Aspects of SUSY and R-Symmetry Breaking

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Motivations

An outstanding puzzle in particle physics is the question of why

$$M_Z/M_P \sim 10^{-17}$$

is so small. A main benefit of having SUSY is the potential to explain this hierarchy of scales.

Unlike other symmetries, if SUSY is unbroken classically, it can only break due to non-perturbative effects. Thus, we may hope for (Witten)

$$M_Z \sim e^{-c/g^2} M_P$$

This is one of the main reasons for interest in theories that break SUSY dynamically.

Motivations

- Many dynamical models of SUSY breaking can be described at low energies by weakly coupled supersymmetric Lagrangians of chiral superfields. This is mostly achieved due to duality or other exact and approximate methods.
- In many cases these models also have an approximately canonical Kähler potential.
- By the argument of Nelson and Seiberg one should expect an R-symmetry.

Theories of chiral superfields with canonical Kähler potential are often referred to as *Wess-Zumino* models.

Motivations

The class of dynamical models reducing to Wess-Zumino models at low energy includes the

- ISS model : massive free-magnetic SQCD (Intriligator et al.),
- ITIY model : uplifted moduli space (Intriligator et al., Izawa et al.) ,

and many variations of these basic building blocks. In fact, these models and their relatives cover a major fraction of the model building work on supersymmetry breaking.

In addition there are the Extraordinary Gauge Mediation models (Cheung et al.), and it is an interesting open problem to embed them in Dynamical SUSY Breaking.

Motivations

One recurring phenomenological problem in these models is the fact that there usually is an R-symmetry. Unbroken R-symmetry is problematic since the R-charge of the gaugino is necessarily one and therefore the mass term

$$\mathcal{L} \supset m_\lambda \lambda\lambda$$

is forbidden (in contradiction with observations). However, we know quite well the conditions for R-symmetry breaking (Shih) and usually in practice it is not hard to deform or modify given models so that R-symmetry is spontaneously broken.

Motivations

It turns out that there is yet another persistent problem in such models. "Empirically" it appears that in many models although SUSY and R-symmetry are broken,

$$m_\lambda / m_{sparticle} \ll 1$$

This is a surprising fact. This is a concern because it results in very heavy scalars and some of the original motivations for SUSY are lost.

This problem plagues models of dynamical SUSY breaking since the very early history. A recent reincarnation of this phenomenological obstacle is in the model building attempts based on ISS, which often turned out to predict unsatisfactorily heavy scalars.

Goal

Our goal is to study general Wess-Zumino models and understand the origin of the anomalously light gauginos.

Along the way we will derive interesting and general results on SUSY-breaking Wess-Zumino models.

Our understandings will shed light on how to think in a different way about SUSY breaking and outline ways to more natural models.

An Example

Let us recall the original O'Raifeartaigh model.

$$W_{O'R} = X(\frac{1}{2}\phi_0^2 - f) + m\phi_2\phi_0$$

$$V = \left|\frac{1}{2}\phi_0^2 - f\right|^2 + m^2\left|\phi_0\right|^2 + \left|X\phi_0 + m\phi_2\right|^2$$

It has a unique R-symmetry $R(X) = R(\phi_2) = 2$, $R(\phi_0) = 0$. For $f < m^2$ the solution $X = \phi_2 = \phi_0 = 0$ breaks SUSY and has no tachyons. ψ_X is a massless fermion, the Goldstino. However, the boson X is massless, $m_X^2 = 0$, and moreover

$$X \to X + a, \qquad \phi_0 = \phi_2 = 0$$

leaves the vacuum energy fixed. Thus, there is a (pseudo)modulus space of solutions.

An Example

The spectrum as a function of X looks like for $X \ll m$ as $m_{\phi_0} \sim m_{\phi_2} \sim m$. For $X \gg m$ we have $m_{\phi_0} \sim X$, $m_{\phi_2} \sim \frac{m^2}{X}$ with small splitting.

The see-saw like spectrum can be understood from the mass matrix of the model

$$\det \begin{pmatrix} X & m \\ m & 0 \end{pmatrix} = f(m)$$

which is X independent. There are never tachyons as a function of X, even if X is very large. Thus, this pseudomodulus space is

locally stable everywhere

Basics of WZ

The starting point is a WZ model with chiral superfields ϕ_i having a canonical Kähler potential and a superpotential

$$W = f_i \phi_i + \frac{1}{2} m_{ij} \phi_i \phi_j + \frac{1}{6} \lambda_{ijk} \phi_i \phi_j \phi_k + \cdots$$

The tree-level scalar potential is

$$V = \sum_i |W_i|^2$$

with $W_i = \frac{\partial W}{\partial \phi_i}$.

Suppose that V has a SUSY-breaking local minimum at some $\phi_i = \phi_i^{(0)}$. This imposes a number of constraints on the theory:

Basics of WZ

- At least one $W_i \neq 0$. (All derivatives of W are evaluated at $\phi_i^{(0)}$)
- V has extremum provided that $W_{ij} W_j^* = 0$. But $(M_F)_{ij} \equiv W_{ij}$ is also the fermion mass matrix, so we see the massless Goldstino. (Pointing in the direction of non-zero F-terms)
- The bosonic mass-squared matrix is

$$\mathcal{M}_B^2 = \begin{pmatrix} M_F^* M_F & F^* \\ F & M_F M_F^* \end{pmatrix}$$

and $F_{ij} \equiv W_k^* W_{ijk}$ is the effect of SUSY-breaking.

In a consistent vacuum, $\mathcal{M}_B^2$ must be positive semi-definite (i.e. there cannot be any tachyons).

Properties of WZ

From the formulae for $\mathcal{M}_B^2$ and $\mathcal{M}_F^2$,

$$\mathcal{M}_B^2 = \begin{pmatrix} M_F^* M_F & F^* \\ F & M_F M_F^* \end{pmatrix}, \qquad \mathcal{M}_F^2 = M_F^* M_F$$

we see

- STr theorem : $\sum_{bosons} m_B^2 = 2 \sum_{fermions} m_F^2$. This is well known (and trivial to see).
- If there is a massless fermion, then its scalar superpartner must be massless too. This is a new result and it will be very useful for us, so we prove it :

Properties of WZ

$$\mathcal{M}_B^2 = \begin{pmatrix} M_F^* M_F & F^* \\ F & M_F M_F^* \end{pmatrix}, \qquad \mathcal{M}_F^2 = M_F^* M_F$$

Suppose v is a massless fermion, i.e. $\mathcal{M}_F^2 v = 0$. Then,

$$\begin{pmatrix} v^\dagger & v^T \end{pmatrix} \mathcal{M}_B^2 \begin{pmatrix} v \\ v^* \end{pmatrix} = v^T F v + c.c.$$

If this is non-zero it can be made negative by rotating the phase of v. This contradicts $\mathcal{M}_B^2$ being positive semi-definite. Thus, $v^T F v = 0$.

However, for positive semi-definite matrices $w^\dagger A w = 0$ implies $Aw = 0$ and therefore the boson $\begin{pmatrix} v & v^* \end{pmatrix}$ is massless. Note that this is the superpartner of the massless fermion.

Flat directions

If SUSY is broken there is a massless fermion, the goldstino $v_i = \langle W_i^* \rangle$. From the result above we conclude that there is a massless boson, the superpartner of the goldstino (they are degenerate although SUSY is broken).

Thus, the deformation

$$\phi_i \to \phi_i + \alpha W_i^*$$

is massless. In fact it follows that it is an exact flat direction for any $\alpha \in \mathbb{C}$. It has also been shown by S. Ray in a different way. Our derivation explains why there is a massless particle and why it is the superpartner of the goldstino.

We conclude that SUSY breaking solutions of WZ models are always accompanied by complex flat directions.

Universal Canonical Form

We can always rotate to a canonical basis where only one field X gets an F-term VEV and shift the ϕ_i to $\phi_i = 0$. Then, the most general superpotential is

$$W = X(f + \frac{1}{2}\lambda_{ab}\phi_a\phi_b) + \frac{1}{2}m_{ab}\phi_a\phi_b + \frac{1}{6}\lambda_{abc}\phi_a\phi_b\phi_c + \cdots$$

Terms like, e.g. $X^2\phi$ and X^3 are missing because they can not be consistent with a flat direction $\phi = 0$ and $X \in \mathbb{C}$.

The set of transformations and the canonical form itself need not respect symmetries of the problem.

SUSY Breaking at Tree-Level

Given that viable classical solutions are always accompanied by flat directions we can ask :

What does it mean for SUSY to be broken at tree-level ?

One should always calculate radiative corrections a-la Coleman-Weinberg

$$V = \frac{1}{64\pi^2} \mathrm{STr} M^4 \log \frac{M^2}{\Lambda^2}$$

and find where, if at all, the pseudomodulus stabilizes. But, there is a natural and extremely useful notion of “SUSY breaking at tree-level."

SUSY Breaking at Tree-Level

Suppose the following two conditions are satisfied :

- The pseudomoduli space is locally stable everywhere
- The radiative potential on the pseudomoduli space rises at infinity everywhere.

The first condition means that there are no tachyons at any point of the pseudomodulus space. (The O'R model is an example)

The second conditions requires some trivial knowledge of the radiative corrections, e.g. it is always satisfied when the pseudomodulus is coupled to some other chiral fields via $X\phi_i\phi_j$.

When these conditions are satisfied we know that SUSY is broken

SUSY Breaking at Tree-Level

This notion of SUSY breaking at tree-level is extremely useful, e.g. the original O'R model satisfies it, as well as the low energy theory of the ITIY model and massive SQCD (ISS).

In addition, if there is an R-symmetry (or any other global symmetry), we will say that it is broken if the pseudomoduli space breaks the symmetry everywhere.

Note that the pseudomoduli space in question need not be the global minimum of the potential; the theory could have multiple disconnected pseudomoduli spaces, SUSY vacua, or runaway directions.

A Determinant Identity

Recall the canonical basis

$$W = fX + \frac{1}{2}(\lambda_{ab}X + m_{ab})\phi_a\phi_b + \frac{1}{6}g_{abc}\phi_a\phi_b\phi_c$$

We ask, under what conditions is the pseudomoduli space spanned by X locally stable for any X?

Since the determinant of $\lambda X + m$ must be a polynomial in X,

$$\det(\lambda X + m) = \sum c_i(\lambda, m)X^i$$

unless it is a constant, there must be places in the complex X plane where it vanishes.

A Determinant Identity

Consider the theory around some such point $X = X_0$ where $\det(\lambda X_0 + m) = 0$, and let v satisfy

$$(\lambda X_0 + m)v = 0$$

Since $\lambda X + m$ is the mass matrix of the fermions, this corresponds to a massless fermion direction.

But from what we proved before, we get that either the corresponding boson $\phi_i = v_i$ must also be massless, or there is a tachyon.

The existence of tachyons contradicts our assumption that the modulus-space is locally stable. Thus, the boson must be massless.

A Determinant Identity

Recalling the bosonic mass matrix

$$\mathcal{M}_B^2 = \begin{pmatrix} (\lambda X + m)^*(\lambda X + m) & f\lambda^* \\ f^*\lambda & (\lambda X + m)(\lambda X + m)^* \end{pmatrix}$$

The massless fermion, as we said, is equivalent to $(\lambda X_0 + m)v = 0$, then, the massless boson implies $\lambda v = 0$ and therefore $mv = 0$ too.

Therefore the mode v is completely decoupled to quadratic order in the canonical form

$$W = fX + \frac{1}{2}(\lambda_{ab}X + m_{ab})\phi_a\phi_b + \cdots$$

A Determinant Identity

In this case, the vanishing of the determinant is identical for any X. We remove the decoupled mode and repeat the proof.

We conclude that an existence of a zero of the polynomial at X_0 leads to a contradiction and therefore the determinant must be an X independent function.

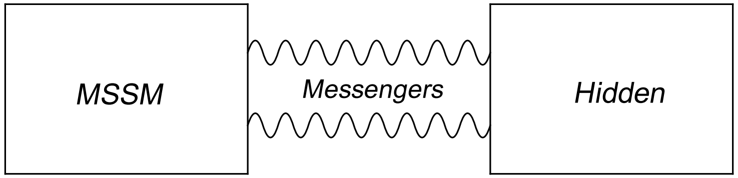
So, if SUSY is broken at the tree-level then

$$\det(\lambda X + m) = \det\, m$$

Keep in mind that this is only a necessary condition for locally stable pseudomodulus space, since there could be some light states as $X \to \infty$. We will not discuss them here.

Application : Gauge Mediation

Gauge Mediation is a way to transmit the supersymmetry breaking occurring in the hidden sector to the visible sector.



The “messengers" are particles in the hidden sector charged under the SM gauge group $SU(3) \times SU(2) \times U(1)$. We refer to it as “direct gauge mediation" if these charged particles play an important role in SUSY breaking. Otherwise, this is a “theory of messengers."

Theories of messengers are in general less aesthetically appealing and require extra ad-hoc structure.

Application : Gauge Mediation

The most general renormalizable theory of messengers, say in the 5 and $\bar{5}$ representations of $SU(5)$ takes the form

$$W = fX + (\lambda_{ij}X + m_{ij})\psi_i\bar{\psi}_j + \cdots$$

Every calculable model with approximately canonical Kähler potential reduces to this form at low energies. By integrating out the messengers $\psi, \bar{\psi}$ we get the gaugino mass

$$m_{\tilde{g}} = f^\dagger \partial_X \ln \det(\lambda X + m) + \mathcal{O}(f^2/m^3)$$

It is also important to mention that the leading order comes from a superpotential term :

$$\int d^2\theta \ln \det(\lambda X + m) W_\alpha^2$$

Application : Gauge Mediation

We have seen that the same determinant appears in two different places. Let us summarize the results so far

- The gaugino mass is (to leading order in f)

$$m_{\tilde{g}} = f^{\dagger}\partial_X \ln\det(\lambda X + m)$$

- Around any vacuum which is part of a locally stable modulus space

$$\det(\lambda X + m) = \det\, m$$

Thus, around vacua belonging to locally stable pseudomoduli spaces

$$m_{\tilde{g}} = 0$$

Application : Gauge Mediation

$m_{\tilde{g}} = 0$ (at leading order) holds :

- For messengers in any representation
- For calculable modes based on ITIY, massive SQCD and various other WZ models, exactly because the work revolved around finding the lowest-lying state ! This strategy is like “shooting yourself in the foot" from our perspective.
- Since the leading order is holomorphic, corrections to the Kähler potential can not actually change the low energy contribution to the gaugino masses. So our main result has relevance beyond WZ models, and is useful in more general situations.

Fine Print

To keep in mind :

- The gaugino mass we calculated is only the leading term in the SUSY breaking expansion. The other terms may be important too for very low scale breaking (such that $f/m^2 \sim 1$), but these terms turn out to be numerically small in all the examples, so in fact one still remains with a numerically large hierarchy of scalars w.r.t. gauginos.
- There may be some heavy thresholds which do not make it to the calculable model and can affect the gaugino masses. This leads to a model in which physics at different scale contributes to different soft masses, remains to see whether can be viable.

Models of Messengers

If one is willing to give up direct mediation, it is quite easy to circumvent the obstruction, but still the main line of thought is very useful. Let us think of Minimal Gauge Mediation

$$W = X\psi\bar{\psi} + M\psi\bar{\psi}$$

with

$$X = \theta^2 f$$

obtained by some other hidden sector which couples to the messengers. We see that there are massless messengers at $X = -M$.

Thus, the usually studied models of messengers naturally live in such “excited pseudomoduli spaces."

Summary and Prospects

Dynamical models which give viable gaugino masses should end up NOT being in the SUSY-breaking ground states and not even approximately the SUSY breaking ground states.

We are led to suggest

A new kind of inevitable metastability

The strategy of model building should be revised and one should look for states which can decay even within the low energy approximation. Longevity is usually not a problem, as the energy difference can be small compared to the distance in fields space.