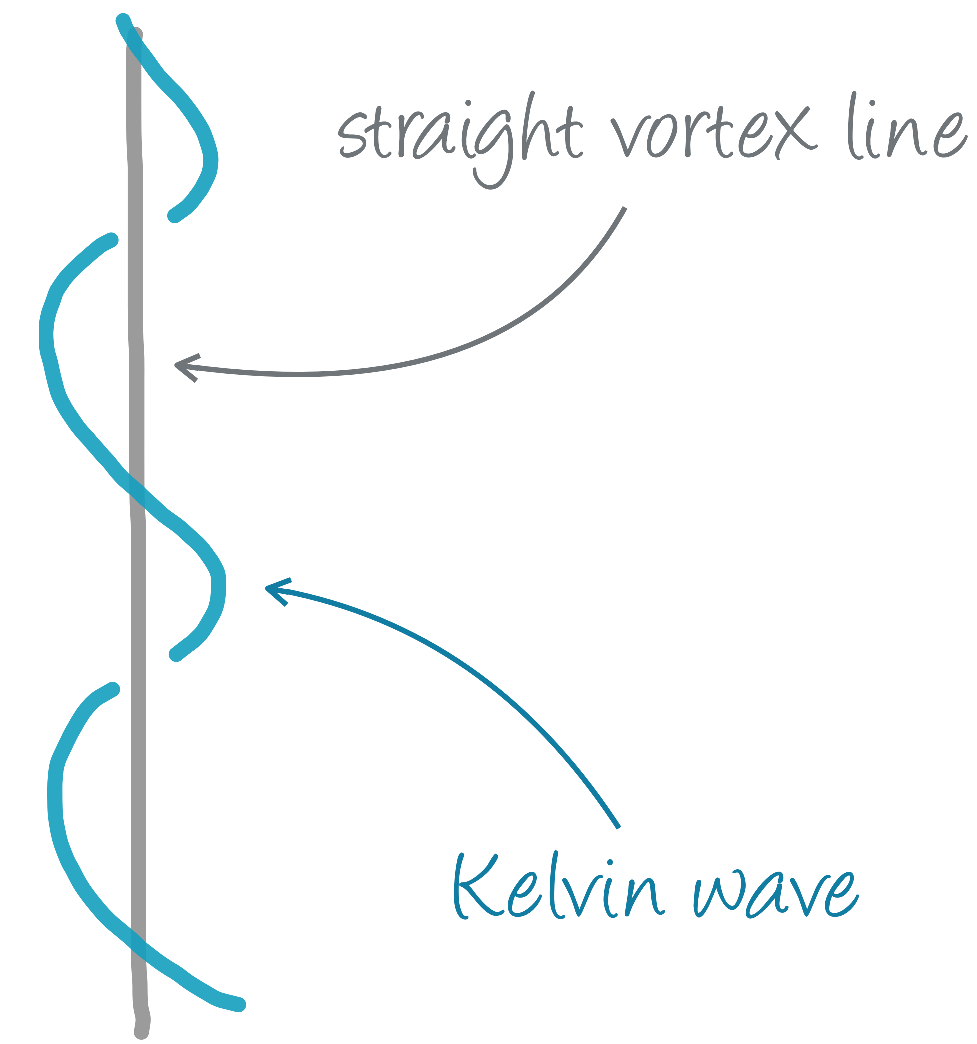


Kelvin Waves

$$\mathcal{L}_{\text{eff}} = i\,n\lambda\,\phi^*\partial_t\phi - T(k)\sqrt{1+2|\partial_z\phi|^2}$$

$$Rk \ll 1$$

$$\omega = -\frac{n\lambda}{4\pi}\,k^2\log(k/\mu_0)$$



[Thomson aka Kelvin, 1880]

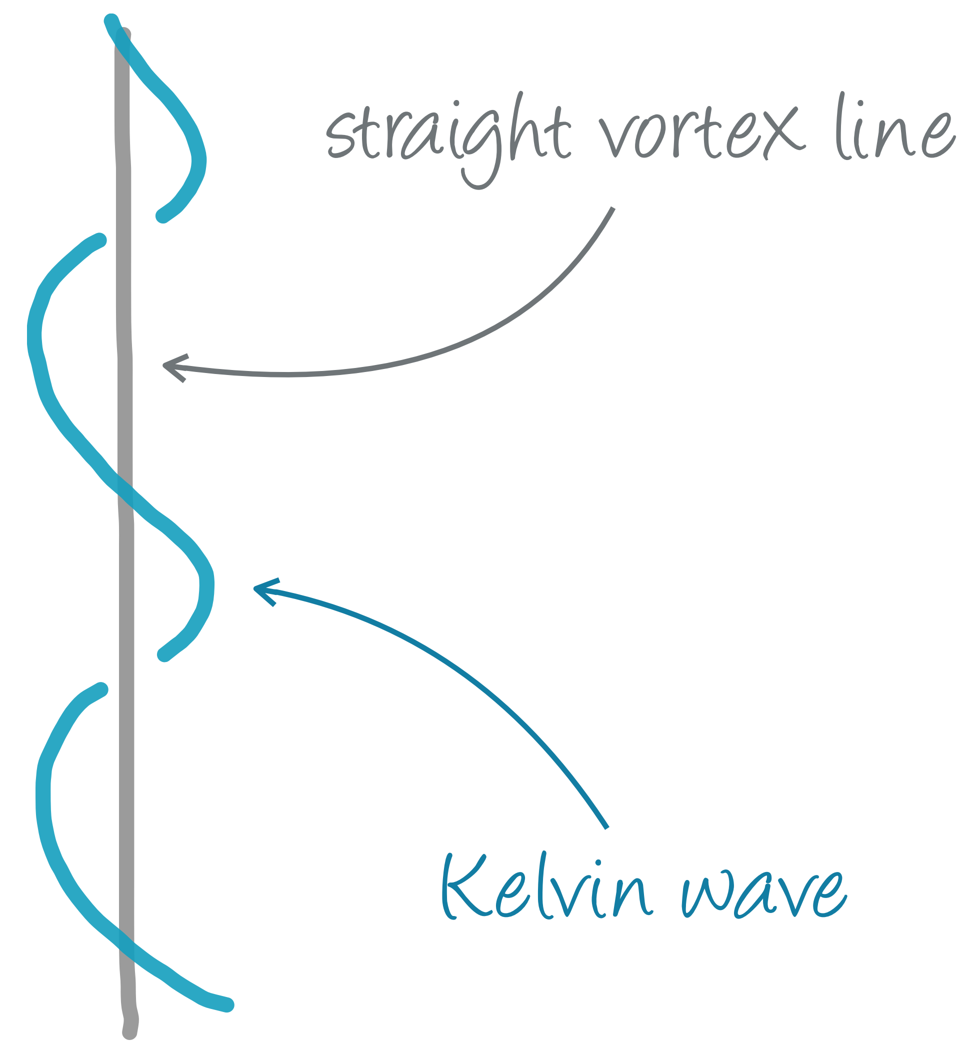
Kelvin Waves

$$\mathcal{L}_{\rm eff} = i\, n\lambda\, \phi^* \partial_t \phi - T(k)\sqrt{1 + 2|\partial_z \phi|^2}$$

Two regimes:

1. small amplitude: $Rk \ll 1$

$$\omega = -\frac{n\lambda}{4\pi}\, k^2 \log(k/\mu_0)$$



[Thomson aka Kelvin, 1880]

Kelvin Waves

$$\mathcal{L}_{\text{eff}} = i\,n\lambda\,\phi^*\partial_t\phi - T(k)\sqrt{1+2|\partial_z\phi|^2}$$

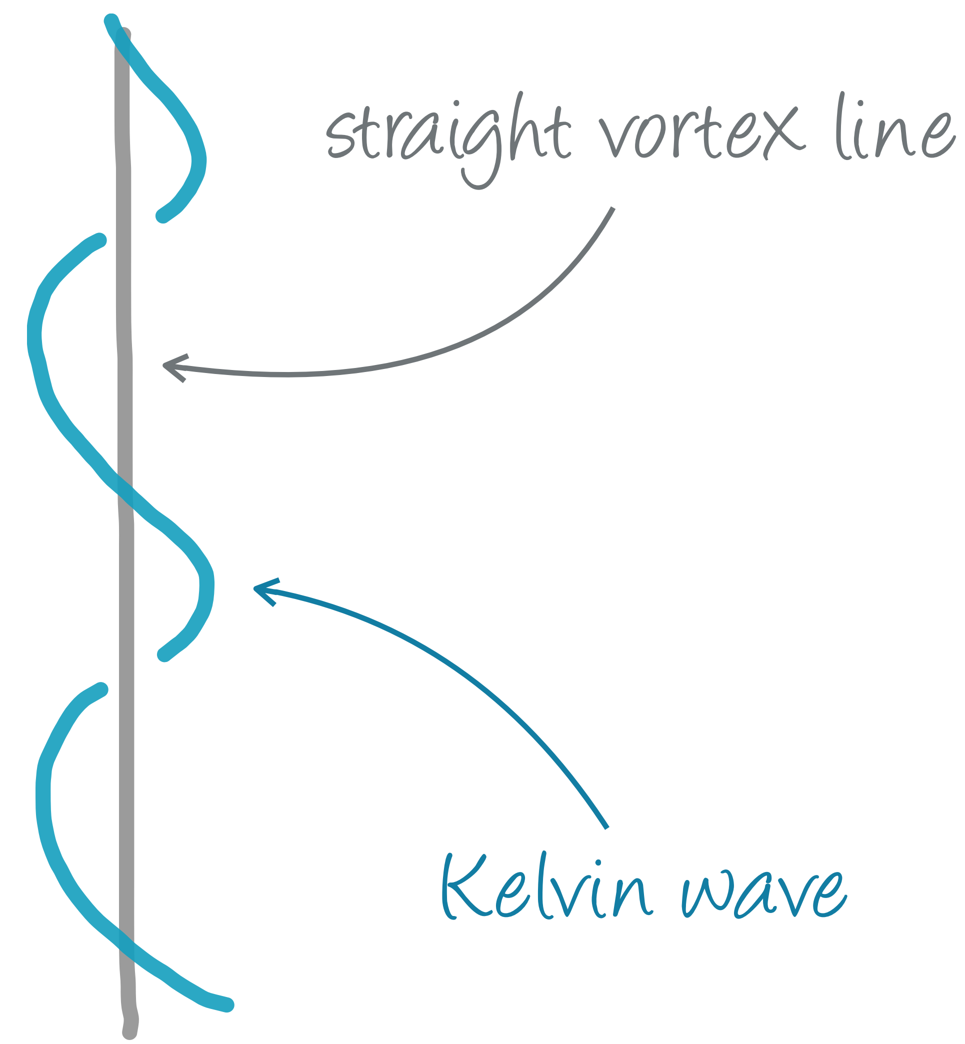
Two regimes:

1. small amplitude: $Rk \ll 1$

$$\omega = -\frac{n\lambda}{4\pi}\,k^2\log(k/\mu_0)$$

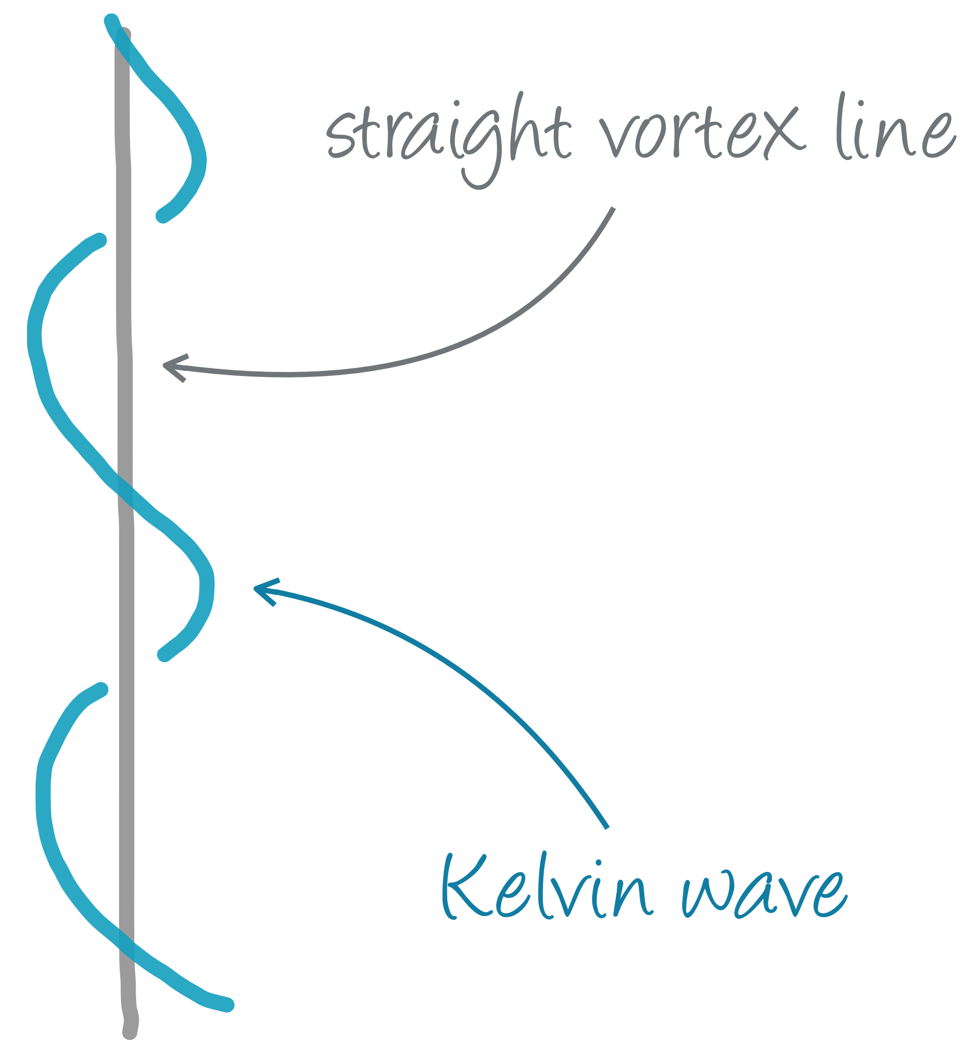
2. large amplitude: $Rk \gg 1$

$$\omega = -\frac{n\lambda}{4\pi}\,\frac{k}{R}\,\log(k/\mu_0)$$



Kelvin Waves

$$\mathcal{L}_{\text{eff}} = i\, n\lambda\, \phi^* \partial_t \phi - T(k)\sqrt{1 + 2|\partial_z \phi|^2}$$



Two regimes:

1. small amplitude: $Rk \ll 1$

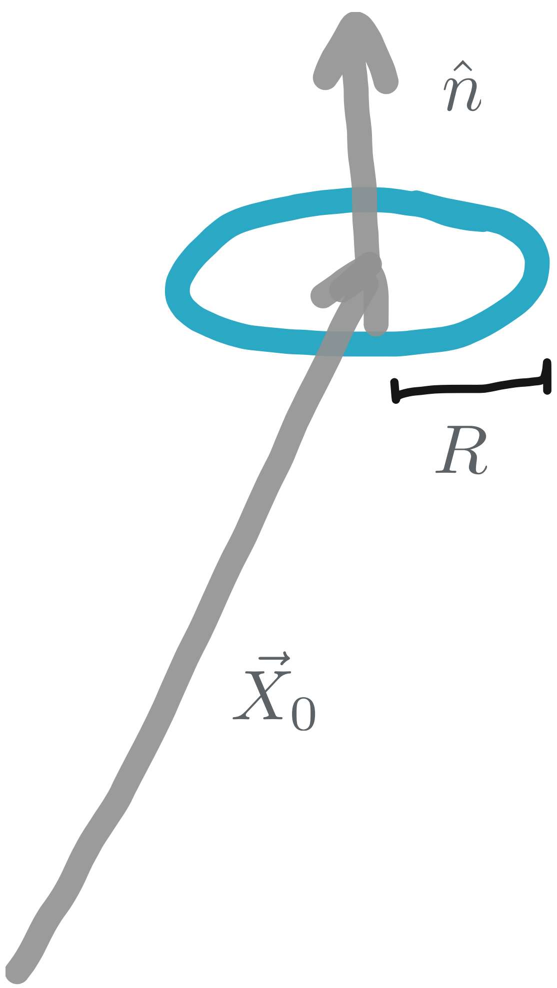
$$\omega = -\frac{n\lambda}{4\pi}\, k^2 \log(k/\mu_0)$$

2. large amplitude: $Rk \gg 1$

$$\omega = -\frac{n\lambda}{4\pi}\, \frac{k}{R}\, \log(k/\mu_0)$$

"self-pipe"

Vortex Rings



$$\vec{X}(t,\sigma) = \vec{X}_0(t) + \delta\vec{X}(\sigma)$$

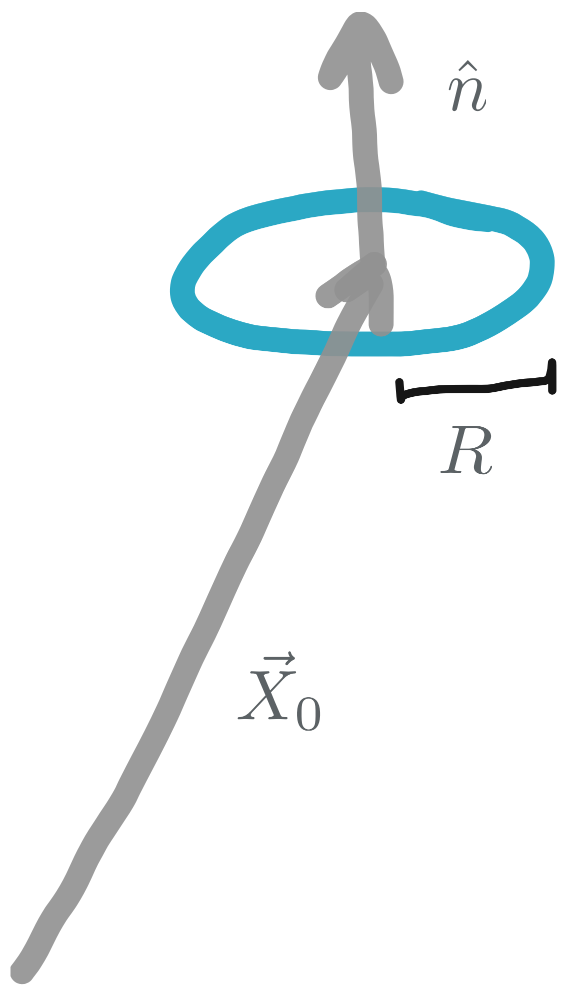
Effective Lagrangian (incompressible):

$$S_{\text{eff}} \simeq \int dt \left[\lambda \bar{n}\, \pi R^2 \hat{n} \cdot \dot{\vec{X}}_0 - 2\pi R\, T(1/R) \right]$$

Equations of motion:

$$\begin{cases} \partial_t \vec{X}_0^{\perp} = 0 \\ \partial_t (R^2 \hat{n}) = 0 \\ \hat{n} \cdot \partial_t \vec{X}_0 = f(R) \end{cases}$$

Vortex Rings



Effective Lagrangian (incompressible):

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Equations of motion:

$$\begin{cases} \partial_t \vec{X}_0^{\perp} = 0 \\ \partial_t (R^2 \hat{n}) = 0 \\ \hat{n} \cdot \partial_t \vec{X}_0 = f(R) \end{cases}$$

$$\vec{X}(t,\sigma) = \vec{X}_0(t) + \delta \vec{X}(\sigma)$$

$$S_{\text{int}} = \bar{n} \lambda \pi R^2 \int dt \left\{ -\partial_t \vec{B} + \vec{\nabla} \times \vec{A} - \dot{\vec{X}}_0 \left(\vec{\nabla} \cdot \vec{B}\right) \right\} \cdot \hat{n} + \cdots$$

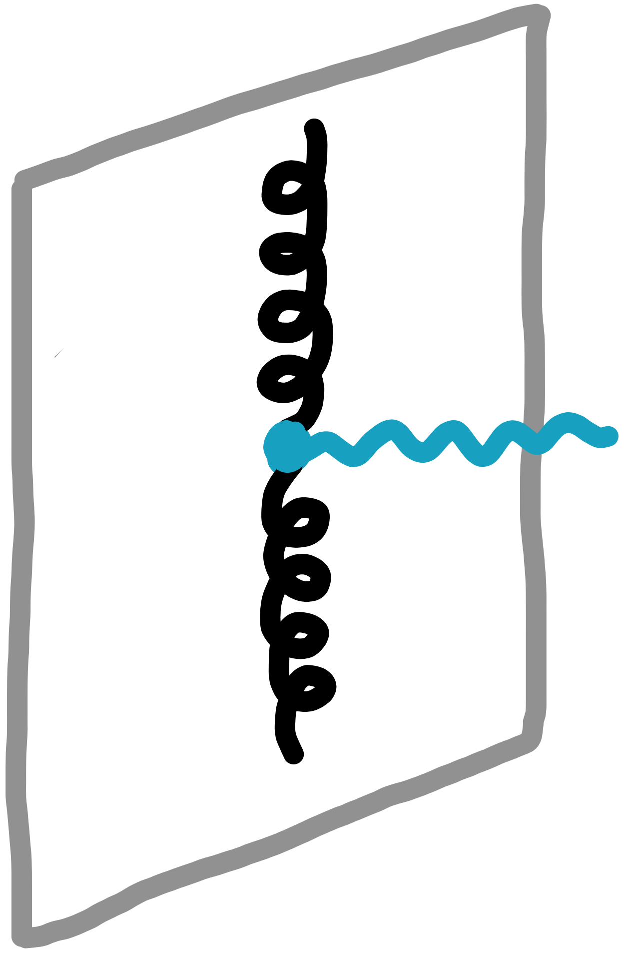
Phonon Absorption

- Leading coupling:

$$S \supset \int dt dz \, n\lambda \, \epsilon_{ab} \pi^a \pi^c \partial_c B^b$$

- Cross-section:

$$\sigma \simeq \mathcal{N} \frac{\sin^4 \theta}{c_s^3} \frac{\omega^{5/2}}{\sqrt{\log(\omega/\omega_0)}}$$



Concluding Remarks

- Our EFT applies also to ordinary fluids*
- Applications outside CM: neutron stars, dark matter, …
- Can we use what we have learned to better understand rotons?

* terms and conditions may apply [Endlich + Nicolis 13]

Thank you,

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