A non-Fermi Liquid for htsc

Andre LeClair
Cornell University

Rutgers, April 7 2009

BASED ON:

* arXiv:0805.4182 with E. Kapit, J. Phys. A. 42(2009)
(so(5) symmetry, d-wave gap eqn.)

* H. Tye arXiv:0804.4200 (pseudogap)

* arXiv:0903.2484 with E. Kapit (pseudogap)

* with D. Robinson, to appear. (resistivity)

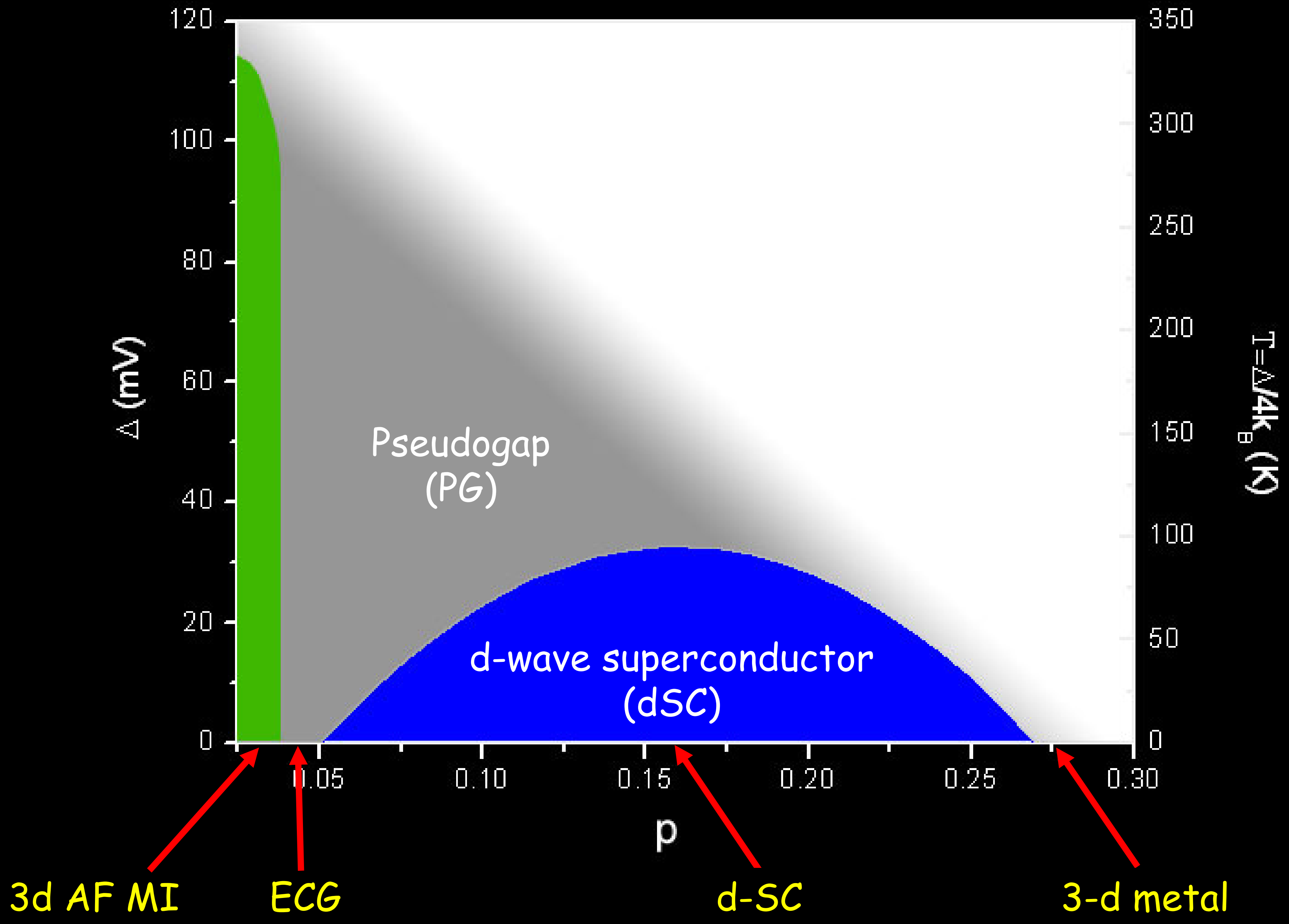
* JHEP 10 (2007) 027 with M. Neubert (2-loop RG)

* preliminary work: AL, arXiv:cond-mat/0610639, 0610816 (unpublished).

Outline

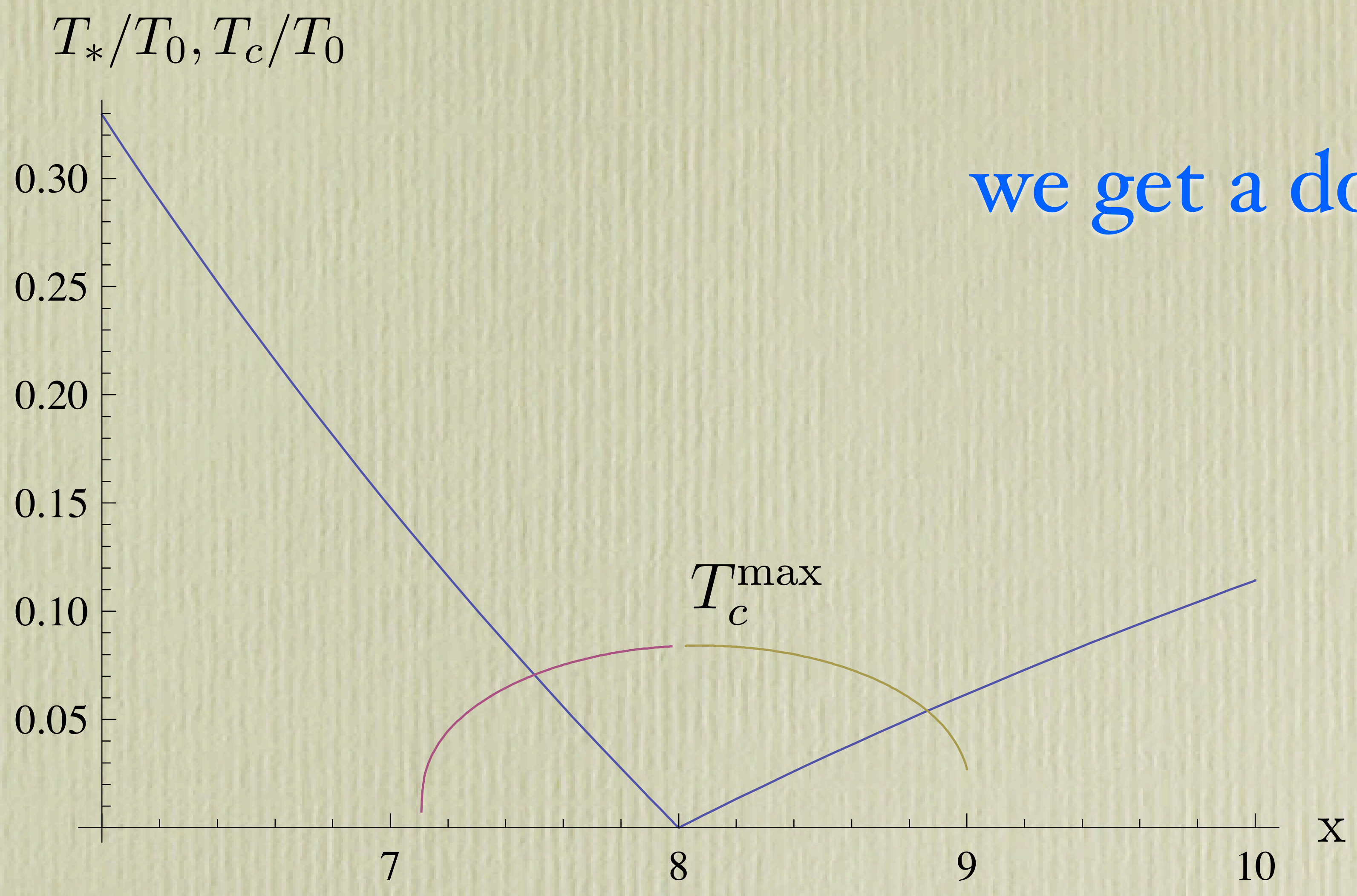
- Review of what we know about HTSC.
- Our model of scalar ``symplectic” fermions.
- Renormalization group and doping.
- Pseudogap
- Resistivity
- Specific heat
- d-wave gap equation and Tc

Schematic phase diagram of hole-doped cuprates



(courtesy of Seaumus Davis)

Numerical solutions to Tc and T*:



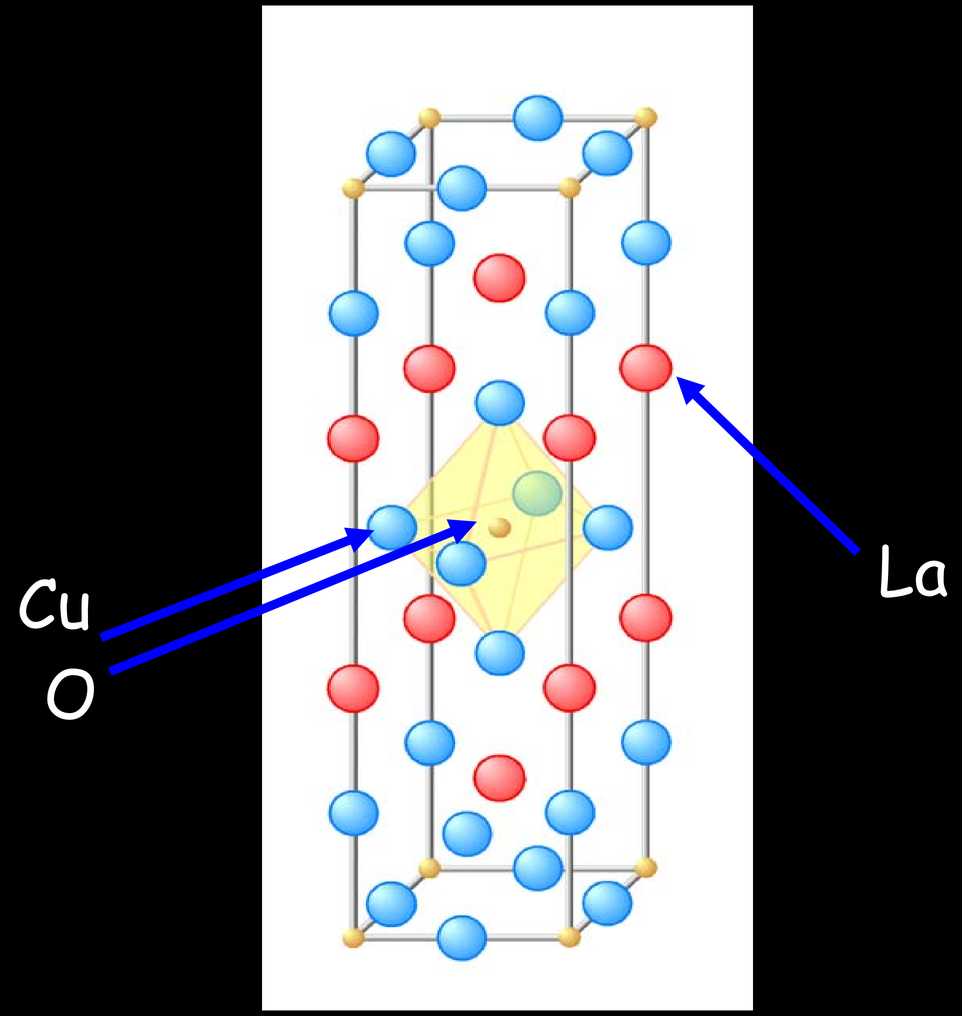
we get a dome!

pseudogap competes with SC, no preformed pairs etc.

$$T_c^{\max} \approx .084 T_0,$$

For LSCO: $T_c^{\max}$ = 90K

$La_{2-x}Ba_xCuO_4$

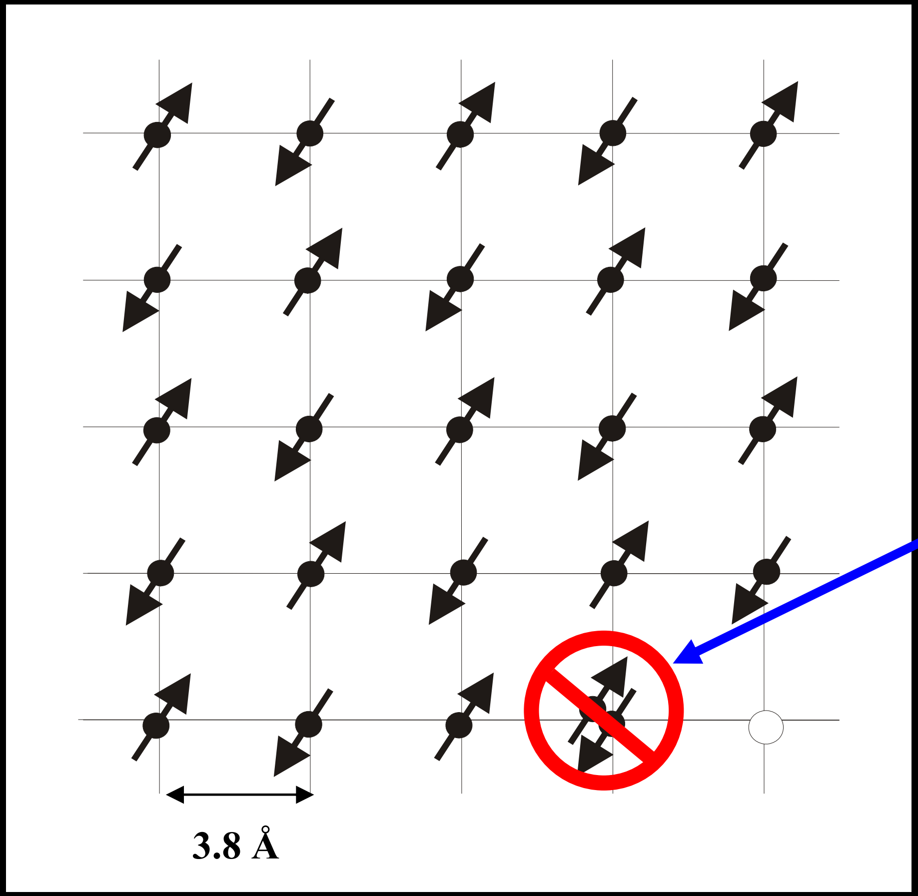


Antiferromagnetic

Mott Insulator

Z. Phys. Rev. **B** **64** 189 (1986)

Mott Insulator: Repulsive Coulomb U~3eV

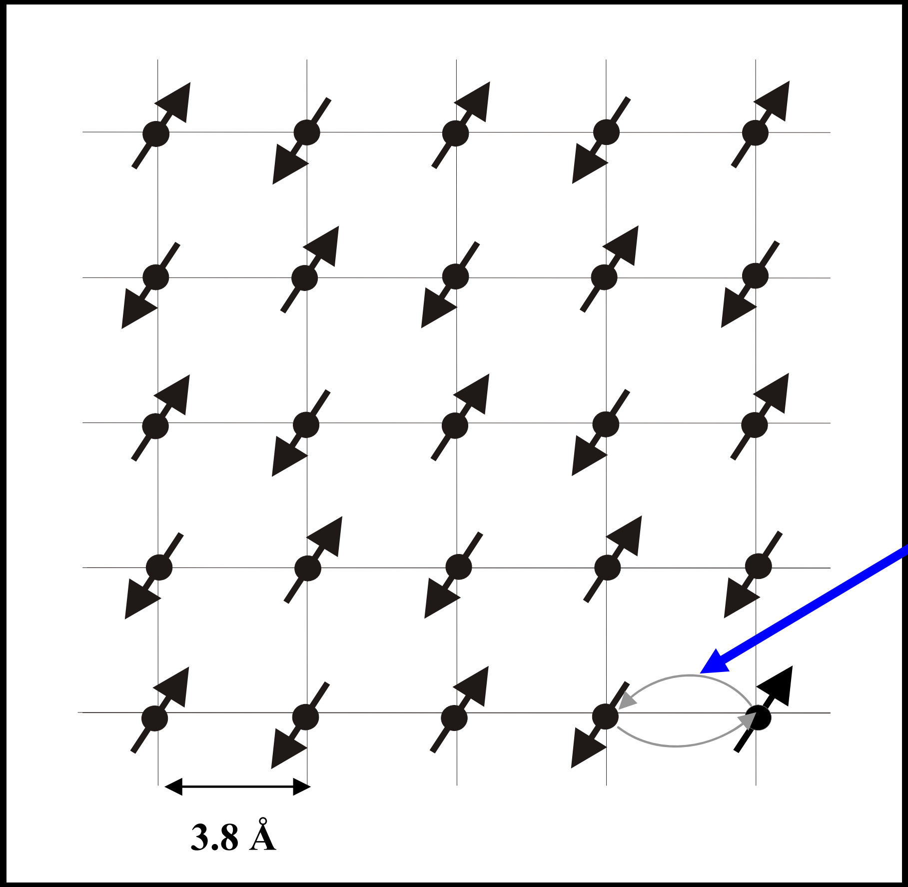


No double
occupancy
allowed..

N.F. Mott, ***Proc. Phys. Soc A62***, 416 (1949)

(courtesy Seamus Davis)

Antiferromagnetic: Superexchange J~0.14eV



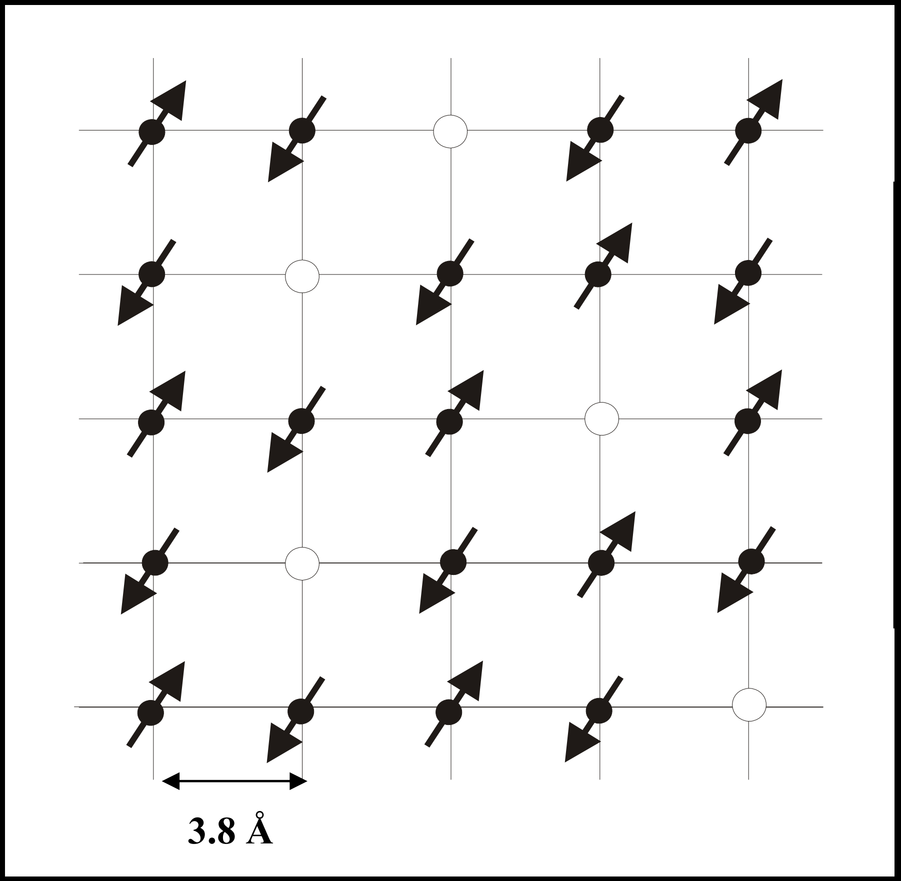
..except as a virtual process.

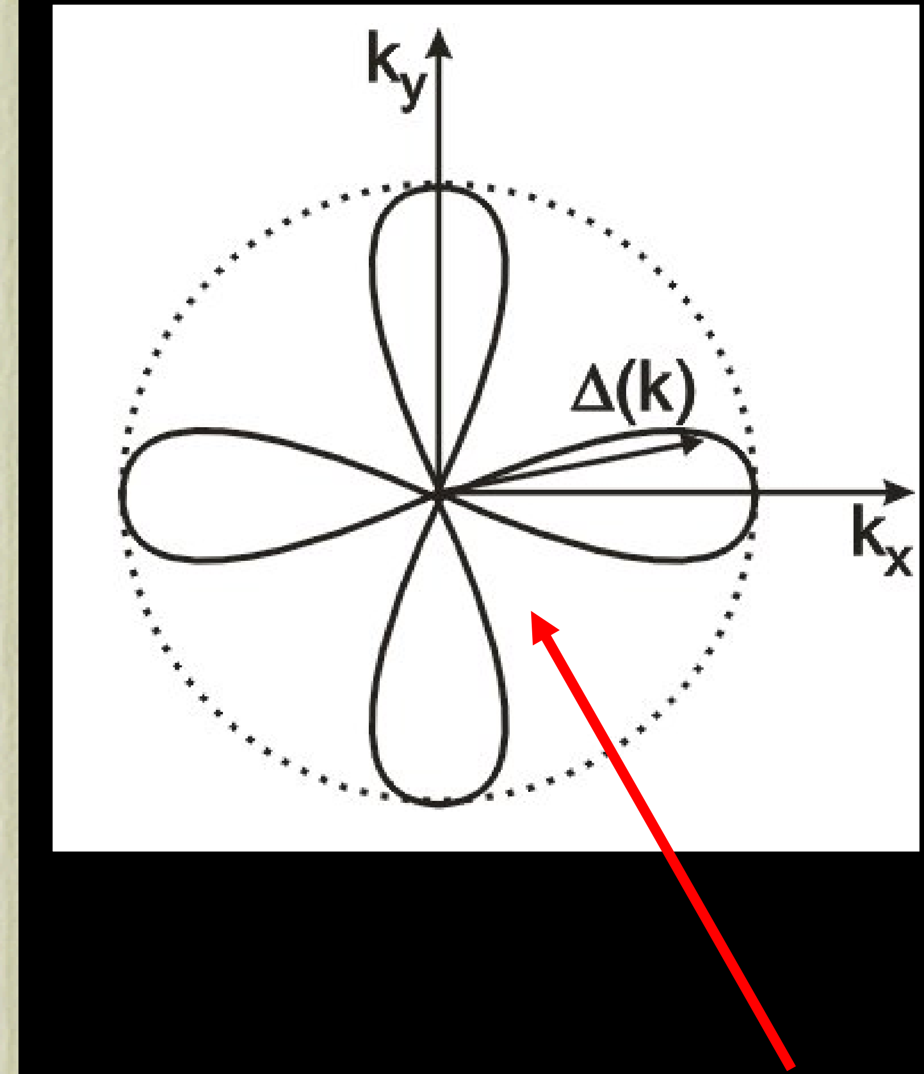
$$H = \frac{4t^2}{U}\sum_{<i,j>}\vec{S}_i \bullet \vec{S}_j - \frac{1}{4}n_i n_j$$

P. W. Anderson, ***Phys. Rev. 115***, 2 (1959)

AF order preferred since it allows virtual hopping

How could this state become superconducting?





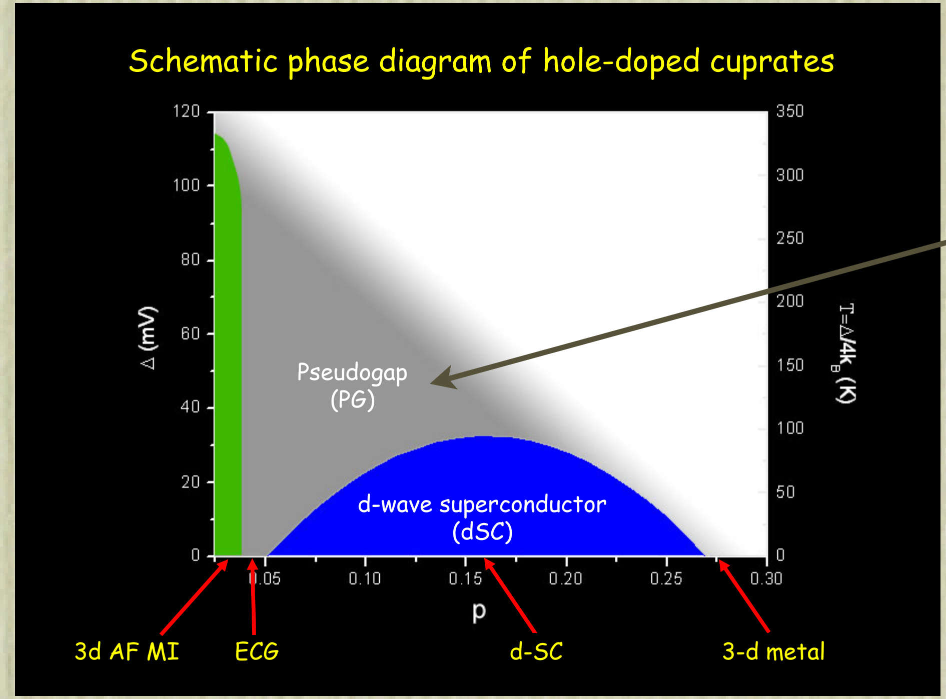
The SC energy gap $\Delta(\vec{k})$ has four nodes.

The SC gap has d-wave symmetry

MANY OPEN QUESTIONS

- What is the basic mechanism that leads to d-wave pairing from repulsive interactions?
- What is the pseudogap? Pre-formed pairs? Intrinsic to 1-particle density of states?
- Does the pseudogap compete or help superconductivity?
- What sets the scale of Tc?

Where to begin?



Here......

Hypothesize a gas of particles that SC condenses out of.

* assume rotational invariance at long wavelengths. (lattice effects absent in the basic model.)

* Need a new kind of non-Fermi liquid with a quantum field theory description. (Like Luttinger in 1d)

Basic requirements on models

- Purely electronic: only repulsive quartic interactions (like the Hubbard model).
- intrinsically 2 dimensional.
- d-wave pairing (attractive).
- non-Fermi liquid properties of normal state: pseudogap in resistivity, specific heat. Most important clues are here.
- prediction of Tc.

Difficult to obtain all in a single model....

THE MODEL

4 fundamental fermionic fields:

+, - = electric charge

$$\boxed{\chi^{\pm}_{\uparrow,\downarrow}}$$

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$$H = \int d^2\mathbf{x} \left(\sum_{\alpha=\uparrow,\downarrow} (\partial_t \chi_\alpha^- \partial_t \chi_\alpha^+ + v_F^2 \vec{\nabla}\chi_\alpha^- \cdot \vec{\nabla}\chi_\alpha^+ + m^2 \chi_\alpha^- \chi_\alpha^+) + 8\pi^2 g\, \chi_\uparrow^- \chi_\uparrow^+ \chi_\downarrow^- \chi_\downarrow^+ \right)$$

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Novelty: the kinetic term second order in time derivatives, with emergent Lorentz symmetry.

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Novelty: the kinetic term second order in time derivatives, with emergent Lorentz symmetry.

Phenomenological motivation: m = pseudogap, specific heat proportional to T^2 at low temperatures.

Motivation from O(3) sigma model description of AF.

non-linear sigma model: $S = \int dt\, d^d\mathbf{x}\, \partial_\mu \vec{\phi} \cdot \partial^\mu \vec{\phi}$

constraint on phi follows from a constraint on chi:

$$\vec{\phi} \cdot \vec{\phi} = -\frac{3}{2}(\chi^- \chi^+)^2$$

Imposing:

$$\chi_\uparrow^- \chi_\uparrow^+ + \chi_\downarrow^- \chi_\downarrow^+ = \text{constant}$$

$$\partial_\mu \vec{\phi} \cdot \partial_\mu \vec{\phi} \;\propto\; \partial_\mu \chi^- \partial_\mu \chi^+ \quad + \text{irrelevant operators}$$

* explains the second order in time derivatives.

Unitarity, spin statistics!?

- spin is a flavor here and thus does not need to be embedded in the Lorentz group.
- The issue is really: can one consistently quantize a fermionic theory that is second order in time derivatives?

Yes……

Canonical quantization:

The momentum expansion of the free fields is

$$\begin{aligned}\chi^-(\mathbf{x},t) &= \int \frac{d^2\mathbf{k}}{(2\pi)^2\sqrt{2\omega_\mathbf{k}}}\left(a^\dagger_\mathbf{k} e^{-ik\cdot x} + b_\mathbf{k} e^{ik\cdot x}\right)\\ \chi^+(\mathbf{x},t) &= \int \frac{d^2\mathbf{k}}{(2\pi)^2\sqrt{2\omega_\mathbf{k}}}\left(-b^\dagger_\mathbf{k} e^{-ik\cdot x} + a_\mathbf{k} e^{ik\cdot x}\right)\end{aligned}$$

$$H_{\text{free}} = \int \frac{d^2\mathbf{k}}{(2\pi)^2} \sum_{\alpha=\uparrow,\downarrow} \omega_\mathbf{k} \left(a^\dagger_{\mathbf{k},\alpha} a_{\mathbf{k},\alpha} + b^\dagger_{\mathbf{k},\alpha} b_{\mathbf{k},\alpha}\right)$$

$$\boxed{\omega_\mathbf{k} = \sqrt{\mathbf{k}^2 + m^2}}$$

The free theory is perfectly hermitian and unitary in momentum space.

Note: m is a gap in the single particle density of states.

Introduce unitary operator that distinguishes particles from holes:

$$CaC = a, \; CbC = -b \qquad C^2 = 1.$$

Then: $\chi^+ = C(\chi^-)^\dagger C.$

pseudohermiticity of interacting theory: $H^\dagger = CHC$

Generally one can prove a pseudohermitian hamiltonian has real eigenvalues and has a unitary time evolution with a suitably defined C-inner product. C-inner product gives negative norm states in the b-particle sector. Low energy effective theory has no negative probabilities since no transitions between states of mixed norm.

SO(5) symmetry

N-component version has Sp(2N) symmetry Sp(4) = SO(5).   SO(5) is hidden, accidental, and due to the fermionic statistics.

5-vector of bilinears can serve as order parameters for both spontaneous symmetry breaking of spin SU(2) (AF) and the charge U(1) for superconductivity:

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$$\vec{\Phi} = (\vec{\phi}, \phi_e^+, \phi_e^-) = (\underset{\text{magnetic}}{\underset{\longleftrightarrow}{\chi^- \vec{\sigma} \chi^+ / \sqrt{2},}} \underset{\text{electric}}{\underset{\longleftrightarrow}{\chi_\uparrow^+ \chi_\downarrow^+, \chi_\downarrow^- \chi_\uparrow^-}})$$

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Singlet: $\chi^- \chi^+ \equiv \sum_{\alpha=\uparrow,\downarrow} \chi_\alpha^- \chi_\alpha^+ .$ ~ pseudogap

Energy scales

There are two zero temperature energy scales in the model, the cut-off and the mass m. Since we will be computing temperature dependence, we convert these to equivalent temperatures:

Cut-off:

$$\boxed{E_0 = \hbar v_F \Lambda_c \equiv k_B T_0}$$

T$_o$ will turn out to be comparable to AF exchange energy, around 1000K.

Mass or "pseudogap:"

$$\hbar v_F m \equiv k_B T^*$$

Renormalization group

The interaction is relevant, in constrast to Dirac fermions.

Λ_c =upper cutoff

Λ = running RG scale

$$g(\Lambda) = \Lambda \widehat{g}(\Lambda)$$

$$-\Lambda \frac{d\widehat{g}}{d\Lambda} = \widehat{g} - 8\widehat{g}^2$$

fixed point: $\widehat{g}_* = 1/8$

fermionic quantum critical point.

Define: $x = 1/\widehat{g}$ $\quad x_* = 8$

Integrate the RG flow:

$$\frac{\Lambda}{\Lambda_c} = \frac{x_* - x}{x_* - x_0}, \qquad (x_* = 8)$$

linear!

Initial condition: $g(\Lambda_c) = \Lambda_c \widehat{g}_0$ $\quad x_0 = 1/\widehat{g}_0$

Hole doping

We vary the doping p by varying the cut-off. One can argue that it is related to a 1-point function:

$$p = \frac{c}{\Lambda_c} \left(\langle \chi^- \chi^+ \rangle_{\Lambda_c} - \langle \chi^- \chi^+ \rangle_{\Lambda} \right)$$

The constant c can be estimated by comparing to Heisenberg model:

$$\boxed{p(x) \approx \frac{\sqrt{2}}{\pi^2} \left(1 - \frac{\Lambda}{\Lambda_c} \right) = \frac{\sqrt{2}}{\pi^2} \left(\frac{x - x_0}{x_* - x_0} \right)}$$

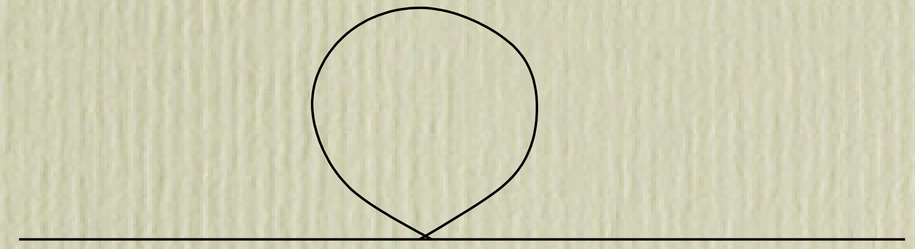
linear!

Main point: plots as a function of p or x simply related by a shift of the origin and overall scale.

Near the critical point, p = 0.14

Dynamical pseudogap generation

If the mass were classically zero, a non-zero mass is generated by quantum fluctuations.



Gap equation for the mass m:

$$m^2 = -8\pi^2 g \int \frac{d^3\ell}{(2\pi)^3} \frac{1}{\ell^2 + m^2}$$

Result:

$$\boxed{\frac{E_{pg}}{E_0} = \frac{T^*}{T_0} = \widehat{m}(x) \frac{\Lambda}{\Lambda_c} = \widehat{m}(x) \frac{x_* - x}{x_* - x_0}}$$

where m^ satisfies the transcendental equation:

$$\widehat{m} = 4\widehat{g} \tan^{-1} 1/\widehat{m}$$

Plot of pseudogap as function of “doping” x:

Quantum critical point

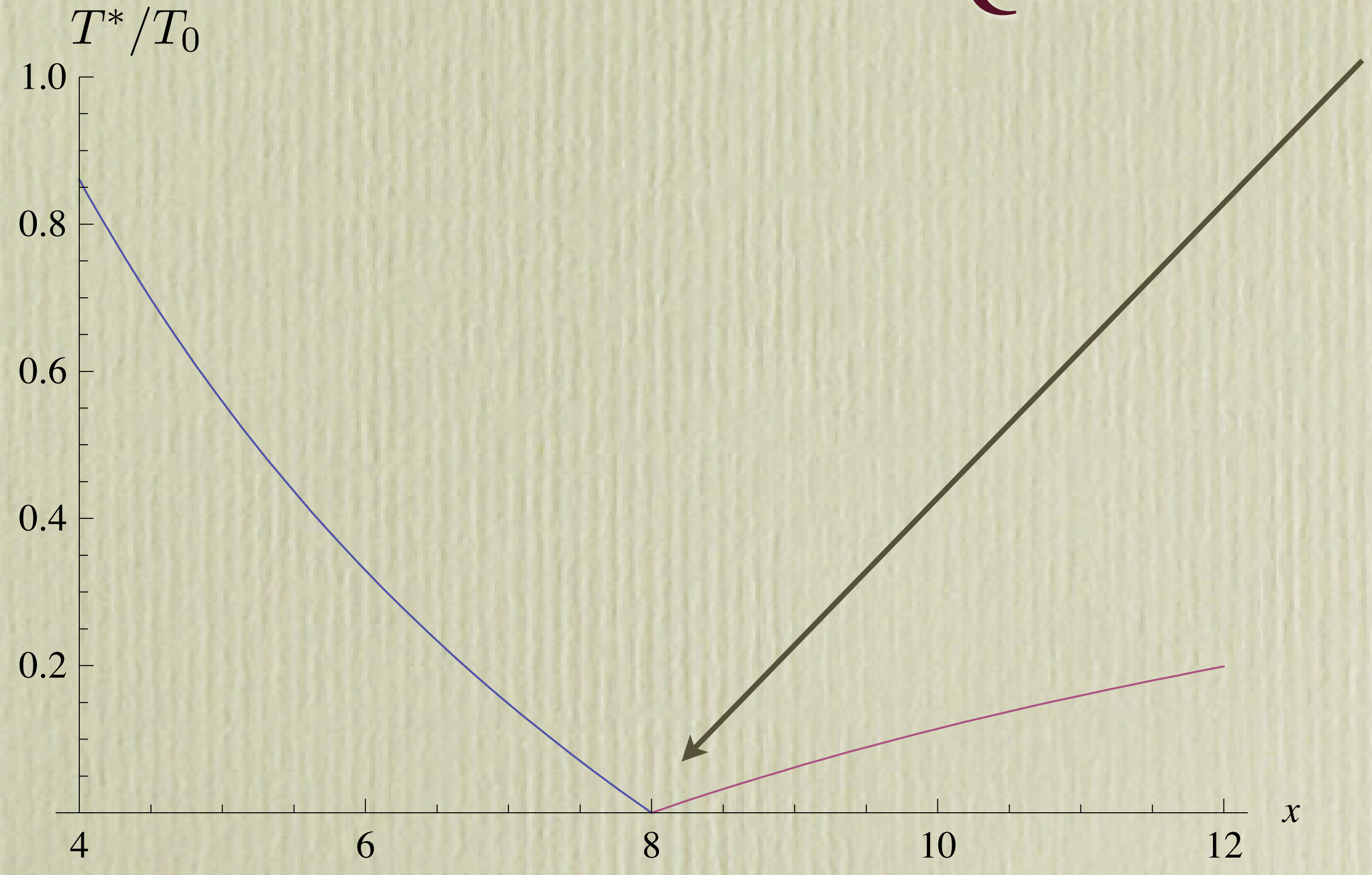


FIG. 3: The pseudogap T^* as a function of x for $x_0 = 4$ and $\widetilde{x}_0 = 16$.

Pseudogap has entirely different origin than SC.

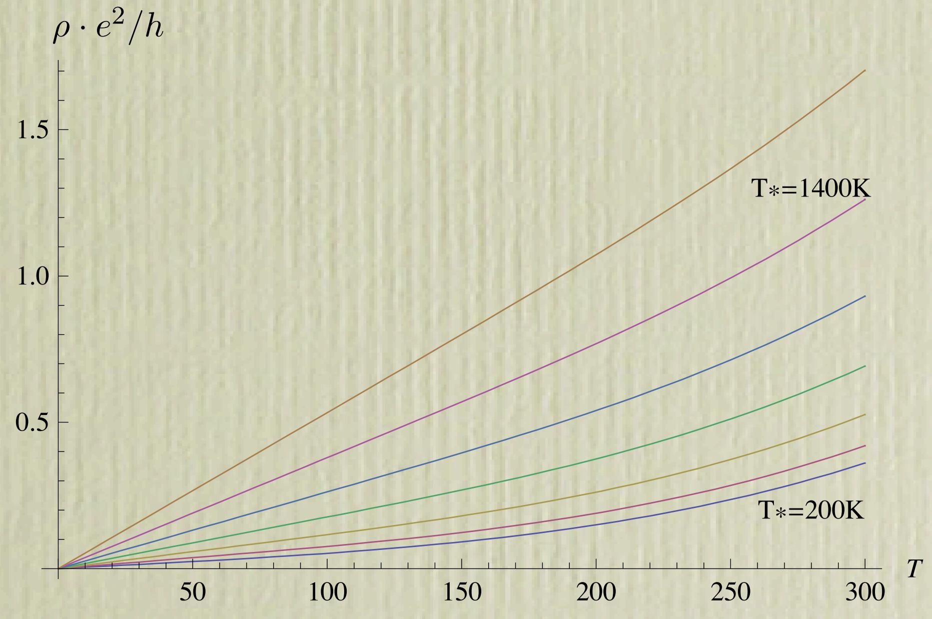
Resistivity

As an approximation: compute resistivity in the free theory, a reasonable approximation near the critical point where T* is small.

Kubo formula gives the following dc conductivity:

$$\sigma(\omega = 0) = \frac{e^2\pi}{Td}\int^{\Lambda_c}\frac{d^d\mathbf{k}}{(2\pi)^d}\frac{k^2}{\omega_{\mathbf{k}}^3}\left[2\tanh(\omega_{\mathbf{k}}/2T) - \frac{\omega_{\mathbf{k}}}{T}\operatorname{sech}^2(\omega_{\mathbf{k}}/2T)\right]$$

There is a linear regime in the resistivity at low T:



(T₀ =1000K)

For a stack of 2-dim'l conducting layers, in the linear regime:

resistivity:

$$\rho = .08b\left(\frac{T}{T_0}\right)\left(\frac{1+2t_*^2}{\sqrt{1+t_*^2}} - 2t_*\right)^{-1} \qquad [\mathrm{m}\Omega\mathrm{cm}]$$

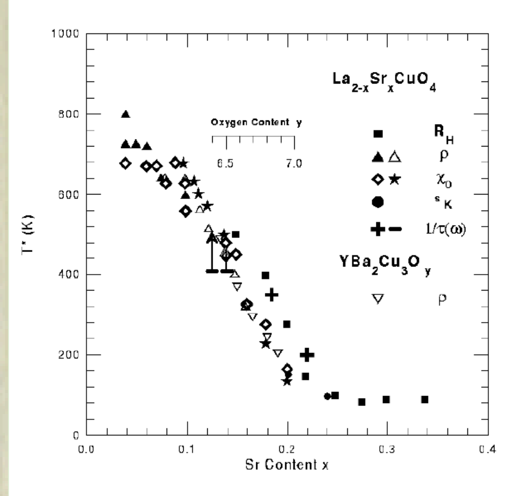
b= interlayer spacing in angstroms=6.4 $t_* = T^*/T_0.$

This formula works surprisingly well, a zero parameter fit!

doping x	T^*	$\rho_{\mathrm{exp}}(300K)[\mathrm{m}\Omega\mathrm{cm}]$	$\rho_{\mathrm{theory}}(300K)[\mathrm{m}\Omega\mathrm{cm}]$
0.24	$0.0K$	0.15	0.15
0.20	$173K$	0.18	0.21
0.15	$390K$	0.28	0.33
0.10	$606K$	0.68	0.52
0.075	$715K$	1.4	0.65

$La_{2-x}Sr_xCuO_4$

Fit the pseudogap data to a straight line:

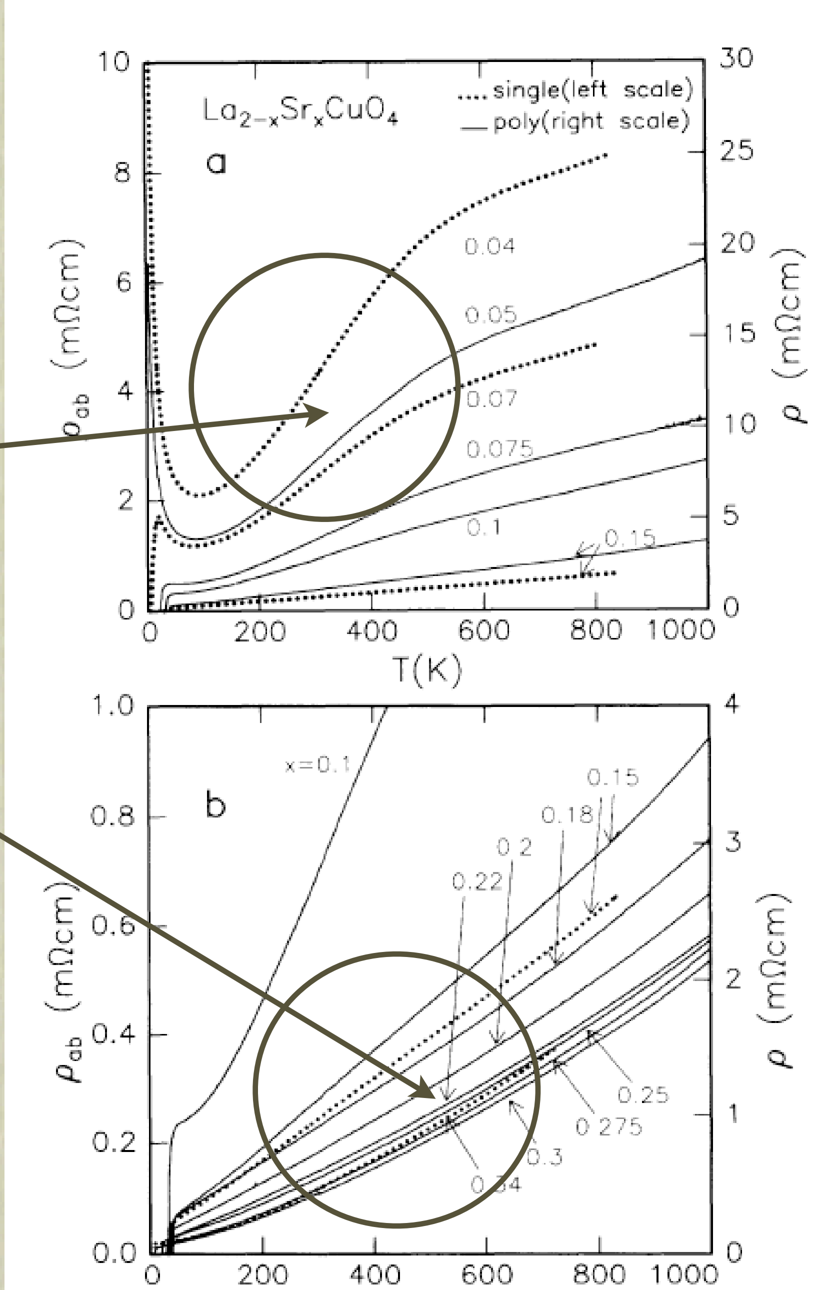


Batlogg et. al. 1994

$$T^*(x) = 1040(1 - x/.24)K$$

$$T_0 = 1040K$$

Linear regime



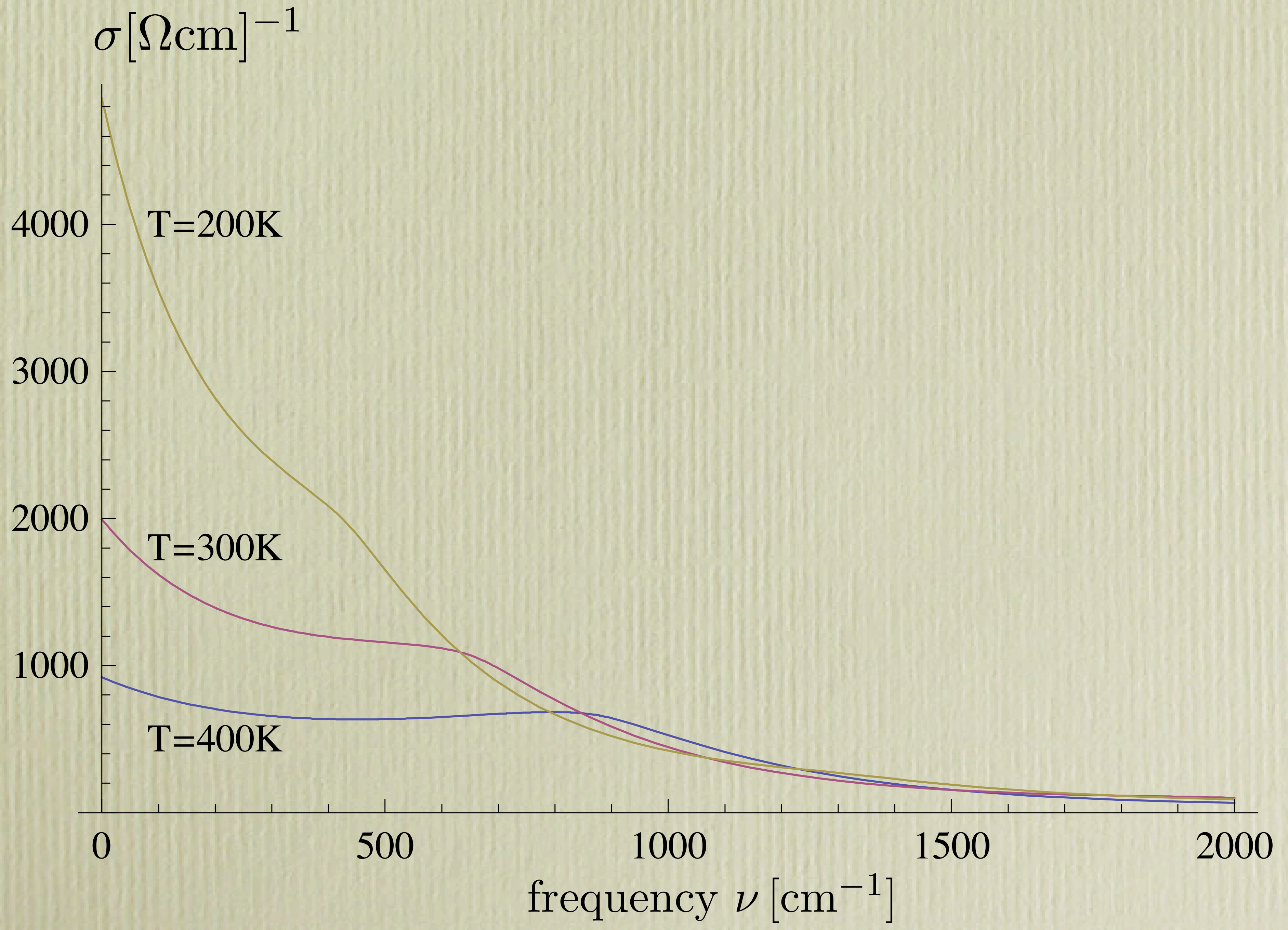
Takagi et. al. 1992

Experimental data on the in-plane resistivity of $La_{2-x}Sr_xCuO_4$ at various doping x.

(not the same x)

Finite frequency:

Our theory:



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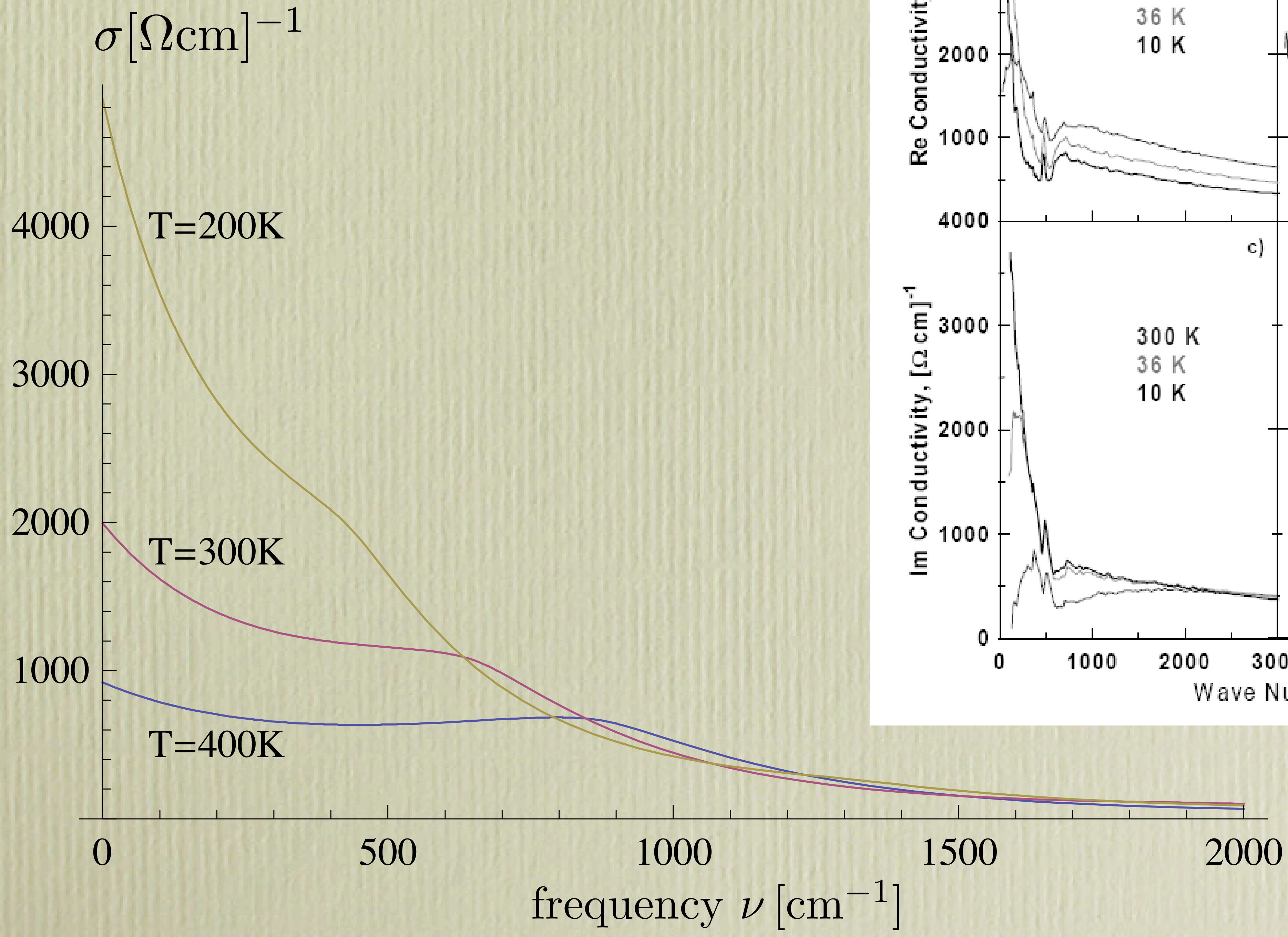
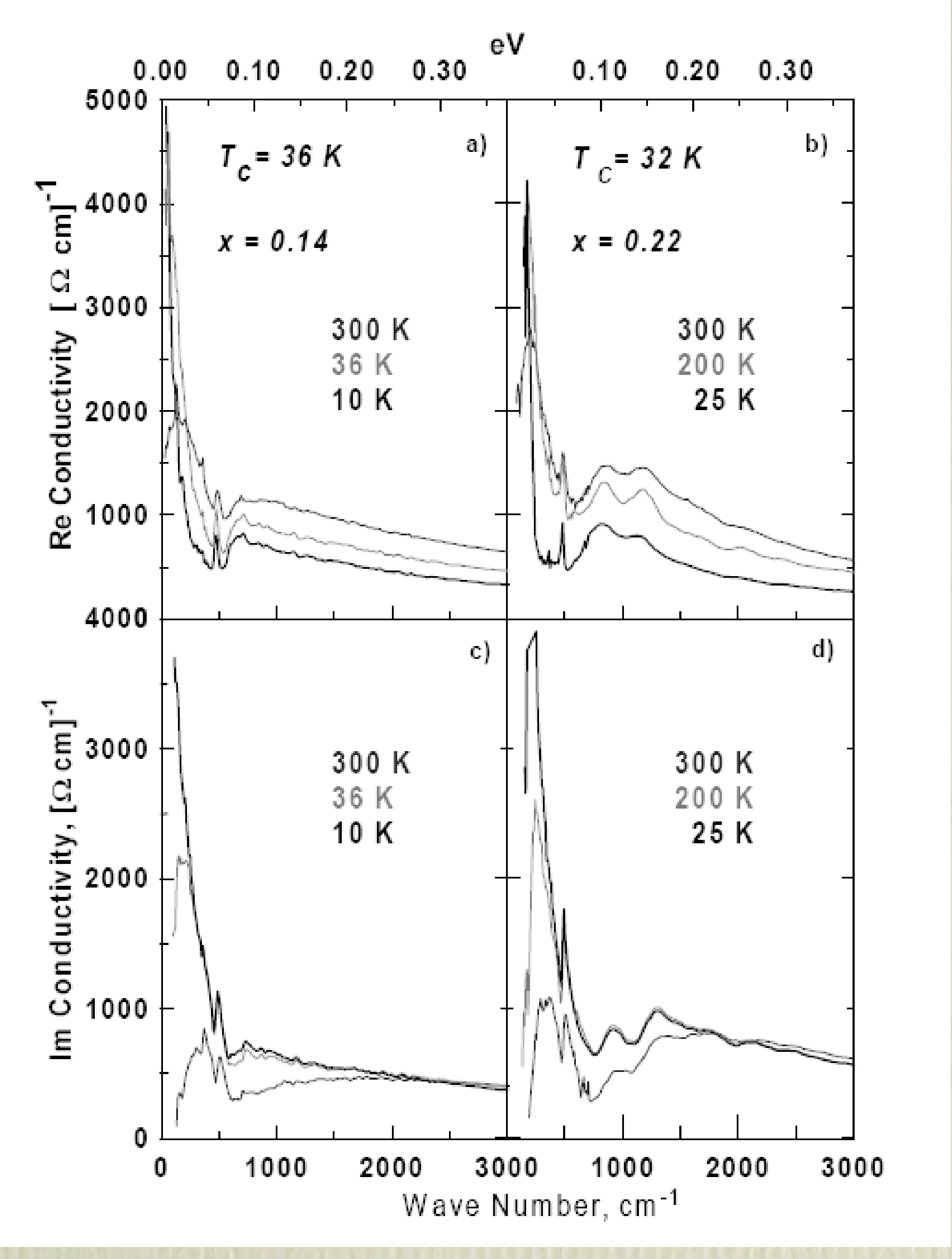


Startseva et. al. 1997



Electronic specific heat

Approximation: same as before, in the free theory with a dynamically generated mass.

Free energy density: $\mathcal{F} = -4T \int_0^{\Lambda_c} \frac{d^2\mathbf{k}}{(2\pi)^2} \log\left(1 + e^{-\omega_{\mathbf{k}}/T}\right)$

$\omega_{\mathbf{k}} = \sqrt{\mathbf{k}^2 + m^2}$

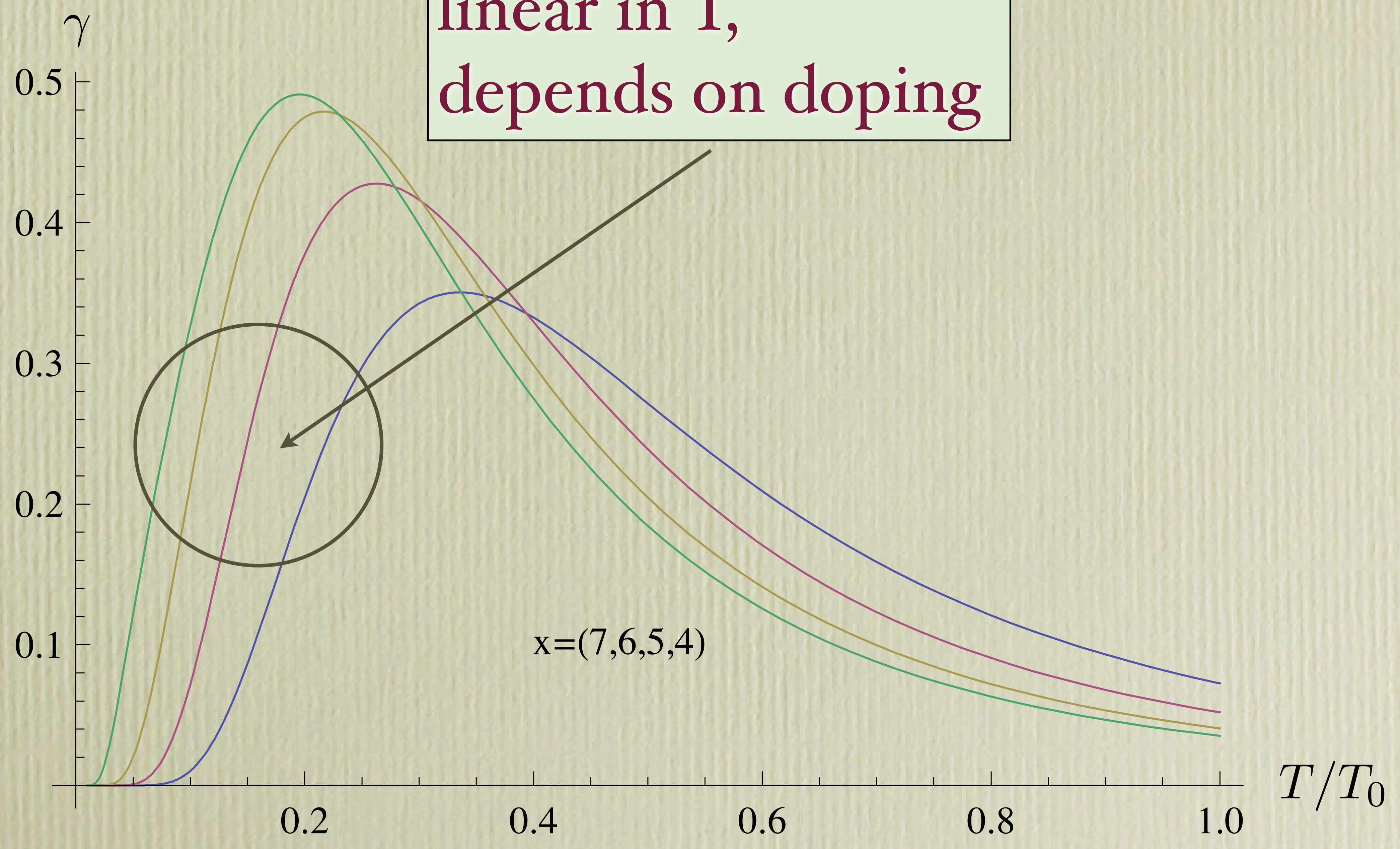
There is a cross-over behavior of the specific heat at the temperature $T^* = m$.

At low T: $\gamma = C/T \quad \propto T$

For a Fermi liquid: γ is a constant.

Specific heat coefficient: $\gamma = C/T$

Calculation:



linear in T,
depends on doping

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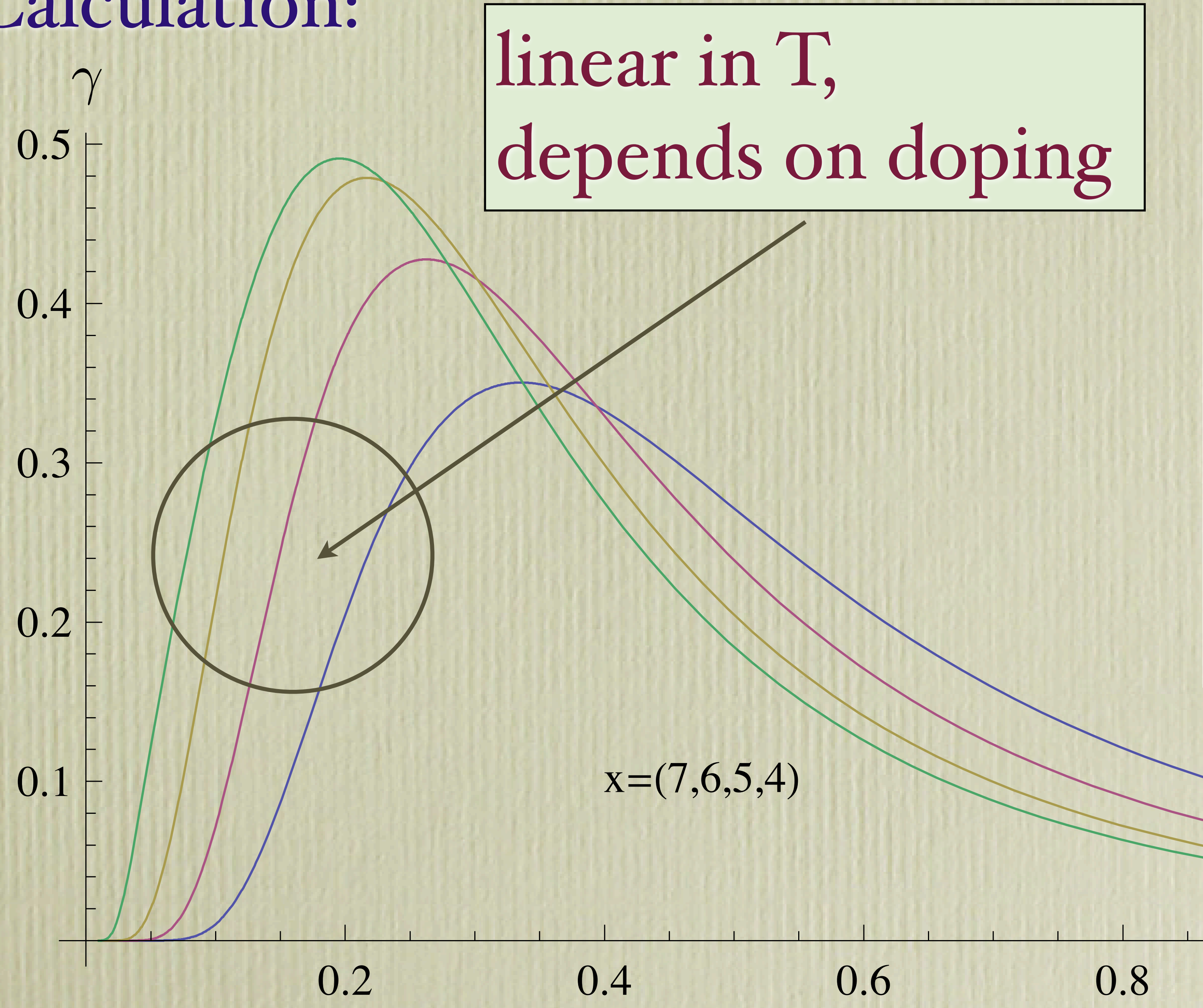

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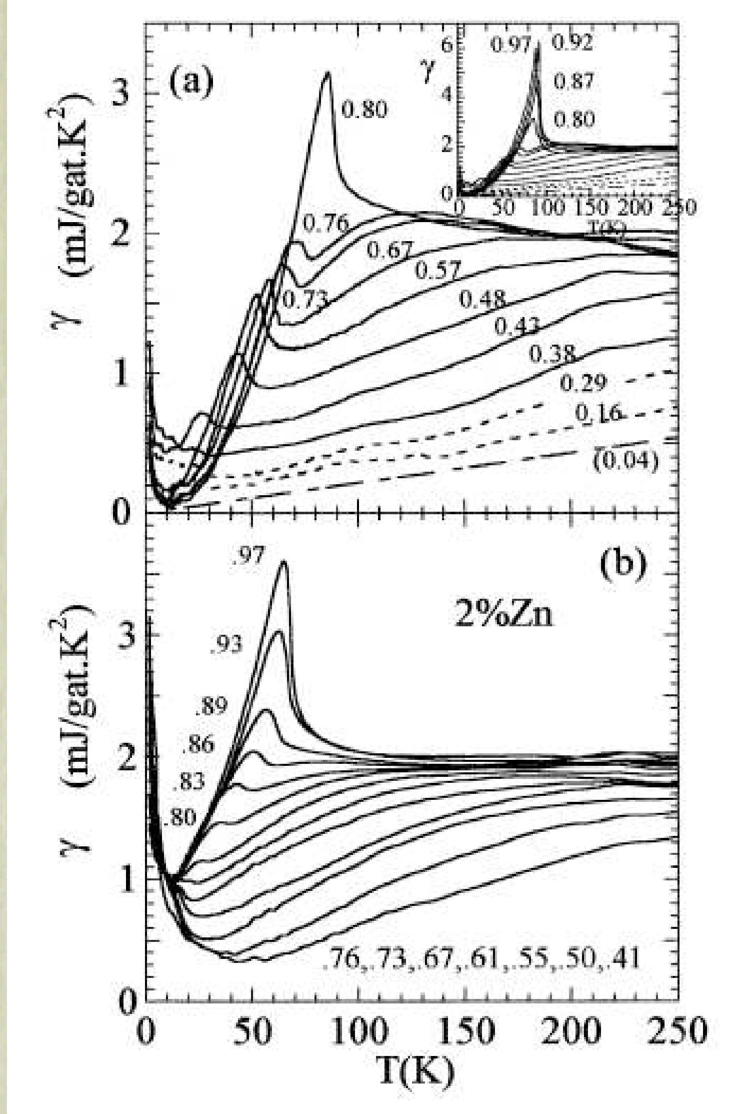


$Y_{0.8}Ca_{0.2}Ba_2Cu_3O_{6+x}$

Loram et.

d-wave Superconductivity

superconductivity implies spontaneous symmetry breaking of the electro-magnetic U(1). We thus consider the charge 2 Cooper pair order parameters:

$$q^{\pm} = \langle \chi_{\uparrow}^{\pm} \chi_{\downarrow}^{\pm} \rangle$$

q(k) = Fourier transform

One can derive the gap equation:

$$\boxed{q(\mathbf{k}) = -\int \frac{d\omega d^2\mathbf{k}'}{(2\pi^3)} G(\mathbf{k}, \mathbf{k}') \frac{q(\mathbf{k}')}{(\omega^2 + \omega_{\mathbf{k}'}^2)^2 + q(\mathbf{k}')^2}}$$

The kernel G(k,k') represents the scattering of Cooper pairs.

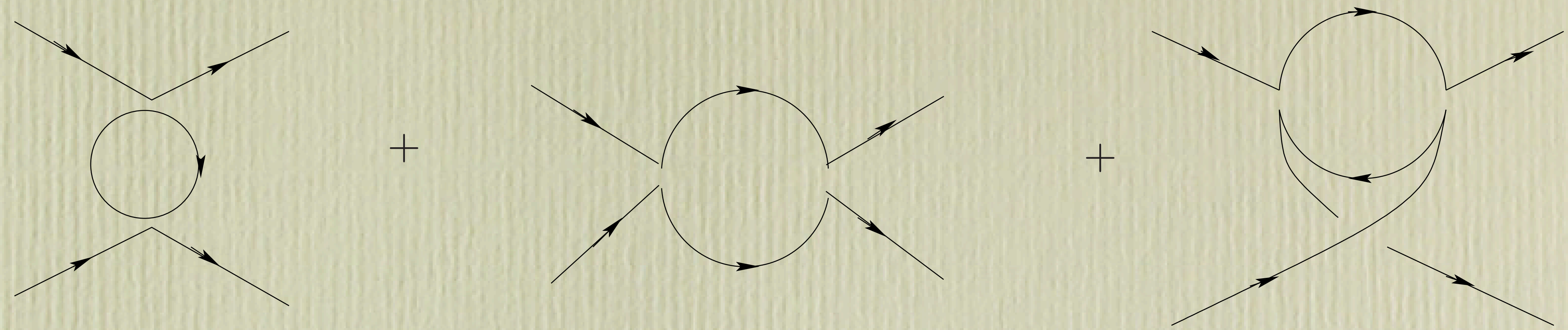
Expand in circular harmonics:

$$
\begin{aligned}
G(\mathbf{k},\mathbf{k}') &= \sum_{\ell=0}^{\infty} G_\ell(k,k')\cos\ell(\theta-\theta') \\
q(\mathbf{k}) &= \sum_{\ell=0}^{\infty} q_\ell(k)\cos\ell\theta
\end{aligned}
$$

s-wave SC (l=0) doesn't occur because the coupling is repulsive.

d-wave SC (l=2) is the first attractive channel in our model, based on momentum dependence of 1-loop scattering.

1- loop scattering



gives: $G_2(k,k') = -8\pi^2 g_2 k^2 k'^2$

$$\widehat{g}_2 = \frac{\pi}{40}\frac{\widehat{g}^2}{\widehat{m}^5}\left(\frac{\Lambda_c}{\Lambda}\right)^3$$

This gives a solution precisely of the d-wave form:

$$\boxed{q(\mathbf{k}) = \delta^2(k_x^2 - k_y^2) = \delta^2 k^2 \cos(2\theta)}$$

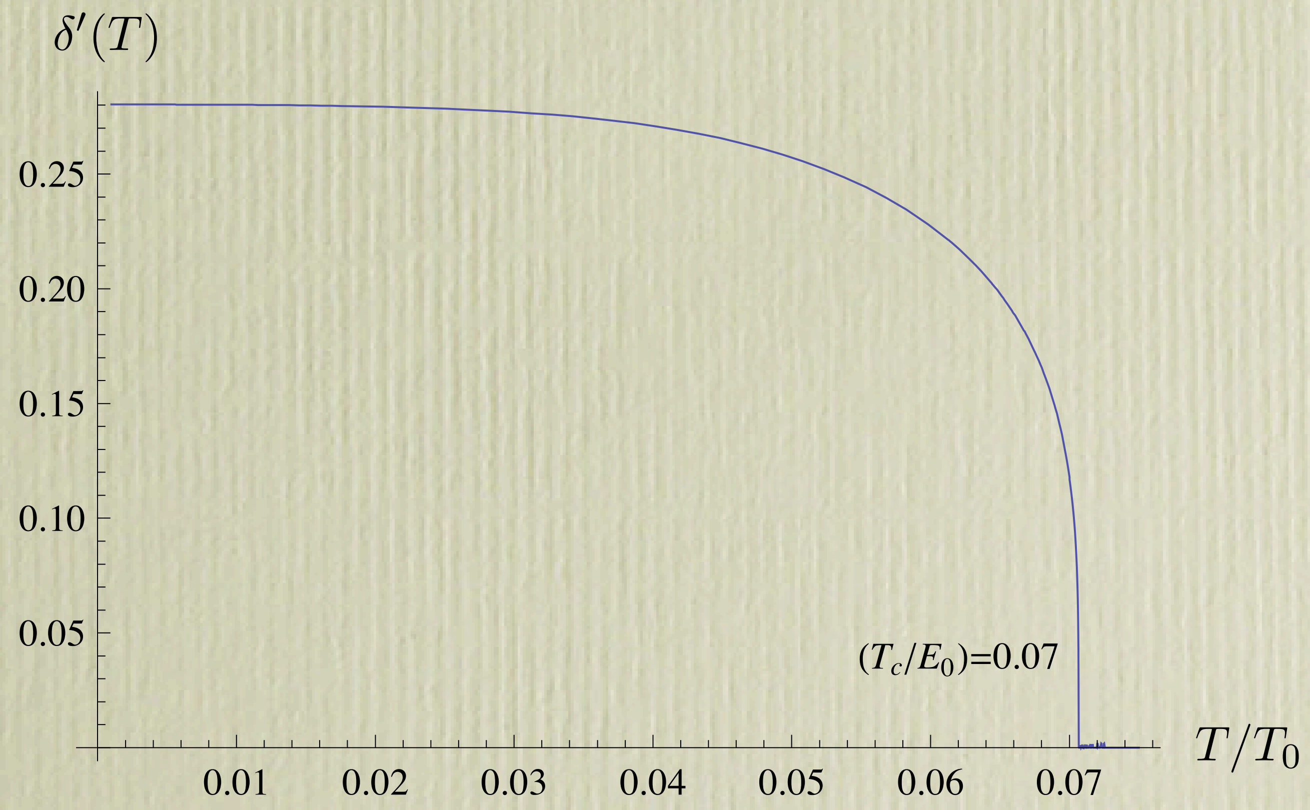
Finite temperature d-wave gap equation:

$$\delta^2 = g_2 \int dk d\theta\ k^3 \cos(2\theta) \mathrm{Im}\left(\frac{1}{\omega_{\mathbf{k},\delta}} \tanh\left(\frac{\omega_{\mathbf{k},\delta}}{2T}\right)\right)$$

where

$$\omega_{\mathbf{k},\delta} = \sqrt{\omega_{\mathbf{k}}^2 - i\delta^2 k^2 \cos 2\theta}$$

Tc is where the gap goes to zero:



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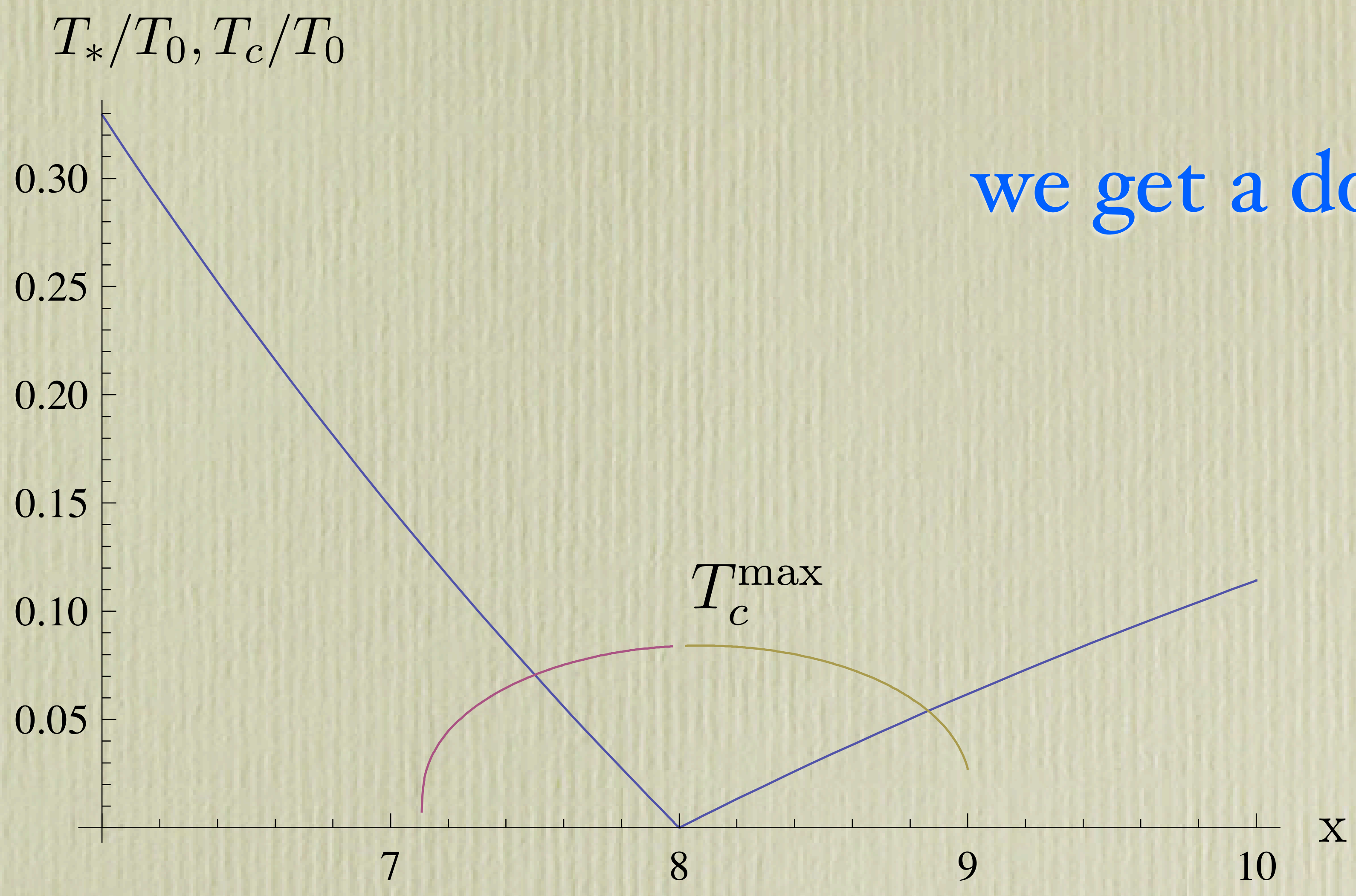

we get a dome!

pseudogap competes with SC, no preformed pairs etc.

$$T_c^{\max} \approx .084 T_0,$$

For LSCO: $T_c^{\max}$ = 90K

Conclusions and open problems

- A simple model that appears to capture the main features of HTSC in a calculable way.
- Clearly identified mechanisms for pseudogap, d-wave superconductivity.
- Gives good quantitative results for resistivity, Tc.
- How to get the model from lattice fermions?
- Lattice effects?