Aspects of 3d, N=2 Chern Simons Theories

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based on work with Nati Seiberg

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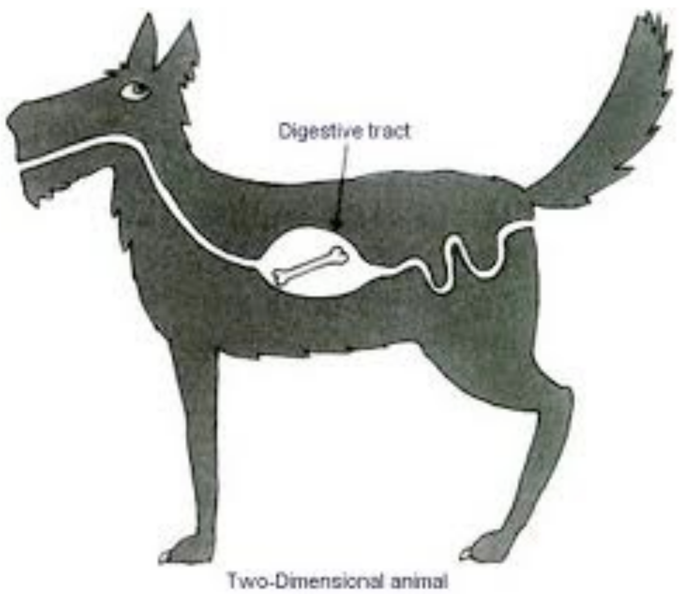
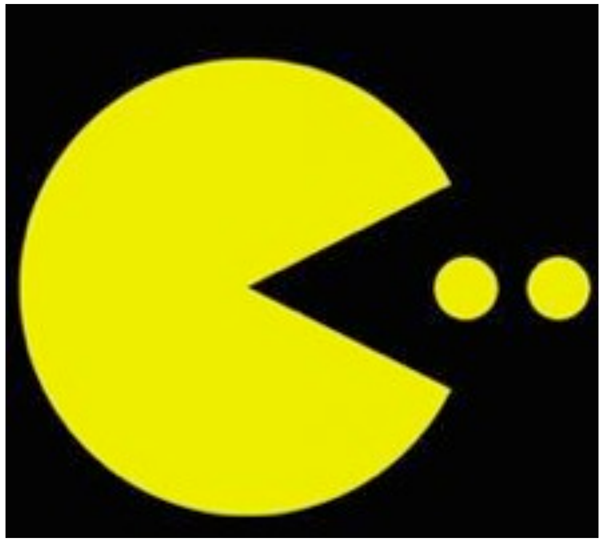
Why study 3d, N=2 theories?

- Related to 4d, N=1 theories on a circle.
- 3d: Enhanced, IR-interesting dynamics.
- Even U(1) theories have interesting duals.
- Vortices / Monopole op. = concrete map: electric description of some dual d.o.f.. Could provide new insight / hints into the still-elusive aspects of 4d dualities.
- Web of dualities. Build up methods.

Life in flatland





hologram

Some history / refs

- '77: Polyakov, 3d instanton confinement
- '82: Affleck, Harvey, Witten, instanton W_{dyn}
- '85-'93: Solitons / vortices in 3d. Ward; Hindmarsh; Lee, Lee, Min; Leese, Samols. Zero mode curiosities.
- '94: Seiberg duality in 4d, N=1.
- '96: Seiberg, Witten 4d N=2 on circle to 3d.
- '96: KI, Seiberg mirror symmetry in 3d, N=4.
- '97: Aharony, Hanany, KI, Seiberg, Strassler, 3d N=2 mirror symmetry. + DeBoer, Hori, Oz.
- '97: Aharony duality. Unusual: monopole ops in L.

Some history / refs cont

- '99: Kapustin, Strassler. More on monopole ops, CS.
- '99-'00: Dorey, Tong; Tong. Compact Coulomb, CS.
- '02: Kapustin and co., monopole operators. /
- Various: AdS_4 / CFT_3
- '08: Giveon, Kutasov duality w/ CS terms (branes).
- '08: ABJM, 3d, N=6 CS SCFT and M2 branes.
- '10: Jafferis exact R_{scft} from F minimization.
- '11-now: exact sphere indices, chiral ops. **Many.**

Here, we explore:

- The rich phase structure of simple 3d, N=2 theories, with real parameters.
- Flat space, as opposed to spheres, e.g. moduli spaces of vacua.
- Vortices, monopole operators, Coulomb branch = monopole's Higgs branch, etc.
- Distinctions between these things.
- Witten index of theories with matter.

review: 4d theories, and susy

- Most 4d theories are IR-free: anything w/o non-Abelian gauge fields is IR-free. Even w/ non-Abelian gauge fields, the asymptotic freedom bound. Even there, many have IR-free duals, confinement.
- Susy theories highly constrained by holomorphy in chiral superfields and parameters. No phase transitions: can have 2nd order <chiral>, but can move around them, no walls separating phases.
- Seiberg duality: SU(N_c) with N_f flavors = (IR) dual to SU(N_f -N_c) with N_f flavors and $W = M_{i\tilde{j}} q^i \tilde{q}^{\tilde{j}}$

3d theories, new elements

- U(1) can have non-trivial IR dynamics.
- **Real** parameters: masses and FI terms, not in chiral superfields, no holomorphy help. Allows for interesting phase transitions.
- Chern-Simons parameters, quantized. Topological phases, vacua.
- Dualize photon, Coulomb branches, vortex particles, monopole operators.

Review: 3d U(1) gauge theory

- Compact U(1) (vs R): quantized electric charges, $n_i \in \mathbf{Z}$. Does not require non-Abelian UV completion; then, no instantons. U(1) vs U(1) $\subset$ SU(2). If U(1), then get:
- A (topological) $U(1)_J$ global symmetry. Current: $J^\mu = \epsilon^{\mu\nu\rho} F_{\nu\rho}/4\pi$ ~ magnetic field. Charges: $q_J = \int F/2\pi = c_1(F) \in \mathbf{Z}$
- ANO vortex particle: Higgs scalar winds in space around U(1): nugget of magnetic flux.

Review: 3d gauge theory, cont.

$$\mathcal{S}_{gauge} = -\frac{1}{4e^2}\int d^3x F^2 \ + \int a(x) d\frac{F}{2\pi}$$

$$da/2\pi = *F/e^2 \qquad a \sim a + 2\pi$$

3d photon = dual to periodic scalar a. $V_{eff}(a) = 0.$

If $U(1)_J$ is spontaneously broken, then a = the NGB.

If U(1) is in an SU(2) UV completion, then $U(1)_J$ is explicitly broken by SU(2)/ U(1) instantons. Without fermions, instanton can generate (Polyakov). $V_{eff} = e^{-S_{inst}} \cos a$
(If fermions, instanton has fermi zero modes.)

3d, N=2 susy: $\{Q_\alpha, \bar{Q}_\beta\} = 2\sigma^\mu_{\alpha\beta} P_\mu + 2iZ\epsilon_{\alpha\beta}$

Chiral operators: $[\bar{Q}_\alpha, \mathcal{O}(x)] = 0$

(For $Z = 0$)

Chiral superfields: $\bar{D}_\alpha X(x, \theta_\alpha, \bar{\theta}_\alpha) = 0$

Real superfields: $V(x, \theta_\alpha, \bar{\theta}_\alpha)$

(real scalar, ~ A_4)

Real linear multiplet: $\Sigma \equiv D\bar{D}V = \sigma + \cdots + \bar{\theta}\sigma^\mu \theta F^{\nu\rho}\epsilon_{\mu\nu\rho}$

$D^2\Sigma = \bar{D}^2\Sigma = 0$

(dual to a)

$U(1)_J$ global super-current: $J_J = \Sigma/2\pi$

Coulomb branch

- Continuous moduli space of susy vacua, vev for $\Sigma \equiv D\bar{D}V = \sigma + \cdots + \bar{\theta}\sigma^\mu\theta F^{\nu\rho}\epsilon_{\mu\nu\rho}$

- When Coulomb branch exists, useful to use

  chiral superfield: $X_\pm = e^{\pm(\sigma/e^2 + ia)}$ (+ quantum corrs.)

$$U(1)_J : \;\; q_J = \pm 1$$

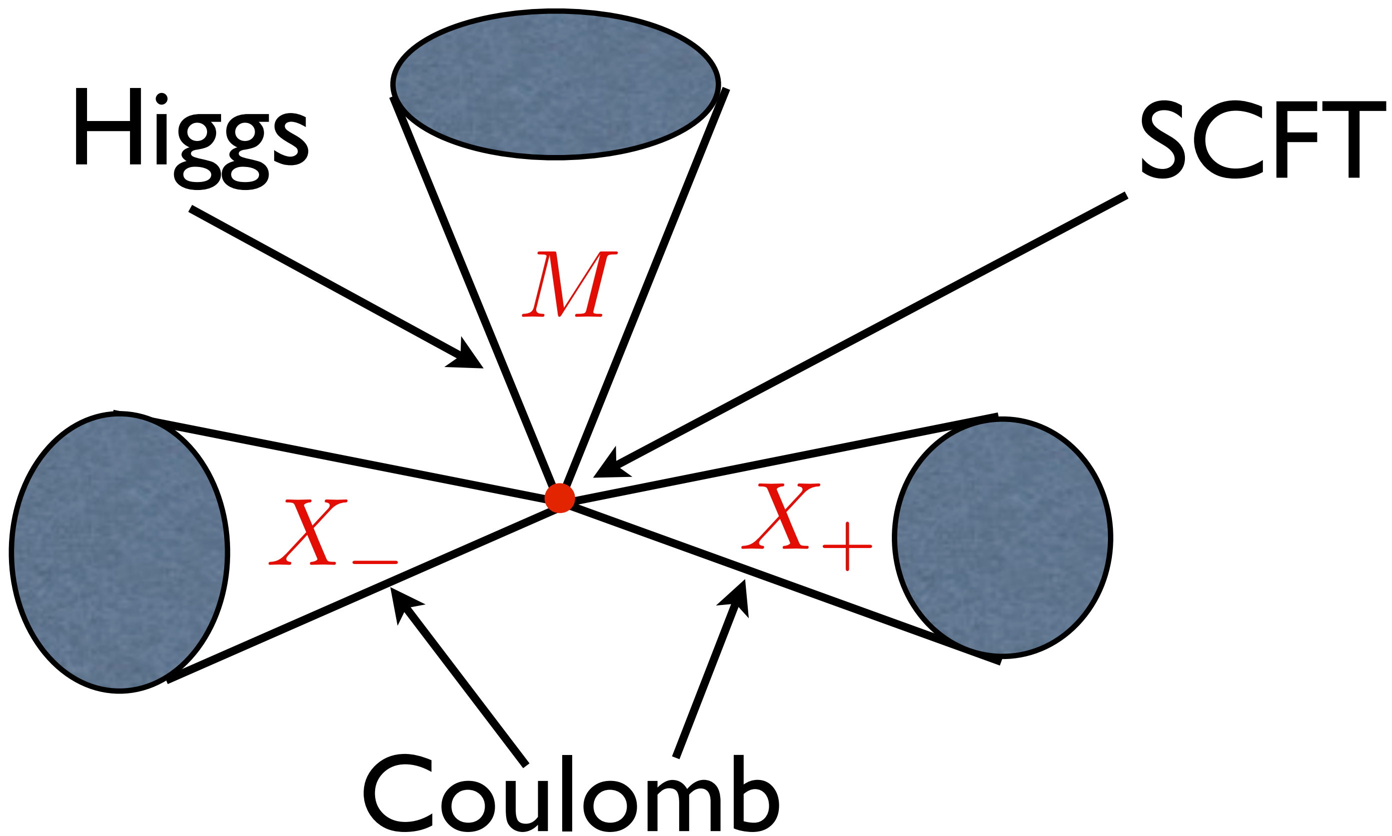
Classical constraint: cylinder $X_+ X_- \sim_{cl} 1$

Quantum theory with matter: entire X plane,

including X=0, where $U(1)_J$ is unbroken.

Example: U(1) with 1 flavor

$M = Q\tilde{Q}$



AHISS:
d'BHO

$$W_{dual} = MX_+X_-$$

AHISS: we mostly avoided chiral matter, real masses, FI parameters, CS parameters. Explore all that here.

3d N=2 U(1) gauge theory

$$\Sigma \equiv D\bar{D}V = \sigma + \cdots + \bar{\theta}\sigma^{\mu}\theta F^{\nu\rho}\epsilon_{\mu\nu\rho}$$

$$\mathcal{L}_{gauge} \supset \int d^4\theta \left(\frac{1}{2e^2}\Sigma^2 - \frac{k}{4\pi}\Sigma V - \frac{\zeta}{2\pi} V \right)$$

Chern-Simons parameter

Fayet Iliopoulos parameter

real mass parameter

$$\mathcal{L}_{matter} \supset \int d^4\theta Q_i^{\dagger} e^{n_i V + m_i \theta\bar{\theta}} Q_i$$

$$V_{cl} = \frac{e^2}{32\pi^2}\left(\sum_i 2\pi n_i |Q_i|^2 - \zeta - k\sigma \right)^2 + \sum_i \overbrace{(m_i + n_i\sigma)}^{\equiv m_i(\sigma)} |Q_i|^2$$

Quantum potential, vacua

Susy vacua: $m_i(\sigma)Q_i = 0$ and $\sum_i 2\pi n_i |Q_i|^2 = F(\sigma)$

$$F(\sigma) \equiv \zeta_{eff} + k_{eff}\sigma = \zeta + k\sigma + \frac{1}{2}\sum_i n_i |m_i + n_i\sigma|$$

i.e. $\zeta_{eff} = \zeta + \frac{1}{2}\sum_i n_i m_i \mathrm{sign}(m_i(\sigma))$ Piecewise const., 1-loop =exact

$$k_{eff} = k + \frac{1}{2}\sum_i n_i^2 \mathrm{sign}(m_i(\sigma))$$

$k_{eff} \in \mathbf{Z}$ From including monopole operators, or considering on general (spin manif.) spacetime.

Quantum potential, vacua

Susy vacua: $m_i(\sigma)Q_i = 0$ and $\sum_i 2\pi n_i |Q_i|^2 = F(\sigma)$

$$F(\sigma) \equiv \zeta_{eff} + k_{eff}\sigma = \zeta + k\sigma + \frac{1}{2}\sum_i n_i |m_i + n_i \sigma|$$

Generically isolated vacua with mass gap. Vacua:

- Higgs: $\sigma = \sigma_{Q_i} \equiv -m_i/n_i, \;\; Q_i \neq 0$ need $n_i F(\sigma_{Q_i}) > 0$
- Coulomb branch: $Q_i = 0, \; F(\sigma) = 0 \quad \zeta_{eff} = k_{eff} = 0$
- Topological vacua: $Q_i = 0, \sigma = \sigma_I = -\zeta_{eff}/k_{eff}$.

Eg: U(1) with chiral matter Q

charge n

One real parameter = FI term.

$$k_{eff} = k + \frac{1}{2} n^2 \mathrm{sign}(\sigma) \qquad \zeta_{eff} = \zeta$$

Coulomb branch (non-compact) if $k = \pm \frac{1}{2} n^2, \ \zeta = 0$

$X = X_{\mp}$ $\quad n = 1$ dual to free chiral superfield X. Dimofte Gaoitto Gukov

$K(X) \sim |X|^2$ near X=0, smooth. $\quad U(1)_J \to U(1)_X$

$n > 1$ dual to a Z_n orbifold. $\quad \zeta \to m_X$

For nonzero FI term, and/or other k: mass gap.

U(1)$_k$ with chiral matter Q

Chern-Simons term   charge 1

Take $\zeta > 0$ $F(\sigma) = \zeta + k\sigma + \frac{1}{2}|\sigma|$ $k \in \mathbf{Z} + \frac{1}{2}$

Vacua: $\sigma = 0,\ 2\pi|Q|^2 = \zeta$ "Higgs" U(1) broken

$Q = 0,\ \sigma = -\zeta/k_{eff}$ "Topological" Q massive

Higgs vacuum has BPS vortices (static, 2d): $z = re^{i\theta}$

$Q = \sqrt{\frac{\zeta}{2\pi}} f(r) e^{i\theta}$ $A_\theta = \frac{g(r)}{2\pi r}$ $r > 1/m_\gamma :\ f, g \to 1.$

(Exp. fast. Flux confinement)

$q_J :\ \pi_1(U(1))$ $m_{vortex} = |q_J \zeta|$ Abrikosov-Nielsen-Olsen

Vortices, monopole operators, Coulomb branch

Duality: "Coulomb branch = Higgs condensate of magnetic d.o.f.." With FI term, U(1) = Higgsed w/ BPS vortex particles. Become massless as FI term is taken to 0, can condense, = (?) the Coulomb branch. AHISS

Coulomb branch operator X = monopole operator, quanta are the magnetic particle vortices. Kapustin et.al. studied it on a spatial sphere S^2. We're here more interested in non-compact spacetime.

Non-compact vs compact

- With (2 or more) non-compact space directions, can have moduli spaces of vacua, e.g. Coulomb and Higgs branches.
- Vs compact space: wavefunction spreads over moduli space, single vacuum. E.g. on sphere: states map to operators in radial quantization, unique groundstate = 1 op.
- Different zero modes, (non) normalizable. New parameters in non-compact cases.

Return to the vortex

Ex: $U(1)_k$ with chiral matter Q

Vortex has one complex (boson + fermion) zero mode: broken translations and half susy generators of BPS particle. Normalizable: particle's position z(t), & fermion zero mode (raising / lowering ops): particle supermultiplet.

		$U(1)_r$	spin
$q_J = 1:$	$\|a\rangle$	$1/2$	$-(k+1/2)/2$
	$\|b\rangle$	$-1/2$	$-(k-1/2)/2$

$k \in \mathbf{Z} + \frac{1}{2}$

spins integer or half-integer

Coulomb branch X_+ exists for k = -1/2, precisely when the vortex has the same quantum numbers as X_+ .

Monopole operator

- Disorder operator: at spacetime point x_0 gives a magnetic flux there: $dF = 2\pi\delta^3(x - x_0)$
- Susy chiral monopole operator insertion:

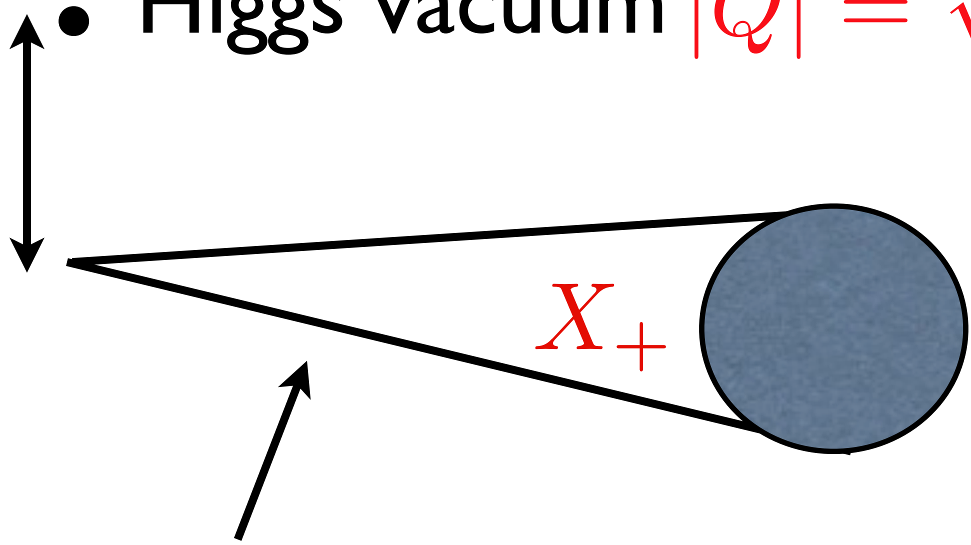
$$\bar{D}^2\Sigma = 2\pi\delta^3(x - x_0)\theta^2 \quad \rightarrow \sigma \sim \frac{1}{r} \quad \text{(3d r)}$$
$$D^2\Sigma = 0$$

Close to operator = far out on Coulomb branch: This shows the local operator is associated with the entire Coulomb branch. Operator ~ related to insertion in functional integral of op: $X = e^{\sigma/e^2 + ia}$

Vortices and monopole ops

Vortices = BPS particles, in Higgs vac., same quantum numbers as X_+ become massless, can condense as $\zeta \to 0$

- Higgs vacuum $|Q| = \sqrt{\zeta/2\pi}$



Monopole operator, spreads over X Coulomb branch. Q is massive away from the origin.

Vortex doublet ~ monopole chiral operator. Vortex has zero modes. (Mon. Op. in rad. quant. does not: different super-multiplet components have different operator dimensions.)

Vortices vs monopole ops

BPS states vs Chiral operators

States = particle worldline z(t) localized in space.

operators = insertion, localized in spacetime.

$$\overline{Q}_\pm = (\overline{Q}_1 \pm i\overline{Q}_2)/2 \qquad Q_\pm = (Q_1 \pm iQ_2)/2$$

$$\{Q_\pm, \overline{Q}_\mp\} = P_0 \pm Z$$

$$Q_-|BPS\rangle_{Z>0} = \overline{Q}_+|BPS\rangle_{Z>0} = 0$$

$Z > 0$

	U(1)$_R$	U(1)$_{spin}$
$\lvert a\rangle \sim \overline{Q}_-\lvert b\rangle$	r	s
$\lvert b\rangle \sim Q_+\lvert a\rangle$	$r-1$	$s+1/2$

BPS states vs chiral operators

$$Q_-|BPS\rangle_{Z>0} = \overline{Q}_+|BPS\rangle_{Z>0} = 0$$

vs $[Q_\alpha, \mathcal{O}(x)] = 0$ ...different conditions.

BPS states:

$$|a\rangle = \lim_{\tau\to\infty} e^{-(H-Z)\tau}\mathcal{O}|0\rangle$$

$$|\bar{a}\rangle = \lim_{\tau\to\infty} e^{-(H+Z)}\bar{\mathcal{O}}|0\rangle$$

(anti)-chiral monopole op. on vacuum

e.g. vortices

project to lowest energy state

QFT 101: create asymptotic particle states from field operators acting on vacuum in far past / future.

Vortices vs monopole ops with non-minimal matter

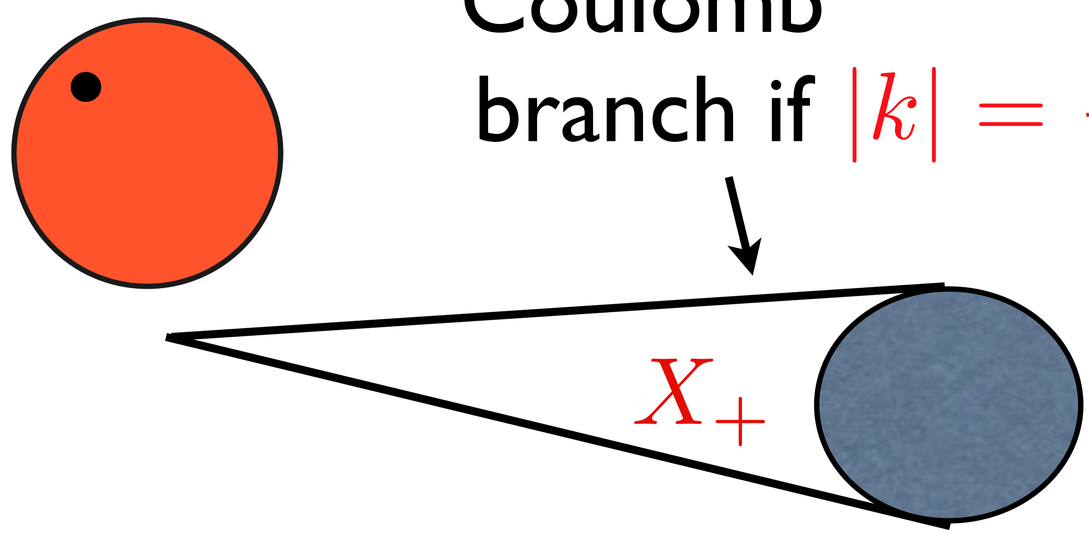
E.g. $U(1)_k$ with N fields Q of charge +1, for FI term >0, vac. manifold = CP^{N-1} non-linear sigma model.

Higgs moduli space

$|Q| = \sqrt{\zeta/2\pi}$

Vacuum pt on space • = super-selection parameter. (Vs 2d)

Coulomb branch if $|k| = \frac{1}{2}N, \ \zeta = 0$



Study of skyrmions / vortices in this model revealed additional super-slection parameters. Play some role in our picture here.

Extra zero modes in this ex.:

Choose Higgs vacuum (by SU(N) rotation): $Q_f = \delta_{f,1}\sqrt{\zeta/2\pi}$

Vortex: $Q_{f=1} = \sqrt{\frac{\zeta}{2\pi}} f(r) e^{i\theta}, \; A_\theta = \frac{g(r)}{2\pi r} \quad r > 1/m_\gamma : \; f, g \to 1.$

N-1 (bose+fermi) zero modes: $Q_{f>1} = \rho_f / r$

$$\psi_{f>1} = \psi_f^0 / r$$

Non-normalizable, so frozen superselection sectors. Interpretation as internal d.o.f. of mirror dual.Quantizing fermi zero modes, non-normalizable ones give separate Hilbert spaces. A vortex has X's quantum #s (spin and global charges) iff $k = \pm N/2$ = same condition for Coulomb branch. <X> = vortex condensate.

Monopole operator charges

	$U(1)_k$	$U(1)_j$	$U(1)_R$	$U(1)_J$
Q_i	n_i	δ_{ij}	0	0
$X_\pm$	$\mp(k \pm k_c)$	$-\frac{1}{2}\lvert n_j\rvert$	$\frac{1}{2}\sum_i \lvert n_i\rvert$	± 1

$$k_{eff}(\sigma = \pm\infty) = k \pm k_c \qquad k_c \equiv \frac{1}{2}\sum_i n_i |n_i|$$

Via induced Chern-Simons terms, including for gauge-gauge, gauge-global, global-global U(1)s.

Same charges for vortices, at end of the day, after accounting for n.n. fermion zero mode effects etc.

Witten index Tr $(-1)^F$

- Defined on non-compact space or T^2
- ill defined if non-compact moduli spaces.
- For generic real parameters, mass gap so it's well defined. We compute it.
- It doesn't have phase transitions as real parameters vary (see explicitly + via compactification to 2d, since then get holomorphy in twisted chiral parameters.)

Witten index: Tr (-1)^F

$$U(1)_k \quad \mathrm{Tr}(-1)^F = |k| + \sum_i n_i^2/2 \qquad |k| \geq |k_c|$$

$$|k_c| + \sum_i n_i^2/2 \qquad |k| \leq |k_c|$$

$k_c \equiv \sum_i n_i|n_i|/2$ Noncompact Coulomb branch at $k = \pm k_c$ for tuned FI parameter. There $Tr(-1)^F = \infty$

Counting the vacua:

- Higgs: $Tr(-1)^F|_{\langle Q_i \rangle} = n_i^2 \longleftarrow Z_{n_i}$ orbifold, unbroken discrete gauge group.
- Topological: $Tr(-1)^F|_{\sigma_I} = |k_{eff}|$

A_i 0 modes ~ 2d quantum particle in B~k, LLL states # =k
Equiv: Witten's CS$\longleftrightarrow$WZW$_k$ rel'n: 3d states$\longleftrightarrow$2d primary ops.

Nonabelian groups similar

- Consider e.g. $SU(N)_k$ and $U(N)_k$
- Instanton corrections and Weyl group modifications vs abelian case.
- Dynamical susy breaking with small k, N_f for generic real parameters. Susy moduli spaces for tuned real parameters.
- $Tr(-1)^F = J_G(k')$ = #WZW primary ops,

$$k' = |k| - h + \sum_f T_2(r_f)/2$$

Checks + extends result of Witten for pure 3d N=1 SYM

E.g. SU(2)$_k$

Recall case of k=0, zero real parameters (AHW; AHISS):

$N_f = 0: \ W_{inst} = 1/Y$ (runaway, no susy vac.)

$N_f = 1: \ W_{dyn} = 0, \ MY = 1$ (quant. def. smooth mod. sp.)

$N_f = 2: \ W_{dyn} = Y\mathrm{Pf}M$ (conf., SCFT at origin)

General, with generic real parameters, mass gap with

$$\mathrm{Tr}(-1)^F = J_{SU(2)}(k') = k' + 1 \quad (k' \geq 0)$$

= 0, susy broken if $k' < 0$

$$k' = |k| - 2 + \frac{1}{2}\sum_i T_2(r_i) \qquad = |k| - 2 + N_f$$

$k' = 0 \leftrightarrow$ confinement

Applications

- Dual theories must have same index. Quick, non-trivial check of dualities.
- Theories with confining duals (no gauge fields) must have k'=0, and index = 1. Unique susy vac with generic real params.
- Some variants, with multiple vacua.

E.g. Jafferis-Yin's appetizer: $SU(2)_1 \quad Q \in \mathbf{3} \qquad k' = 1 - 2 + 4/2 = 1$

$Tr(-1)^F = J_{SU(2)}(k') = k' + 1 = 2$ two vacua, (conf. & oblique conf.)

Dual: free field M $\times U(1)_2 \quad = \quad Tr(-1)^F = 1 \times 2 \leftarrow \times U(1)_2$

$M = Q^2 : SU(2)_1 \to U(1)_2$ Quick check, shows need for topo sector $\langle M \rangle$

Aharony & Giveon-Kutasov

Aharony: $U(N_c)_0 \leftrightarrow U(N_f - N_c)_0$ unusual!

Dual lagrangian contains its own monopole ops. $W_{dual} = Mq\tilde{q} + X_+\tilde{X}_- + X_-\tilde{X}_+$

Many nontrivial checks in literature. Another simple check: Witten indices of both with real mass deformations agree, by WZW(level k') level-rank duality.

Giveon-Kutasov: $U(N_c)_k \leftrightarrow U(N_f + |k| - N_c)_{-k}$

$W_{dual} = Mq\tilde{q} \leftarrow$ more conventional.

Giveon Kutasov; Benini, Closset, Cremonesi: assuming Aharony, derive Giveon-Kutasov via adding real masses, inducing CS terms.

Witten indices again agree, by WZW (k') level-rank duality.

Flow: from Giveon-Kutasov to Aharony duality

The monopole operators in the Lagrangian of Aharony dual is bizarre. “Q: Is that allowed?”

We see the answer is “Yes”: embedding it in a standard, renormalizable, QFT. The monopoles of the electric side are composite, approximately free fields on the magnetic side. The coupling to monopole operators in the Lagrangian arise dynamically, from an instanton, from associated Higgsing U(N) to U(N-1)xU(1)

Giveon-Kutasov to Aharony

E.g. start with $U(N_c)_{-1}$ with $N_f + 1$ flavors. Give one flavor a real mass, flow to $U(N_c)_0$ with N_f

Dual to $U(N_f - N_c + 2)_1$ with $N_f + 1$ flavors, with real mass for one, flows to $U(N_f - N_c)_0 \times U(1)_{\frac{1}{2}} U(1)_{\frac{1}{2}}$

$X_+ \quad X_-$

$$W_{dual} = Mq\tilde{q} + X_+\tilde{X}_- + X_-\tilde{X}_+$$

Monopole ops of dual. Couplings induced by instanton in UV thy.

Conclude

- 3d QFTs are rich theory-playgrounds for exploring rich structure of QFT, develop new methods, intuition, duality webs.
- Probably more to say about monopole operators in non-compact spacetime. Hope to give new insights into magnetic dual d.o.f., also in 4d.
- Methods + results useful for upcoming work on 4d/3d duality inter-connections. Aharony, Razamat, Seiberg, Willet to appear.

Thank you!