Higher Spin Black Holes from 2d CFT

Tom Hartman
Institute for Advanced Study

Rutgers Theory Seminar
January 17, 2012

Simplified Holography

A goal

- Find a holographic duality simple enough to solve, but complicated enough to look like gravity in d>2.

This talk

- Simple bulk: 3d higher spin gravity
- Simple boundary: 2d CFT with $\mathcal{W}_N$ symmetry

Higher Spin Gravity

Gravity plus large (or infinite) number of massless fields,

$$A_{\mu_1 \cdots \mu_s}$$

with spins

$$s = 0, 1, 2, 3, 4, \cdots ,$$

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with spins

$$s = 0, 1, 2, 3, 4, \cdots \ ,$$

Spin-2 = graviton. Massless higher spin fields mean very large gauge symmetry extending diffeomorphism invariance.

Consistent interacting theory exists for $\Lambda \neq 0$

Fradkin and Vasiliev, 1987
Vasiliev, 1990

Toy model for string theory in the stringy limit

Higher Spin Holography

Higher Spin Gravity in AdS $\xleftrightarrow{\text{dual}}$ Solveable CFT

$$A_{\mu_1 \ldots \mu_s} \longleftrightarrow J_{\mu_1 \ldots \mu_s}$$

Higher Spin Dualities

d=4 gravity

Fronsdal '79
Witten
Sundborg
Mikhailov
Sezgin & Sundell
Giombi & Yin...

- Vasiliev gravity in $AdS_4 \longleftrightarrow O(N)\ CFT_3$
- N free scalars or the interacting critical point

Klebanov & Polyakov

- Similarly, N free anticommuting scalars -- the $Sp(N)\ CFT_3$ -- is dual to Vasiliev gravity in 4d de Sitter space

Anninos, TH, Strominger

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d=3 gravity

- Vasiliev gravity in $AdS_3 \longleftrightarrow W_N\ CFT_2$
  - Gravity side is simpler than 4d
  - Rich, interacting, solveable CFT duals
  - Precise duality is not known at finite N

Campoleoni et al
Henneaux & Ray
Gaberdiel & Gopakumar

Outline

✓ Introduction

- Extended conformal symmetry in 2d
- 3d Higher Spin Gravity
- The duality
- Black hole entropy

Extended Conformal Symmetry

3d CFT

- Maldacena and Zhiboedov 2011: A single conserved current with spin >2 implies an infinite set of conserved higher spin currents, and the <JJJ...> correlators are those of a free theory.

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2d CFT

- There is no such theorem. Interacting theories with a finite set of higher spin currents can be constructed explicitly.
- Definition of W-algebra: An extension of the Virasoro algebra by higher spin currents.

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- Example: $\mathcal{W}_3$
  - Extended chiral algebra: spin-2: $T(z)$ spin-3: $W(z)$

Zamolodchikov ’85

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  - Extended chiral algebra: spin-2: $T(z)$ spin-3: $W(z)$

Zamolodchikov '85

- Example: $\mathcal{W}_N$
  - Higher spin currents of all spins s=2,...,N

W_N Minimal Models

W_N specifies the symmetries. Most of this talk will be universal, relying only on the algebra. But for a full theory, we must specify the matter content. The simplest possibility is a “minimal model”:

$$\frac{SU(N)_k \times SU(N)_1}{SU(N)_{k+1}} \qquad\qquad c < N - 1$$

(N=2 gives the c < 1 Virasoro minimal models -- Ising, etc.)

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These are solveable CFTs, ie we can compute:

- Dimensions of primary operators
- Zero mode eigenvalue
- W-algebra commutation relations
- All correlation functions
- Partition function

... at least in principle

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3d Gravity/Chern-Simons

AdS$_3$ gravity is related to Chern-Simons gauge theory

Achucarro, Townsend '86
Witten '88

$$g_{\mu\nu} \to \omega, e \to A_{\pm} = \omega \pm e$$

$$S_{Einstein} = \frac{k}{4\pi} \int \mathrm{tr} \left(A_- dA_- + \frac{2}{3} A_-^3 \right) - \frac{k}{4\pi} \int \mathrm{tr} \left(A_+ dA_+ + \frac{2}{3} A_+^3 \right)$$

$$A_{\pm} \in SL(2, R) \qquad\qquad k = \frac{\ell_{AdS}}{4G_N}$$

Comments:

- This sector of the theory is topological. Any extra matter is not.
- These theories are inequivalent. For our purposes, the CS action is just a rewriting of the Einstein action in convenient variables, and should not be thought of as a gauge theory.

Higher Spin Gravity

Enlarge the gauge group

$$sl(2,R) \to g$$

Finite N

$$g = sl(N)\ , \quad \text{spins} = 2, 3, \ldots, N$$

Infinite N (all spins >1)

$$g = sl(\infty)$$

$$\boxed{g = hs(\lambda)} \ \leftarrow \text{non-integer } sl(\lambda)$$

Fradkin, Vasiliev '80s
Blencowe '88
etc.

Higher Spin Gravity

Comments

- We will discuss the one-parameter family of higher spin theories based on the infinite Lie algebra $hs(\lambda)$

- Translation back to metric-like variables is known implicitly, but complicated

- In d > 3, action is unknown, and truncation to any finite number of higher spins is impossible

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The Conjecture

A duality between

Gaberdiel, Gopakumar '10

- The 3d higher spin gravity theory based on $hs(\lambda)$ plus two additional scalar matter fields $\phi_\pm$ with masses

$$M^2 = -1 + \lambda^2$$

- The 2d W_N minimal model CFT at level k, with large N

with the tunable 't Hooft-like parameter

$$\lambda = \lim_{N,k\to\infty} \frac{N}{N+k} \quad , \quad 0 < \lambda < 1$$

Evidence

Symmetries

Spectrum of Primaries

Correlation functions

Black holes

Symmetries in ordinary AdS_3/CFT_2

- Bulk isometries are

$$sl(2,R) \times sl(2,R)$$

- Near the boundary, these are enhanced to the asymptotic symmetries

$$L_m \ , \overline{L}_m \quad \boxed{\text{Virasoro} \times \text{Virasoro}}$$

Brown, Henneaux, '86

- Now we want to repeat this for the higher spin theories

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"global" or "wedge"

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Asymptotic Symmetries of Higher Spin Theories

More generally,

$$g \xrightarrow{\text{at boundary}} \hat{g} \xrightarrow[\text{boundary conditions}]{\text{impose AdS}} W(\hat{g})$$

- Each higher spin field leads to a boundary current. Combined with the Virasoro enhancement, this means we get a W-symmetry.

- Examples:

$$sl(N) \to W_N$$

Campoleoni et al '10

Henneaux, Rey '10

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- For the bulk theory dual to minimal models,

$$hs(\lambda) \to W_\infty[\lambda]$$

Gaberdiel, TH '11

  - This is a particular W-algebra with known commutation relations, with a central charge c and another parameter λ

$\mathcal{W}_\infty(\lambda)$

$$[L, L] \sim L + c$$

$$[L, W^{(s)}] \sim W^{(s)} \qquad\qquad s = 3, 4, \ldots, \infty$$

$$[W^{(3)}, W^{(3)}] \sim W^{(4)} + L + \frac{1}{c}(L)^2 + c$$

$etc.$

Symmetry Recap

ordinary gravity	higher spin gravity
$sl(2)$	$hs(\lambda)$
Virasoro	$\mathcal{W}_\infty(\lambda)$

Back to the Duality...

For the duality to hold, we need

- Full symmetries

$$W_\infty[\lambda] = \lim_{N,k\to\infty} W_N \quad \text{with} \quad \lambda = \frac{N}{N+k}$$

- "Wedge" symmetries
  - wedge part of W_N becomes $hs(\lambda)$ in the 't Hooft limit

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This is apparently true, though unproven

- Checked by comparing spin-2,3,4 charges of degenerate reps
- Can also be motivated (proved?) by fractional level-rank duality

Gaberdiel, TH
Gaberdiel, Gopakumar, TH, Raju

Matching symmetries

To summarize

- The symmetries of the bulk and CFT match in a nontrivial way
- $hs(\lambda)$ is hiding inside the W-algebras at large N

Matter Spectrum & Correlators

Quick Summary

- The CFT spectrum is known exactly
- The bulk scalars, and multiparticle states thereof, match a subset of the CFT primaries
  Gaberdiel & Gopakumar
  Gaberdiel, Gopakumar, TH, Raju
- These primaries decouple as $N \to \infty$
- Certain correlators of these fields have been computed in Vasiliev theory in the bulk, and match the CFT Chang & Yin

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  Chang & Yin
- Other light primaries are *missing* in the bulk, so the duality is incomplete at finite N
- Some of the "missing" light states were recently found as new solitonic classical solutions of higher spin gravity

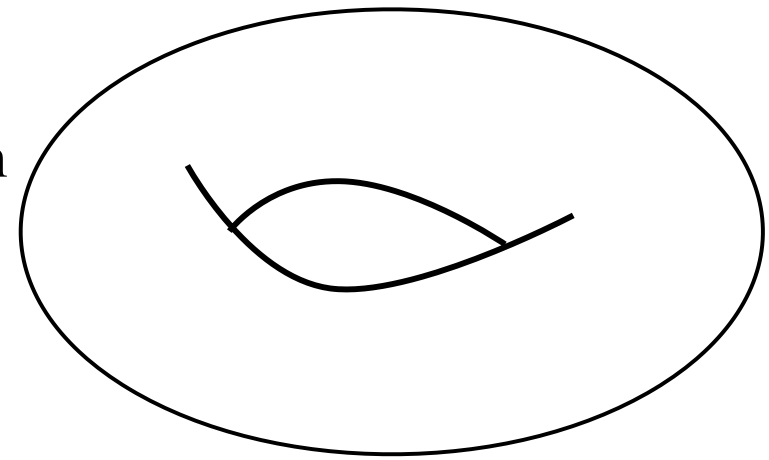
  Castro, Gopakumar, Gutperle, & Raeymaekers

Outline

- ✔ Introduction
- ✔ Extended conformal symmetry in 2d
- ✔ 3d Higher Spin Gravity
- ✔ The duality
- Black hole entropy

Black Holes in Higher Spin Gravity

What is a black hole?



- Classical solution with a smooth horizon
- In Euclidean signature: solid torus

However,

- The metric and gauge fields mix under higher spin gauge transformations

- Thus Ricci and causal structure are *not* gauge invariant.

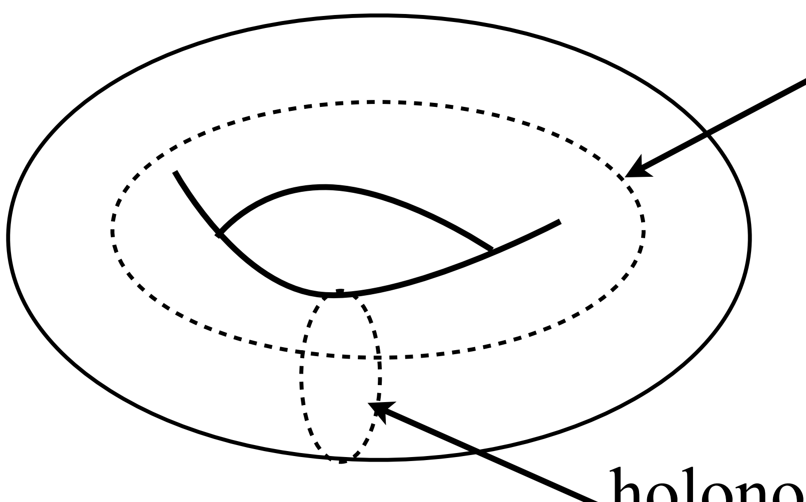
  - What is "smooth"? What is a "horizon"?

Black Holes in Higher Spin Gravity

- The Chern-Simons description is useful to define black holes in a gauge-invariant way
- Gauge invariant data = holonomies of the Chern-Simons gauge field $A \in hs(\lambda)$

$$\mathrm{Pexp} \oint A$$

“Black hole” = flat connection on a torus, with vanishing holonomy around one cycle



non-trivial holonomy encodes mass, ang. mom., and higher spin charges

holonomy = 0

Spin-3 Black Hole

The black hole in $hs(\lambda)$-gravity carrying spin-3 charge has been constructed explicitly.

Gutperle & Kraus '11
Ammon, Gutperle, Kraus, & Perlmutter
Castro, Hijano, Lepage-Jutier, & Maloney
Kraus & Perlmutter

Parameters

$$L_0, \overline{L}_0 = \text{mass} \pm \text{ang. mom.}$$

$$Q_3, \overline{Q}_3 = \text{spin-3 charges}$$

Potentials

$$\tau, \overline{\tau} = \text{inverse Hawking temperature}$$

$$\mu, \overline{\mu} = \text{spin-3 potential}$$

Smoothness

- The zero-holonomy condition relates charges to potentials,

$$L_0 = L_0(\tau, \mu; \lambda) \qquad Q_3 = Q_3(\tau, \mu; \lambda)$$

Black Hole Entropy

Horizon area / Wald entropy is not higher-spin-gauge invariant. But Wald entropy was designed to integrate the first law of thermodynamics, so we might as well just do this directly:

$$Z = \mathrm{Tr}\, e^{2\pi i(\tau L_0 + \mu Q_3)}$$

$$L_0 \sim \partial_\tau \log Z$$

$$Q_3 \sim \partial_\mu \log Z$$

Plugging in charges found by solving the zero-holonomy condition,

$$\log Z = \frac{i\pi c}{12\tau}\left[1 - \frac{4}{3}\frac{\mu^2}{\tau^4} + \frac{400}{27}\frac{\lambda^2-7}{\lambda^2-4}\frac{\mu^4}{\tau^8} - \frac{1600}{27}\frac{5\lambda^4-85\lambda^2+377}{(\lambda^2-4)^2}\frac{\mu^6}{\tau^{12}} + \cdots\right]$$

Kraus & Perlmutter

$$\text{entropy} = (1 - \tau\partial_\tau - \mu\partial_\mu)\log Z$$

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Goal: Reproduce this formula by a microscopic CFT computation

(work in progress with M. Gaberdiel and K. Jin)

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Warm-Up

Virasoro

$$Z(\tau) = \mathrm{Tr}\, e^{2\pi i \tau L_0}$$

is a modular-invariant function. Therefore at high temperature,

$$Z(\tau) = Z(-1/\tau) = e^{\frac{2\pi i}{\tau}\frac{c}{24}}$$

This is the Cardy formula, which reproduces the entropy of ordinary BTZ black holes in AdS_3.

U(1) Charge

The partition function with a spin-1 potential is a weak Jacobi form under modular transformations. Thus a similar trick gives a simple formula for the entropy of a spin-1-charged black hole.

Goal: Generalize the Cardy formula for the growth of states in CFT to an arbitrary chiral algebra.

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Entropy with Spin-3 Charge

The thermodynamics of 2d CFT at high temperature is thus fixed by the chiral algebra. However, the spin-3-charged partition function

$$Z(\tau, \mu) = \mathrm{Tr}\, e^{2\pi i(\tau L_0 + \mu Q_3)}$$

does not have any (known) nice modular properties.

Instead, compute order by order:

$$Z = \mathrm{Tr}\, q^{L_0} + \frac{1}{2!}\oint\oint \mathrm{Tr}\, W(z_1)W(z_2)q^{L_0} + \frac{1}{4!}\oint\oint\oint\oint \mathrm{Tr}\, W(z_1)W(z_2)W(z_3)W(z_4)q^{L_0} + \cdots$$

$$q = e^{2\pi i\tau}\ , \qquad Q_3 = \oint W(z)$$

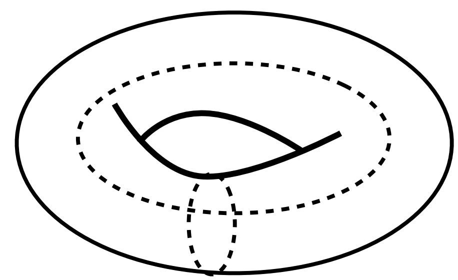
Entropy with Spin-3 Charge

The corrections are torus correlation functions,

$$G_n(z_1, \ldots, z_n; \tau) \equiv \mathrm{Tr}\, W(z_1) W(z_2) \cdots W(z_n) q^{L_0}$$

This is a trace over the whole Hilbert space. However, correlators have a nice modular transformation,

$$\oint \cdots \oint G_n(\tau) \sim \int_C \cdots \int_C G_n(-1/\tau)$$



Integration cycle is swapped

After a modular transformation, high temperature implies

$$\tilde{q} \ll 1$$

which projects the trace onto the vacuum.

Computing Correlators

Torus correlators can be computed by a recursion formula. Schematically,

$$\mathrm{Tr}\, V_1 \ldots V_n q^{L_0} = \sum P\left(\frac{z_2}{z_1}\right) \mathrm{Tr}\,(V_1 \times V_2) V_3 \ldots V_n q^{L_0} + \cdots$$

Weierstrass function

Now,

- Do the recursion, using the (nonlinear) OPEs of $\mathcal{W}_\infty[\lambda]$
  - Roughly (n-1)! terms appear from contracting various currents
- Integrate Weierstrass functions over the torus
- Compare to the black hole...

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Zhu 1990

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Matching CFT to Gravity

$$\log Z = \frac{i\pi c}{12\tau}\left[1 - \frac{4}{3}\frac{\mu^2}{\tau^4} + \frac{400}{27}\frac{\lambda^2-7}{\lambda^2-4}\frac{\mu^4}{\tau^8} - \frac{1600}{27}\frac{5\lambda^4-85\lambda^2+377}{(\lambda^2-4)^2}\frac{\mu^6}{\tau^{12}} + \cdots\right]$$

They match,

$$\log Z_{bulk} = \log Z_{cft}$$

Comments

- The match has been checked to order μ^4 (μ^6)
- The CFT answer relies on the detailed nonlinear commutation relations of the chiral algebra, $\mathcal{W}_\infty[\lambda]$. The procedure generalizes to an arbitrary chiral algebra.
- The entropy is therefore reproduced by microstate counting. However, detailed microsopics are needed to check validity of the Cardy approximation in any particular CFT. Do the minimal models meet the criteria, in the appropriate regime?

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Matching CFT to Gravity

A question

- The bulk entropy was compute by solving a zero-holonomy condition for Chern-Simons on a donut. This was reproduced order by order by CFT correlators.

- These procedures look very different. Why are they the same?

Conclusions

Summary

- 3d higher spin theory = CFT with W_N symmetry
- Both theories are tightly constrained by the higher spin algebra $hs(\lambda)$ and its W-algebra cousin, $\mathcal{W}_\infty(\lambda)$

Questions

- Refine the duality at finite N (missing states)

- What can higher spin holography be used for?
  - Tackle some thorny issues in holography, like RG, de Sitter space, information paradox, etc.

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Jevicki and Jin
Douglas, Mazzucato, & Razamat
etc.