Pseudo-Chern-Simons Terms in the Standard Model

(with Applications)

Rutgers, Dec. 5, 2007

Based on

- Baryon-Number-Induced Chern-Simons Couplings of Vector and Axial-Vector Mesons in Holographic QCD, PRL 99,14 (2007); arXiv:0704.1604 w/ Sophia Domokos
- Anomaly mediated neutrino-photon interactions at finite baryon density; arXiv:0708.1281 w/ Chris Hill and Richard Hill.
- Pseudo-Chern-Simons Terms in the Standard Model; arXiv:07mm.xxxx w/ Chris Hill and Richard Hill
- Work in preparation

"Pseudo-Chern-Simons Term" is an awkward phrase for interaction terms of the form

$$S_{pCS} = \int A_1 \wedge A_2 \wedge dA_3 = \int d^4x\ \epsilon^{\mu\nu\lambda\rho} A_{1\mu} A_{2\nu} \partial_\lambda A_{3\rho}$$

with the A_i vector fields.

This talk will be about either the origin of such terms or their applications, but I want to first make a few general comments:

- If A_1 is a background vector field (e.g. the baryon current) then in its rest frame $A_1 = (A_1^0, \vec{0})$ we have

$$S_{pCS} \sim \int A_1^0 \underbrace{\epsilon^{ijk} A_{2i} \partial_j A_{3k}}_{\text{CS term in 3 dimensions}}$$

- In a Parity invariant theory an odd number of the A's must be axial-vector rather than vector fields.

- S_{pCS} is not gauge invariant, so for fundamental gauge fields A_1 and A_2 must be massive (Stuckelberg) gauge fields or bkgnd vector fields.

Main Points

- Such terms arise in the low-energy effective theory of QCD coupled to the electroweak theory and are part of the Standard Model.
- These terms are linked to anomalies, familiar from $\pi_0 \to \gamma + \gamma$
- One such term may explain the MiniBoone excess at low-energies and should have astrophysical implications.
- Other pCS terms make new predictions for vector meson couplings.

Outline

1. pCS terms in a toy model
2. pCS terms in the Standard Model
3. Sanity check: $f_1 \rightarrow \rho + \gamma$
4. Application to the MiniBoone excess
5. pCS terms in AdS/QCD (time permitting)
6. Conclusions

A Toy Model with pCS Terms

		B_L	B_R	L_L	L_R
"quarks"	q_L	1	0	0	0
	q_R	0	1	0	0
"leptons"	ℓ_L	0	0	1	0
	ℓ_R	0	0	0	1

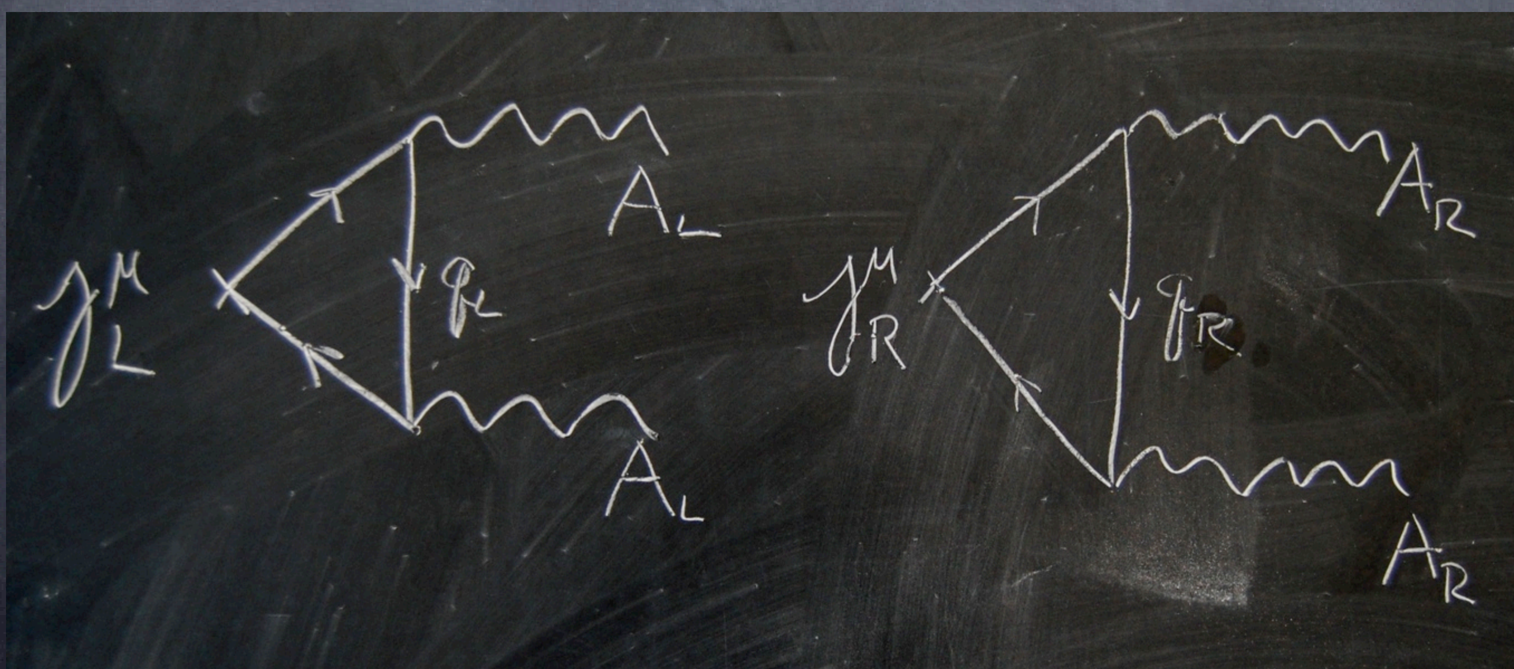
We gauge $U(1)_L \times U(1)_R$ with generators

$$
\begin{aligned}
Q_L &= B_L - L_L \\
Q_R &= B_R - L_R
\end{aligned}
$$

The action is

$$
\begin{aligned}
S = \quad & \int d^4x \; \bar{q}_L (i\partial\!\!\!/ + A\!\!\!/_L) q_L + \bar{q}_R (i\partial\!\!\!/ + A\!\!\!/_R) q_R \\
& + \bar{\ell}_L (i\partial\!\!\!/ - A\!\!\!/_L) \ell_L + \bar{\ell}_R (i\partial\!\!\!/ - A\!\!\!/_R) \ell_R \\
& - \tfrac{1}{4} (F_L)^2 - \tfrac{1}{4} (F_R)^2
\end{aligned}
$$

The quark sector by itself has triangle anomalies



$$\delta S_q^{eff} = \frac{1}{24\pi^2}\int\Big[-\epsilon_L dA_L dA_L + \epsilon_R dA_R dA_R\Big]$$

But this anomaly is cancelled by the leptons so that we can consistently gauge $U(1)_L \times U(1)_R$

We now generalize this model in two ways:

- We add a quark mass term and integrate out the quarks.
- We add in a background field coupling to the (anomalous) vector baryon current.

To add a mass term consistent with gauge invariance we add a Higgs field with a non-zero vev $\Phi = ve^{i\phi/f}$ coupled to the quarks leading to

$$m_q e^{i\phi/f}\bar{q}_L q_R + h.c. \qquad \phi/f \sim \pi^0/f_\pi \text{ in QCD}$$

The $U(1)_L \times U(1)_R$ acts as

$$\delta_L q_L = i\epsilon_L q_L, \; \delta_L A_L = d\epsilon_L, \; \delta_L \phi/f = \epsilon_L$$

$$\delta_R q_R = i\epsilon_R q_R, \; \delta_R A_R = d\epsilon_R, \; \delta_R \phi/f = -\epsilon_R$$

Now consider the low-energy theory after integrating out the massive quarks. This is a function of ϕ, A_L, A_R and since the anomalies must still cancel, it must have an anomalous variation equal to that of S_q^{eff} (D'Hoker and Farhi). This "WZW" contribution is

$$\begin{aligned}\Gamma_{WZW} = & - \frac{1}{24\pi^2}\int\Big(A_L A_R dA_L + A_L A_R dA_R \\ & + \frac{\phi}{f}\left[dA_L dA_L + dA_R dA_R + dA_L dA_R\right]\Big)\end{aligned}$$

We now add a new ingredient: a background field coupled to the baryon current (like the omega meson in QCD).

$$S_q \to \int d^4x \; \bar{q}_L(i\partial\!\!\!/ - A\!\!\!/_L - \omega\!\!\!/\,)q_L + \bar{q}_R(i\partial\!\!\!/ - A\!\!\!/_R - \omega\!\!\!/\,)q_R$$

This leads to new terms in the variation of the WZW term proportional to omega:

$$\delta\Gamma_{WZW} = -\frac{1}{24\pi^2}\int \quad \epsilon_L\left(2dA_L d\omega + d\omega d\omega\right)$$
$$-\epsilon_R\left(2dA_R d\omega + d\omega d\omega\right)$$

Since omega doesn't couple to leptons, the anomaly no longer cancels. But, as one might expect, this can be fixed by adding a local counterterm whose variation cancels the omega dependent terms in $\delta\Gamma_{WZW}$:

$$\Gamma_c = \frac{1}{24\pi^2}\int \left(2\omega A_R dA_R - \omega A_R d\omega - (R \leftrightarrow L)\right)$$

The effective action then has the anomaly related terms

$$\begin{aligned}\Gamma_{eff} &= \Gamma_{WZW}(\phi, A_L+\omega, A_R+\omega)+\Gamma_c \\ &= \underbrace{\Gamma_{WZW}(\phi, A_L, A_R)}_{\text{Variation cancelled by leptons}}+\Gamma_{pCS}\end{aligned}$$

where in terms of $A_L = Z + A, \quad A_R = A$

$$\Gamma_{pCS} = \frac{1}{8\pi^2}\int \left(\omega\underbrace{[2dA + dZ]}_{\substack{\text{gauge invariant}\\ \text{massless field}\\ \text{strength}}} + \omega d\omega\right)\underbrace{\left(Z - d\phi/f\right)}_{\substack{\text{gauge invariant}\\ \text{massive vector}}}$$

This term correctly generates the anomaly in the baryon current by varying $\delta\omega = d\epsilon_B$

One can apply the same procedure to the SM starting with $\Gamma_{WZW}(U, A_L, A_R)$ with A_L, A_R gauging $SU(2)_L \times U(1)_Y$ and then adding a background of QCD vector and axial-vector mesons ρ, ω, a_1, f_1 .

This gives us $\Gamma_{WZW}(U, A_L, B_L, A_R, B_R)$ where for two flavors,

$$B_L + B_R = \begin{pmatrix} \rho^0 + \omega & \sqrt{2}\rho^+ \\ \sqrt{2}\rho^- & -\rho^0 + \omega \end{pmatrix}$$

$$B_L - B_R = \begin{pmatrix} a_1^0 + f_1 & \sqrt{2}a_1^+ \\ \sqrt{2}a_1^- & -a_1^0 + f_1 \end{pmatrix}$$

This leads to a large number of pCS terms:

$$\Gamma_{pCS} = \mathcal{C}\int dZZ\left[\frac{s_W^2}{c_W^2}\rho^0+\left(\frac{3}{2c_W^2}-3\right)\omega-\frac{1}{2c_W^2}f\right]+dAZ\left[-\frac{s_W}{c_W}\rho^0-\frac{3s_W}{c_W}\omega\right]+dZ\left[W^-\rho^++W^+\rho^-\right]\frac{s_W^2}{c_W}$$

$$+dA\left[W^-\rho^++W^+\rho^-\right](-s_W)+(DW^+W^-+DW^-$$

$$+\mathcal{C}\int Z\Big\{d\rho^0\left[-\frac{3}{2c_W}\omega-\frac{s_W^2}{c_W}a^0+\left(-\frac{3}{2c_W}+3c_W\right)f\right]+d\omega\left[-\frac{3}{2c_W}\rho^0\right.$$

$$+da^0\left[\frac{s_W^2}{c_W}\rho^0+\left(\frac{3}{2c_W}-3c_W\right)\omega-\frac{1}{2c_W}f\right]+df\left[\left(\frac{3}{2c_W}-3c_W\right)\right.$$

$$+dA\Big\{s_W\rho^0a^0+3s_W\rho^0f+3s_W\omega a^0+s_W\omega f\Big\}+dZ\Big\{-\frac{s_W^2}{c_W}(\rho^+a$$

$$\left.+)\right\}\Big\}$$

$$+\frac{3}{2}\left[W^+D\rho^-+W^-D\rho\right.\qquad a^-)+W^-(-\rho^++a^+)]\,d\omega$$

$$+\frac{1}{2}\left[W^+Da^-+W^-Da^+\right.\qquad a^-)+W^-(-3\rho^+-a^+)]\,df\,,$$

$$+\mathcal{C}\int 2\Big[(\rho^-f+\omega a^-)D\rho^++(\omega a^++\rho\qquad D\rho^0+(\rho^+a^-+\rho^-a^++\omega f+\rho^0a^0)\,d$$

$$+\mathcal{C}\int i\Big\{W^+W^-\Big[3c_WZ\Big]\omega+W^+W^-\Big[\left(c_W+\frac{1}{2c_W}\right)Z\Big]f\Big\},$$

$$+\mathcal{C}\int i\Big\{W^+W^-\left[\frac{3}{2}(\rho^0+a^0)\omega-\frac{1}{2}(\rho^0-a^0)f\right]$$

$$+W^+Z\left[\frac{3c_W}{2}\rho^-f-\frac{3c_W}{2}\rho^-\omega-\frac{c_W}{2}a^-f+\frac{3c_W}{2}\omega a^--\frac{1}{c_W}\rho^-f\right]$$

$$+W^-Z\left[-\frac{3c_W}{2}\rho^+f+\frac{3c_W}{2}\rho^+\omega+\frac{c_W}{2}a^+f-\frac{3c_W}{2}\omega a^++\frac{1}{c_W}\rho^+f\right]\Big\},$$

$$+\mathcal{C}\int i\Big\{W^+\Big[\rho^-\rho^0(\omega-2f)-\rho^-\omega a^0+\rho^0\omega a^-+\omega a^-a^0\Big]$$

$$+W^-\Big[\rho^+\rho^0(-\omega+2f)+\rho^+\omega a^0-\rho^0\omega a^+-\omega a^+a^0\Big]$$

$$+Z\Big[\rho^+\rho^-\left(\frac{1}{c_W}\omega+\left(-4c_W+\frac{2}{c_W}\right)f\right)+\rho^+\omega a^-\left(-2c_W+\frac{1}{c_W}\right)$$

MiniBoone
Excess

$f_1 \to \rho + \gamma$

What do we do with such couplings?

- Integrate out the massive $W^{\pm}, Z$ to get couplings for light fields (e.g. $\gamma, \nu, \bar{\nu}$) in the presence of background fields (e.g. baryon number)

- Treat the QCD mesons as fundamental fields in the spirit of Vector Meson Dominance.

The first is more clearly justified, the second involves an approximation which is not under good control, but often works reasonably well, and receives some justification from AdS/QCD.

The decay $f_1 \to \rho + \gamma$ provides a useful sanity check of this analysis. It is observed with a 5% branching ratio $\Gamma(f_1 \to \rho + \gamma) = 1.32\mathrm{MeV}$

Our coupling leads to

$$\Gamma = \frac{\lambda^2}{96\pi} \frac{(m_f^2 - m_\rho^2)^3}{m_f^2} \left[\frac{1}{m_f^2} + \frac{1}{m_\rho^2} \right]$$

with $\lambda = 3g_\rho^2/4\pi \simeq 9$ we find reasonable agreement with the overall rate.

More importantly, we get the observed helicity structure. There are two amplitudes:

$$A_0^{(\gamma)} = (J_z(f) = 0) \rightarrow (J_z(\rho) = -1) + (J_z(\gamma) = +1)$$

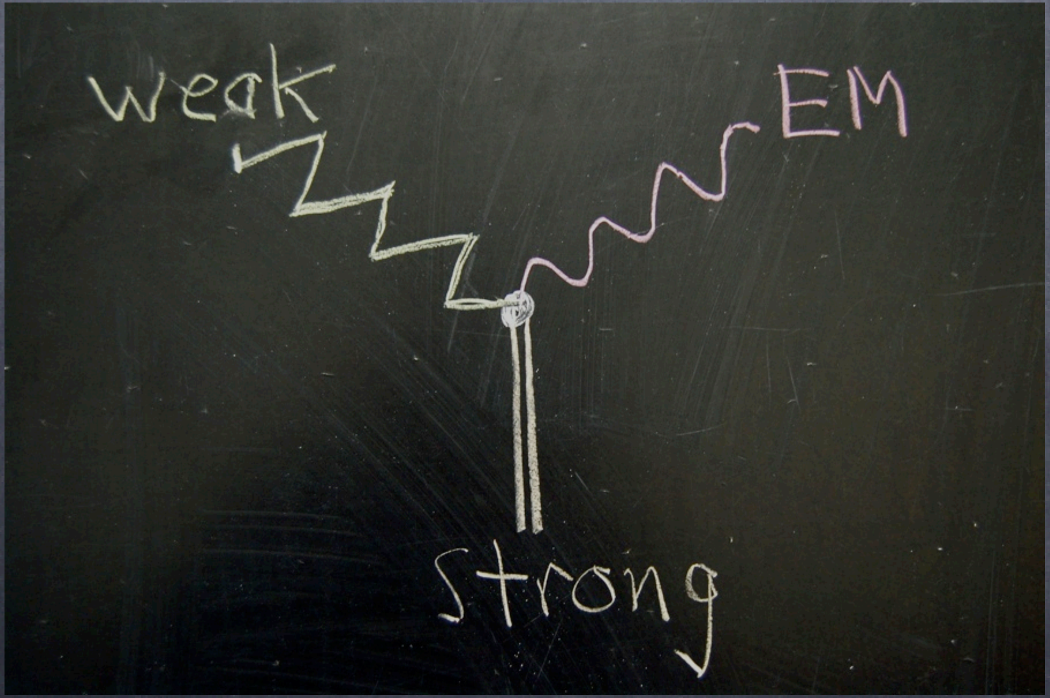
$$A_1^{(\gamma)} = (J_z(f) = +1) \rightarrow (J_z(\rho) = 0) + (J_z(\gamma) = +1)$$

We predict $A_1^{(\gamma)} = 0$ (transverse rho) in contrast to an earlier quark model analysis of Babcock and Rosner. We also predict the rate for $f_1 \rightarrow \omega + \gamma$ down by a factor of 9 from the above and not seen in the PDG.

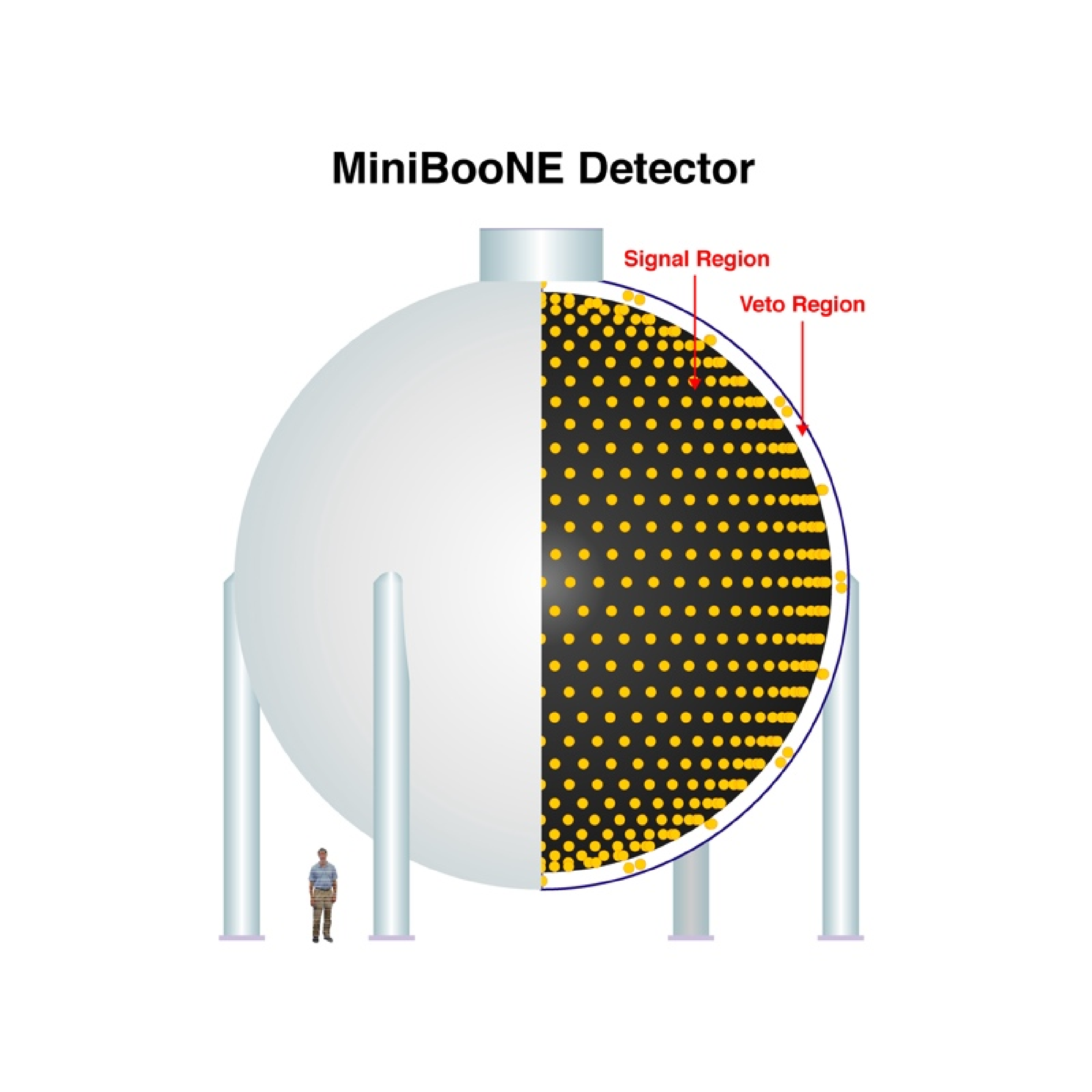
I now want to focus on the term

$$\frac{N_c}{48\pi^2}\frac{e g_\omega g_2}{\cos\theta_W}\epsilon_{\mu\nu\rho\sigma}\omega^\mu Z^\nu F^{\rho\sigma}$$

which gives rise to a "321 widget" which links the strong, weak, and EM interactions:



This interaction may be relevant to the results of the MiniBooNE experiment looking for $\nu_\mu \to \nu_e$ oscillations.



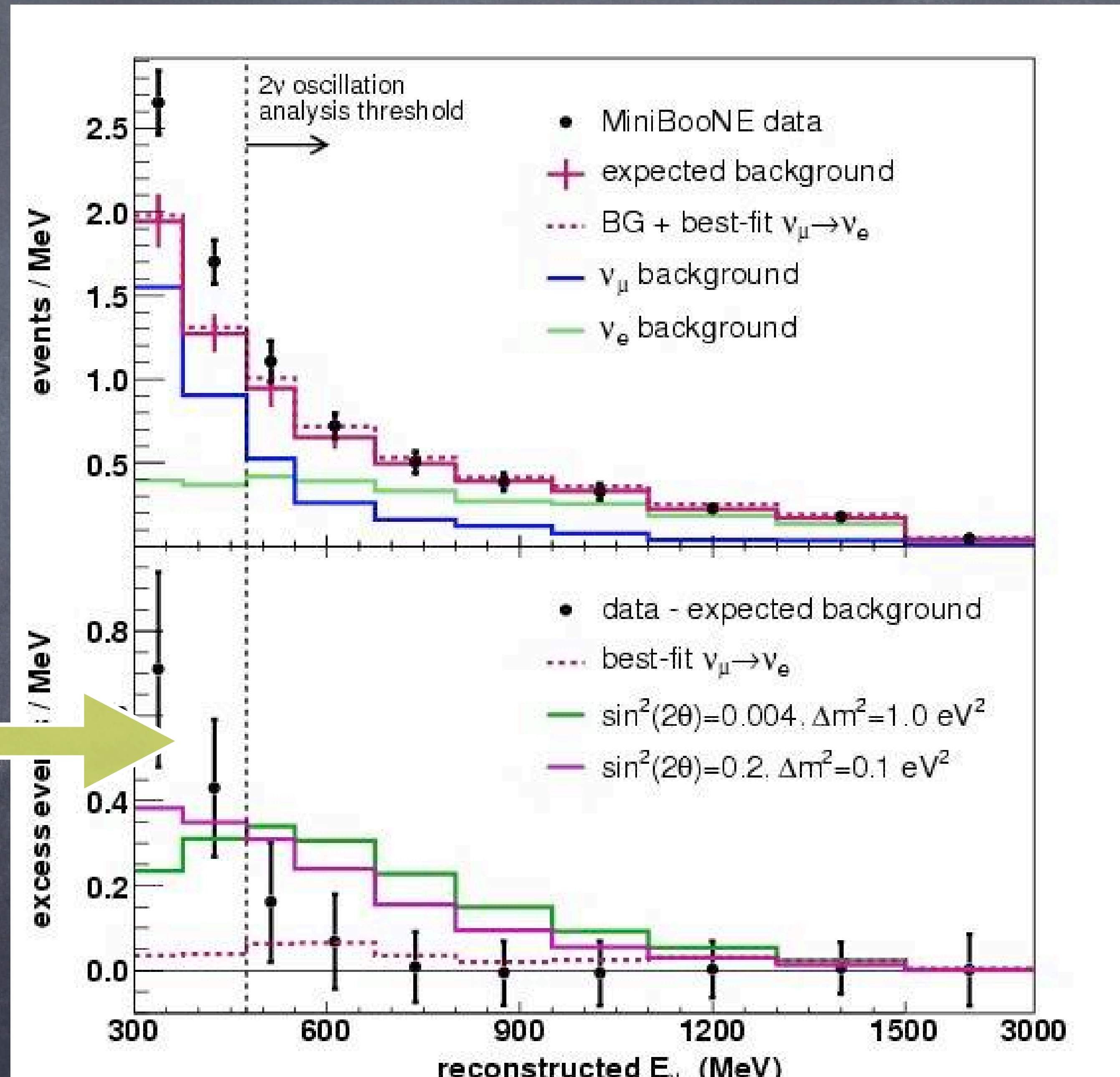
MiniBoone sees an excess at low energies:

$$300 < E_\nu < 475\mathrm{MeV}$$

$$96 \pm 17 \pm 20 \mathrm{events}$$

$$\mathrm{excess} \quad 3.7\sigma$$

(MiniBoone presentation at Lepton-Photon '07)

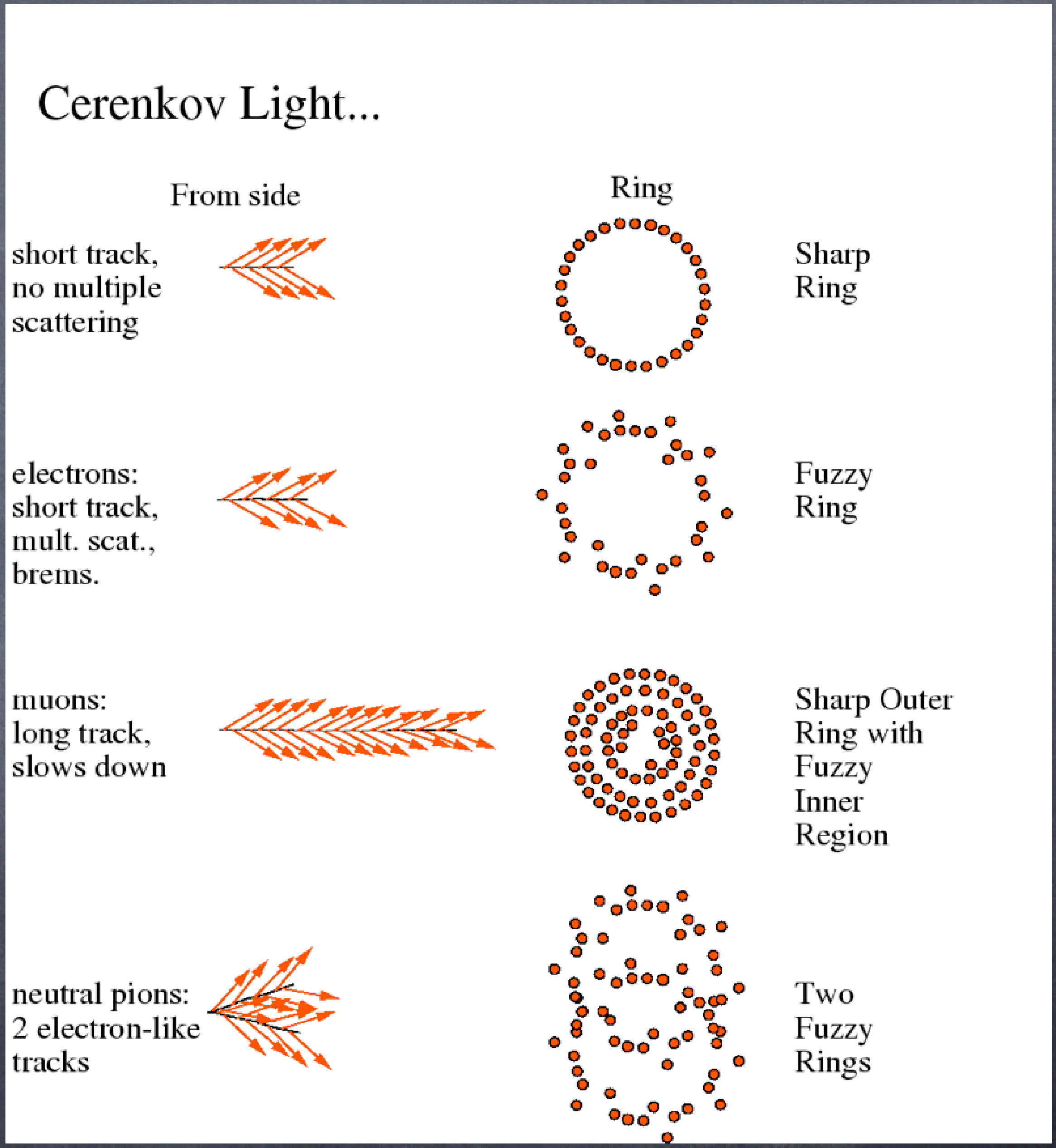


MiniBooNE distinguishes electrons from muons, but cannot discriminate between final state photons and electrons:

Electron

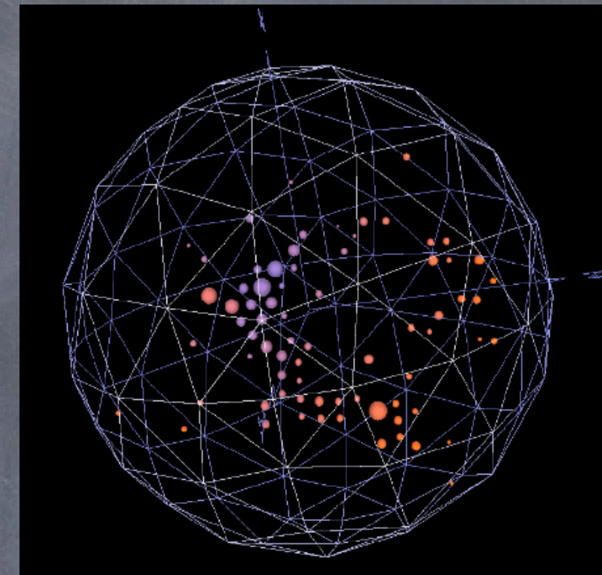
Muon

Two Photons

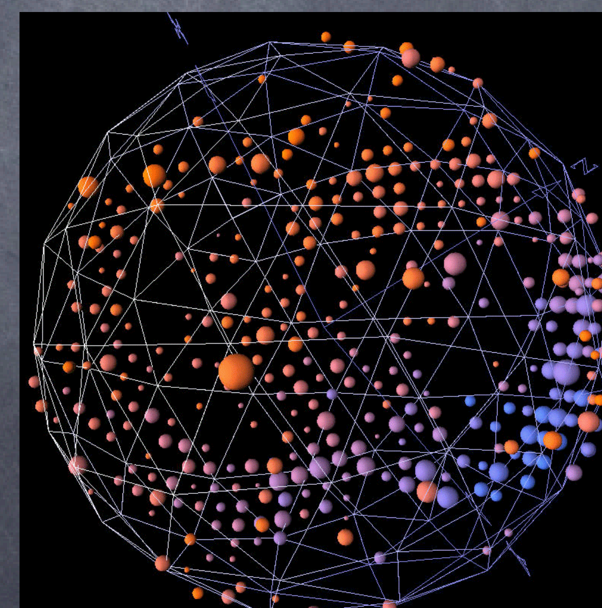


Thanks to Fermilab for graphics.

Michel Electron

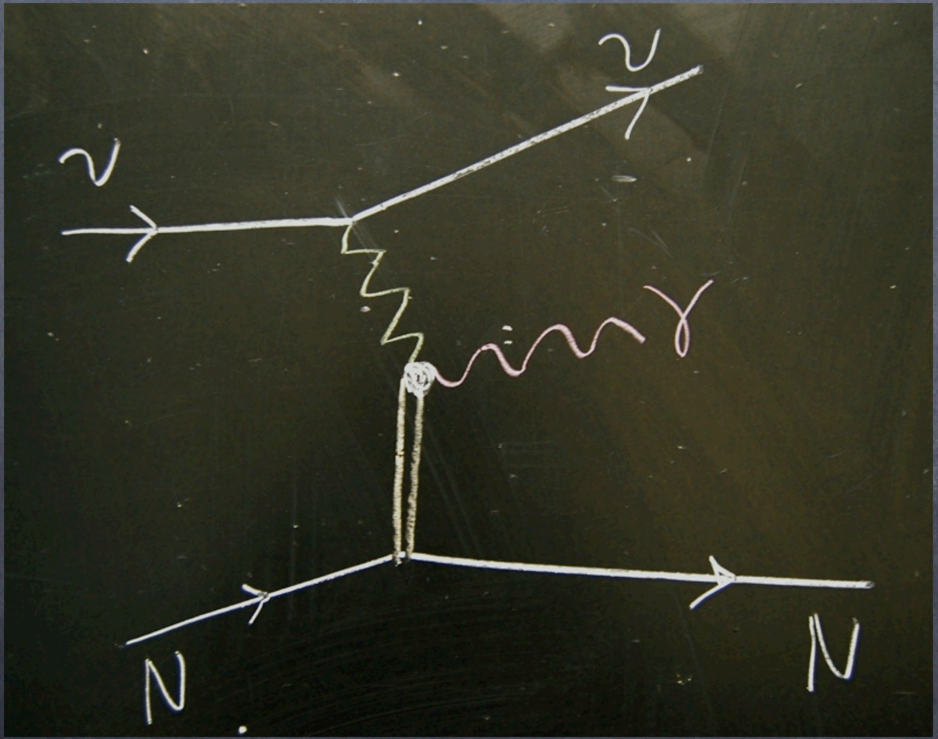


Two Photons from pi0 decay



Thanks to Fermilab for graphics.

Thus the $Z-\omega-\gamma$ vertex gives a background to the charged current events ($\nu_e + N \to e^- + N'$) MiniBooNE is looking for:



The most naive estimate ignores recoil, form factors, nuclear physics effects (Fermi motion, Pauli blocking), and replaces the neutrino beam by a mono-energetic beam at the peak energy of 700 MeV. This gives

$$\sigma \simeq \frac{1}{480\pi^6} G_F^2 \alpha \frac{g_\omega^4}{m_\omega^4} E_\nu^6$$

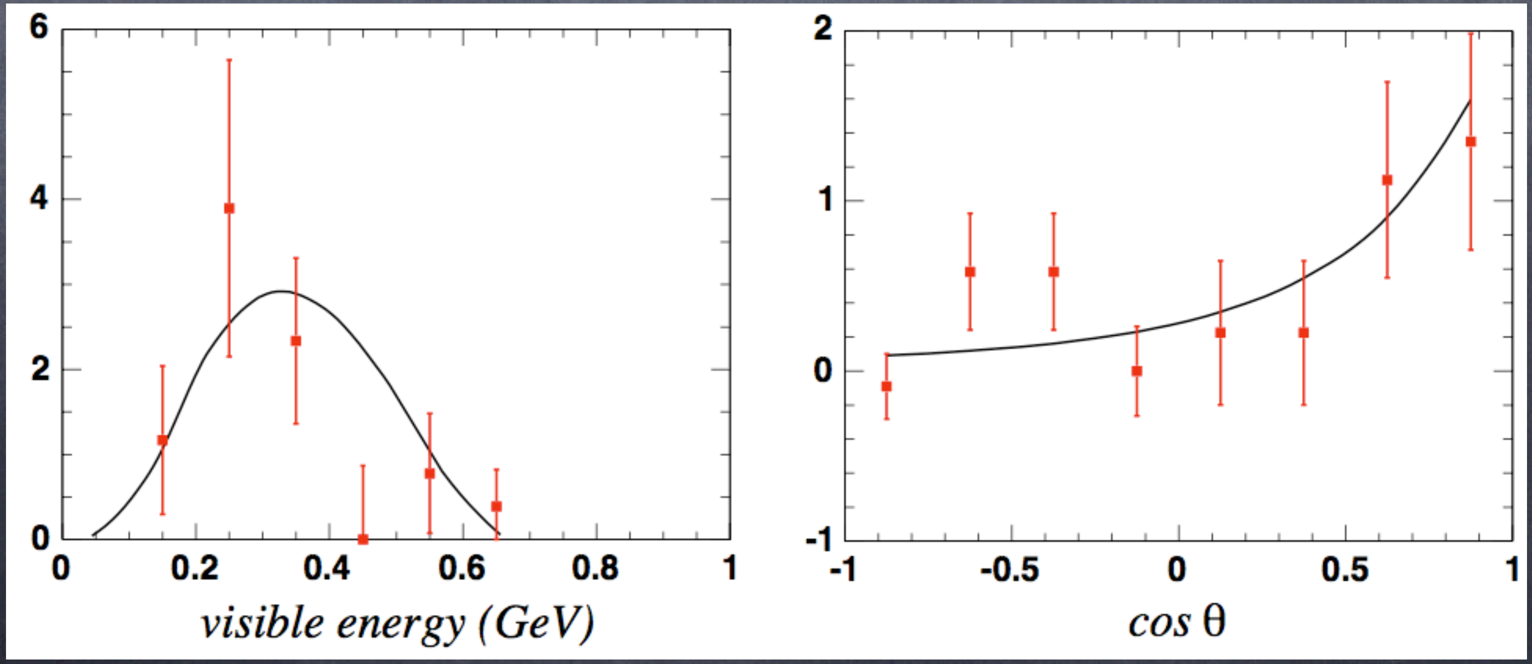
Which for every 2x10^5 CCQE events gives

$$\sim 140 \; (\frac{g_\omega}{10})^4$$

events from the anomaly-induced neutrino-photon interaction.

We are working on a more detailed comparison. Including some of the simpler effects leads to reasonable fits to data. Including nuclear recoil and a simple choice of form factor, but using a mono-energetic beam and scattering off of nucleons rather than nuclei gives, up to normalization.

Events



Note that MiniBoone plots the number of events vs. the reconstructed neutrino energy, assuming a two body final state (electron + nucleon).

The previous graphs plots the number of events vs. the (visible) photon energy, which is shared roughly equally with the final state neutrino in a three body final state (neutrino + nucleon + photon). A neutrino beam with a distribution of neutrino energies peaked at 700 MeV is shifted to a distribution of events with final state photons with the photon energy peaked at ~ 350 MeV.

pCS terms in AdS/QCD

5D AdS with IR cutoff: $$ds^2 = \frac{1}{z^2}\left(-dz^2 + dx^\mu dx_\mu\right)$$

$$0 < z \leq z_m$$

UV IR scale

Fields:

$$A_L^{a,\mu} \sim j_L^{a,\mu} = \bar{q}_L t^a \gamma^\mu q_L$$
$$A_R^{a,\mu} \sim j_R^{a,\mu} = \bar{q}_R t^a \gamma^\mu q_R$$
$$X^{\alpha,\beta} \sim \bar{q}^\alpha q^\beta$$

We will gauge $U(N_f)_L \times U(N_f)_R$ and focus on $N_f = 2$

$$S = \int d^4x dz \sqrt{|g|} Tr \left[|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2) \right]$$

with $g_5^2 = 12\pi^2/N_c$

One finds the pi, rho, a1 etc. by expanding around the tachyon solution

$$X_0(z) = \frac{1}{2} m_q \mathbf{1} z + \frac{1}{2} \sigma \mathbf{1} z^3$$

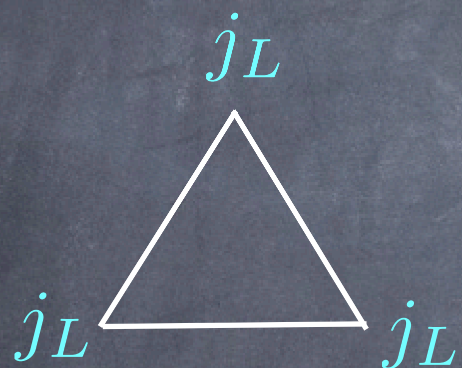
Coeff. of $\bar{q}q$ in S_{QCD}

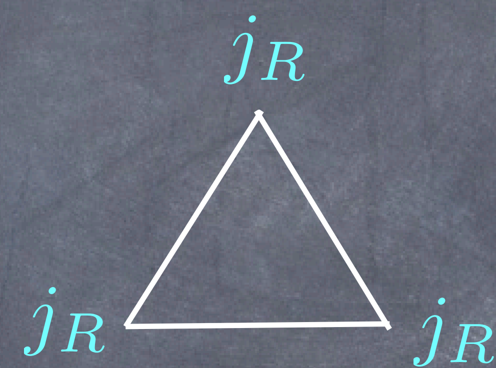
Exp. value $\langle \bar{q}q \rangle$

The model is defined by three parameters: z_m, m_q, σ

Add Chern-Simons terms

In QCD there are anomalies:





The AdS dual involves terms which are gauge invariant in bulk, but vary on the bndy in the same way that QCD would if coupled to fictitious flavor gauge fields.

$$S_{CS} = \frac{N_c}{24\pi^2}\int \omega_5(A_L) - \omega_5(A_R)$$

with
$$\begin{aligned} d\omega_5 &= TrF^3 \\ \delta\omega_5 &= d\omega_4^1 \end{aligned}$$

This can also be understood in the S-S model where such couplings arise from a term on the D8-branes

$$S_{anom} = \int_{\Sigma_p} C \wedge ch(F) = \int_{\Sigma_9} C_3 \wedge TrF^3 + \cdots$$

which in the D4-background with $\int_{S^4} G_4 = 2\pi N_c$ gives the same couplings

Restrict to lightest isoscalar modes:

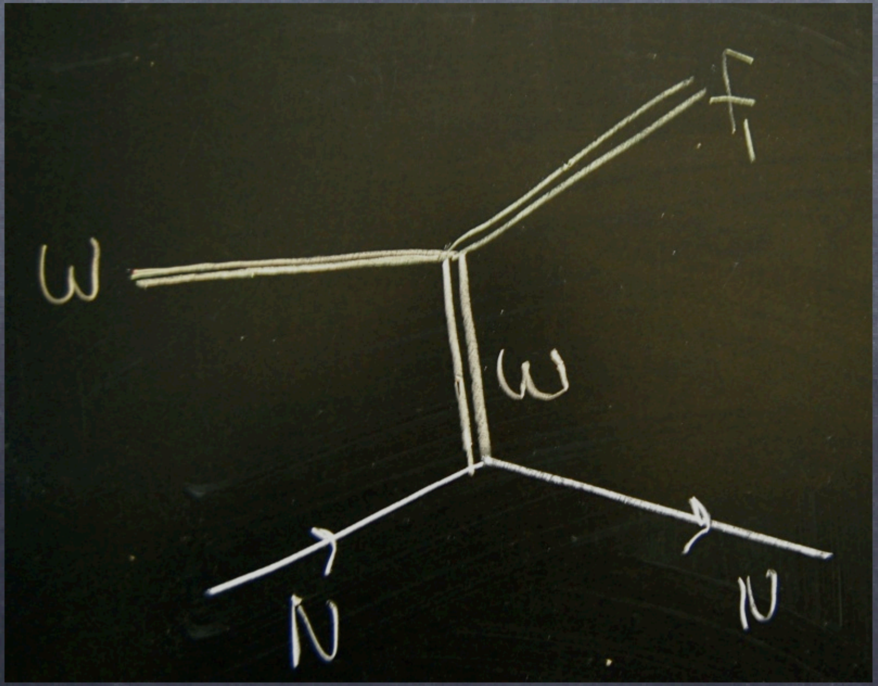
$$
\begin{aligned}
\pi^a(x,z) &= \pi^a(x)\psi_\pi(z) \\
V_\mu(x,z) &= g_5\, \omega_\mu(x)\psi_\omega(z) \\
A_\mu(x,z) &= g_5\, f_\mu(x)\psi_f(z)
\end{aligned}
$$

one finds a pCS term (among others)

$$
S_{f\omega\omega} \sim g_{f\omega\omega} \int d^4x \epsilon^{\mu\nu\lambda\rho} f_\mu \omega_\nu \partial_\lambda \omega_\rho
$$

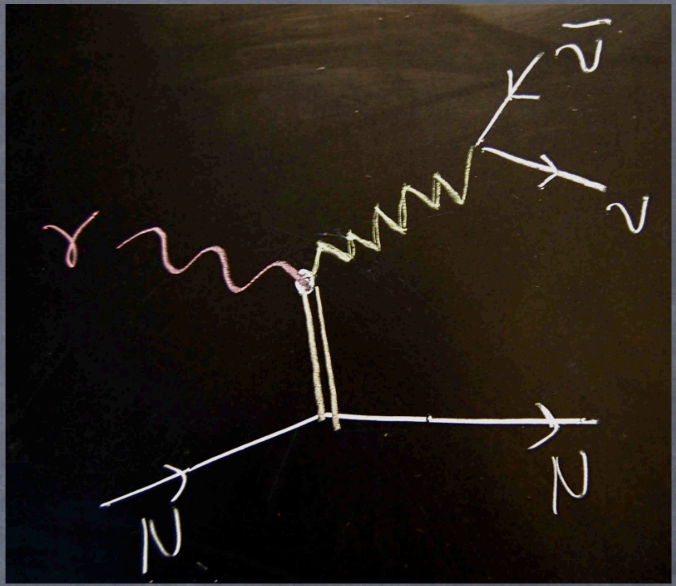
with $g_{f\omega\omega}$ determined in terms of integrals over z and the parameters of the model, leading to $g_{f\omega\omega} \sim 9$.

This might be visible in polarized photon-proton scattering experiments planned for Hall D at JLab $\gamma + p \to f_1 + p$ with $\gamma \to \omega$ using VMD and measurement of the f_1 polarization used to distinguish omega exchange from other effects.



Other Applications:

Neutron Star Cooling,
Supernova dynamics?



Detection of coherent neutrino scattering off of nuclei.

Conclusions

There are a set of SM couplings which involve both the SM gauge fields and the vector fields of QCD which are distinguished by their violation of "natural parity."

The couplings give a reasonable prediction for certain vector meson decays and may account for the MiniBoone excess at low-energies.

These couplings should have a variety of other applications in astrophysics and nuclear physics/QCD.

The cure is well known. The theory also contains a WZW term:

$$\Gamma_{WZW} = \frac{1}{80\pi^2}\int d^4x\ \epsilon^{\mu\nu\alpha\beta}\left[\mathrm{Tr}[\pi\partial_\mu\pi\partial_\nu\pi\partial_\alpha\pi\partial_\beta\pi] + \frac{e^2}{16\pi^2 f_\pi}\pi^0 F_{\mu\nu}F_{\alpha\beta}\right] + \cdots$$

This term is related to anomalies. One cannot consistently gauge the full global symmetry and the coupling of the π^0 to electromagnetism reflects an anomaly in the axial current.

In the SM we want to gauge $SU(2)_L \times U(1)_Y$ and we will have couplings to $\gamma, W^\pm, Z^0$. There are anomalies that are cancelled by the leptons.