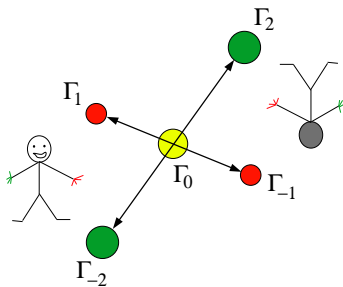


# Orientiholes



Frederik Denef, Mboyo Esole and Megha Padi, arXiv:0901.2540

# Outline

Motivation and basic idea

Review of  $\mathcal{N} = 2$  black hole bound states

Type IIA orientifolds

# Motivation and basic idea

## The IIB (F-theory) landscape

- Central in modern string phenomenology (Fenomenology)
  - GUT model building [Beasley-Heckman-Vafa,...]
  - models of inflation [Baumann-Dymarsky-Klebanov-McAllister,...]
  - moduli stabilization [Kachru-Kalosh-Linde-Trivedi,...]

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- Missing: **global** picture
  - genuine, fully consistent compactifications
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- In principle simple [Vafa]:

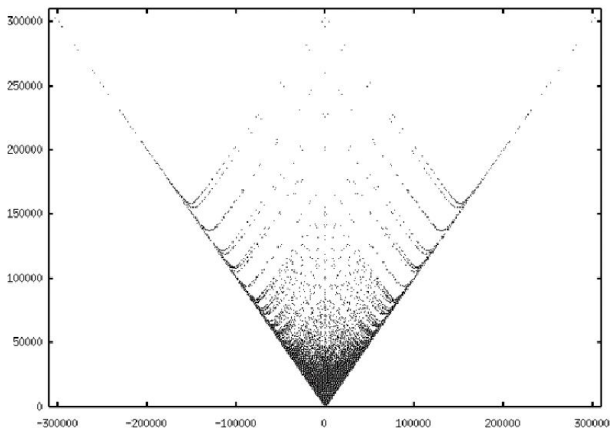
F-theory compactification with 4 susies in 4 dim

$\Leftrightarrow$

elliptically fibered CY 4-fold + 4-flux

## All CY4 hypersurfaces in weighted $\mathbb{C}P^5$

Total number = 1,100,055 [Lynker-Schimmrigk-Wisskirchen]:

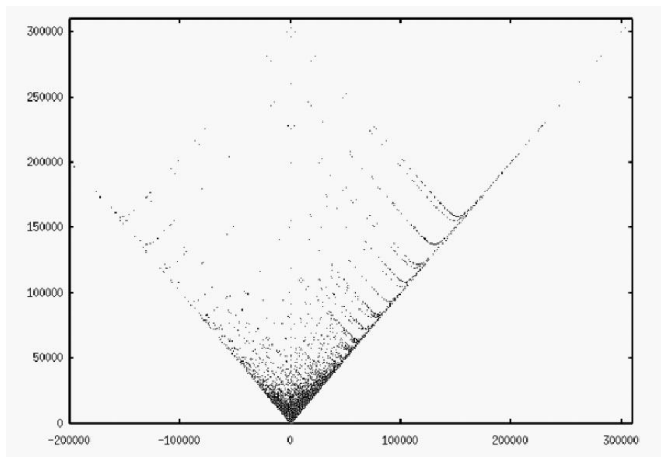


$$h_+ \equiv h^{3,1} + h^{1,1} \text{ versus } h_- \equiv h^{3,1} - h^{1,1}$$

$$N_{D3} = \frac{\chi}{24} \approx \frac{h_+}{4}.$$

## Elliptically fibered subset

At least 49,751:



$$h_+ \equiv h^{3,1} + h^{1,1} \text{ versus } h_- \equiv h^{3,1} - h^{1,1}$$



## Number of flux vacua

Continuum estimate for number of vacua for *fixed* CY4 within region  $\mathcal{M}$  of complex structure moduli space [Ashok-Denef-Douglas]:

$$N_{vac} = \text{Vol} \left[ S^{b_4} |_{R^2 = \frac{\chi}{12}} \right] \int_{\mathcal{M}} e(D)$$

where

$$\text{Vol} \left[ S^b |_{R^2 = \frac{\chi}{12}} \right] = \frac{(\pi R^2)^{\frac{b}{2}}}{(\frac{b}{2})!}$$

Example with largest  $\chi$ :

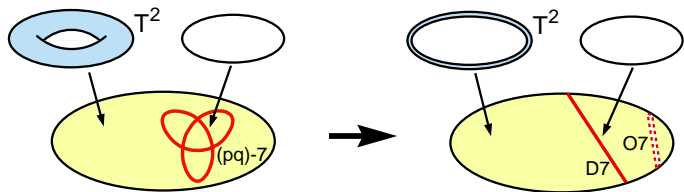
$$w = (1, 1, 84, 516, 1204, 1806), \quad h^{3,1} = 303148, \quad h^{1,1} = 252, \\ b_4 = 1, 819, 942, \quad \chi = 1, 820, 448$$

has

$$N_{vac} \propto \text{Vol} = 10^{139598}.$$

## Weakly coupled IIB picture

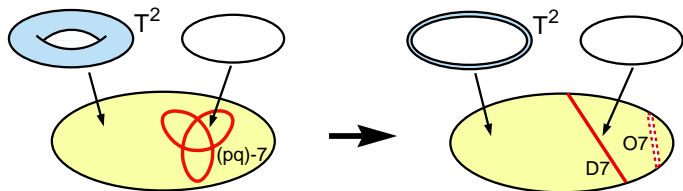
[Sen]



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  - $O7 + O3$
  - $D7 + D3$
  - $RR + NSNS$  3-flux + worldvolume 2-flux

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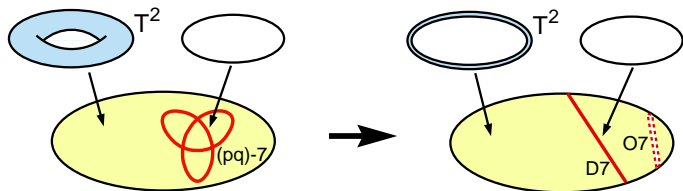
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  - $D7 + D3$
  - $RR + NSNS$  3-flux + worldvolume 2-flux
- Virtually all vacuum degeneracy arises from D-brane d.o.f.
- Constructing all = intractable. Instead: **how many** D-brane vacua in different sectors? [Douglas]

## Problems with conventional approach

- ADD-formula far outside of regime of asymptotic validity  
 $Q_{D3} \gg b_4$  (because  $Q_{D3} \sim \frac{b_4}{24}$ )
- D7-D3 bound states not taken into account
- No systematic enumeration of different sectors of D7 configuration space
- Even more basic issues such as K-theory constraints and D-term stability have not systematically been addressed.

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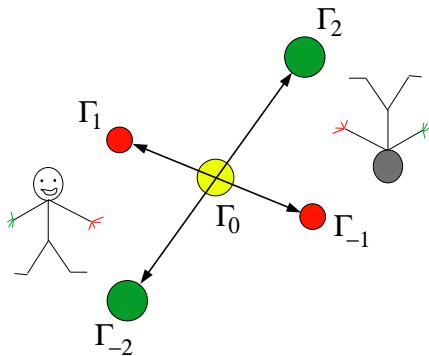
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- 5 Take  $g_s^{(4)}$  up again

## Result: Orientiholes



Key fact:

Witten index vacua  $\Leftrightarrow$  index of BPS states

## “Experimental” landscapeology

- Estimate numbers of vacua in various sectors landscape by “measuring” (refined) Bekenstein-Hawking entropy of various mesoscopic black hole configurations:  $N_{\text{vac}} \sim e^{S_{BH}}$

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- Finer enumeration from multiparticle quantum states (the fuzzball approach to landscapeology)
- brane-brane open string indices  $\Leftrightarrow$  angular momenta
- Subtle  $\mathbb{Z}_2$  “tadpoles” on IIB side = charge measurable by Aharonov-Bohm experiment



## Other motivations

- 1 funky spacetimes, where you can take a walk around the center of the universe and come back as your mirror image.
- 2 new invariants and associated modular forms, wall crossing formulae, ...
- 3 new version of the OSV conjecture:  $\mathcal{Z}_{OH} \sim \mathcal{Z}_{top}$  (linear!)

# Review of $\mathcal{N} = 2$ black hole bound states

## Single centered black holes

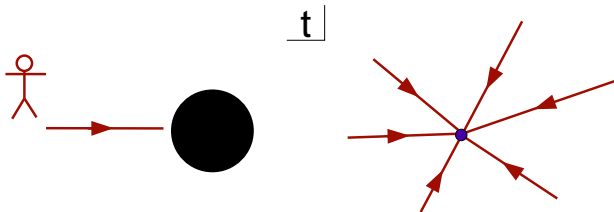
Spherically symmetric BPS black hole of charge  $\Gamma \equiv (p^\Lambda, q_\Lambda)$ :

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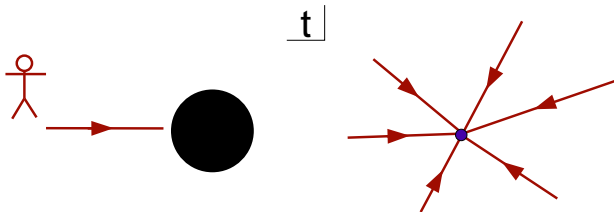


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Solutions  $\Leftrightarrow$  attractors [Ferrara-Kallosh-Strominger]:

Radial inward flow of vector multiplet moduli  $t^A(r)$  is gradient flow of central charge  $|Z(\Gamma, t)|$ .

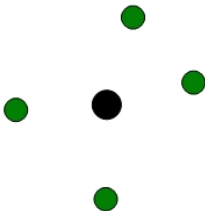
BH entropy:

$$S(\Gamma) = \pi \min_t |Z(\Gamma, t)|^2$$

## BPS black hole molecules

More general BPS solutions exist: multi-centered **bound states**:

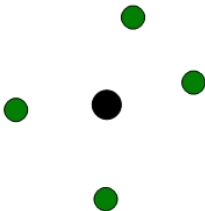
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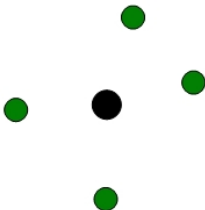


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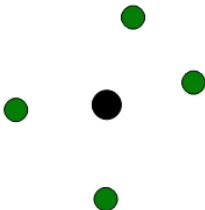
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- Bound in the sense that positions are **constrained** by gravitational, scalar and electromagnetic forces.
- Stationary but with intrinsic **spin** from e.m. field

## Explicit multicentered BPS solutions

- $N$ -centered solutions characterized by harmonic function  $H(\vec{x})$  from 3d space into charge space:

$$H(\vec{x}) = \sum_{i=1}^N \frac{\Gamma_i}{|\vec{x} - \vec{x}_i|} + H_\infty$$

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- All fields can be extracted completely explicitly from the entropy function  $S(\Gamma)$  on charge space, e.g.

$$e^{2U(\vec{x})} = \frac{\pi}{S(H(\vec{x}))}$$

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- Spin:

$$J = \frac{\langle \Gamma_1, \Gamma_2 \rangle}{2}$$



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- Instead: realized as bound state of single D6 with  $U(1)$  flux  $F = P/2$  and anti-(single D6 with flux  $F = -P/2$ ):

$$D6[P/2] \bullet \quad \bullet -D6[-P/2]$$

Stable for  $\text{Im } t > \mathcal{O}(P)$ .

## Transition between $g_s|\Gamma| \gg 1$ and $g_s|\Gamma| \ll 1$ pictures

- Mass squared lightest bosonic modes of open strings between  $\Gamma_1$  and  $\Gamma_2$ :

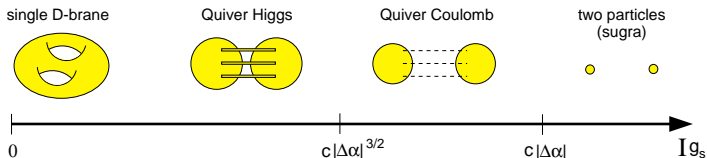
$$\begin{aligned} M^2/M_s^2 &\sim \frac{|\vec{x}_1 - \vec{x}_2|^2}{\ell_s^2} + \Delta\alpha \\ &= c(t)g_s^2 + \Delta\alpha \end{aligned}$$

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- On stable side of MS wall  $\Delta\alpha < 0$ , so if  $g_s$  gets sufficiently small, open strings become tachyonic and branes condense into single centered D-brane.

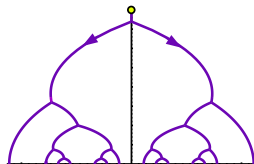
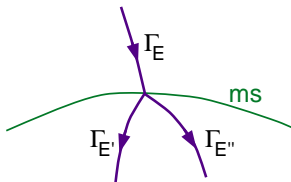


## The flow tree - BPS state correspondence

- Establishing existence of multicentered BPS configurations not easy: position constraints,  $S(H(\vec{x})) \in \mathbb{R}^+ \forall \vec{x}, \dots$

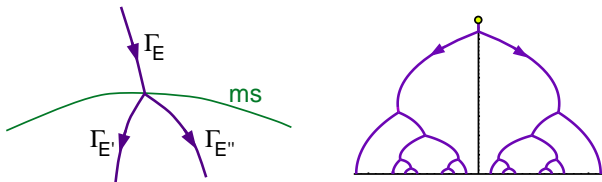
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- **Much simpler to check & enumerate!**



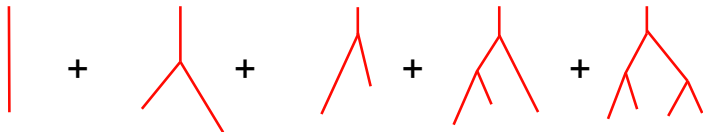
## Flow tree decomposition of BPS Hilbert space

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- $\Rightarrow$  Hilbert space of BPS states of charge  $\Gamma$  in background  $t$  has canonical decomposition in attractor flow tree sectors:

$$\mathcal{H}(\Gamma, t) =$$



## The BPS index

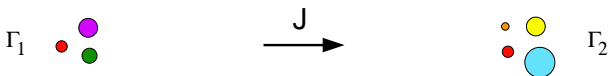
Hilbert space of BPS states in 4d  $\mathcal{N} = 2$  theories:

$$\mathcal{H}(\Gamma, t) = \left(\frac{1}{2}, 0, 0\right) \otimes \mathcal{H}'(\Gamma, t)$$

Index:

$$\Omega(\Gamma, t) = \text{Tr}_{\mathcal{H}'(\Gamma, t)} (-1)^{2J'_3} = (-1)^{\dim_{\mathbb{C}} \mathcal{M}} \chi(\mathcal{M}).$$

## Wall crossing formula for primitive splits

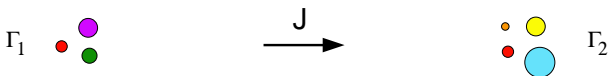


- Near marginal stability wall  $\Gamma \rightarrow \Gamma_1 + \Gamma_2$  (with  $\Gamma_1$  and  $\Gamma_2$  primitive), the decaying part of  $\mathcal{H}'(\Gamma, t)$  has following factorized form:

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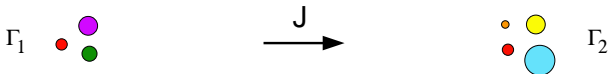
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  - macroscopically from intrinsic angular momentum monopole-electron system ( $-1/2$  from spin-magnetic coupling)
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- Implies index **jump**

$$\Delta \Omega = (-)^{2J} (2J + 1) \Omega(\Gamma_1, t_{\text{ms}}) \Omega(\Gamma_2, t_{\text{ms}}).$$

# Type IIA orientifolds

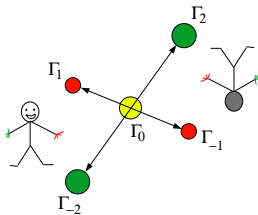
## Solutions

$= \mathcal{N} = 2$  solutions invariant under  $\tau'$ . Two cases:

$$\tau_{04/00} = \Omega \sigma^* \mathcal{P}^*$$

$$\tau_{06/02} = \Omega (-1)^{F_L} \sigma^* \mathcal{P}^* .$$

where  $\mathcal{P} : \vec{x} \rightarrow -\vec{x}$  and  $\sigma$  is internal involution.



E.g. 04/00 one modulus case:

$$\Gamma_1 = (P^0, P^1, Q_1, Q_0)_1, \quad \Gamma_{-1} = \Gamma'_1 = (-P^0, P^1, -Q_1, Q_0)_1, \\ \Gamma_0 = (0, P^1, 0, Q_0)_0 .$$

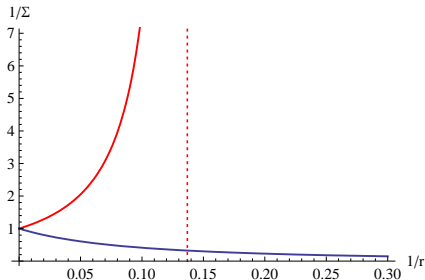


## Main difference

Phase  $\alpha_\infty$  is **fixed** by choice of orientifold projection:

$$\alpha_\infty = 0 \quad (O4/O0), \quad \alpha_\infty = -\frac{\pi}{2} \quad (O6/O2)$$

Consequence: if  $\alpha_\infty = \pi + \arg Z$ : neg. mass, grav. repulsive, inverted attr. flow, attr. point  $\rightarrow$  repulsor point, sol. singular.



## Basic bound state

- Simplest possibility:

$$\Gamma = \Gamma_1 + \Gamma_0 + \Gamma'_1$$

i.e. bound state of charge with its own image (+ charge in the middle of the universe)

- From integrability constraint:

$$\frac{I(\Gamma_1, \Gamma_0)}{|\vec{x}_1|} = 2\text{Im}[e^{-i\alpha} Z_1]_\infty.$$

where

$$I(\Gamma_1, \Gamma_0) := \frac{\langle \Gamma_1, \Gamma'_1 \rangle}{2} + \langle \Gamma_1, \Gamma_0 \rangle = 2J$$

- $\Rightarrow$  Stability condition:

$$I(\Gamma_0, \Gamma_1) \text{Im}[e^{-i\alpha} Z_1]_\infty > 0.$$

## Wall crossing formula

- Index counting orientifold invariant BPS states:

$$\Omega_{\text{inv}}(\Gamma, t) = \text{Tr}_{\mathcal{H}'_{\text{inv}}(\Gamma, t)} (-)^{2J'_3} = (-1)^{\dim_{\mathbb{C}} \mathcal{M}_{\text{inv}}} \chi(\mathcal{M}_{\text{inv}}).$$

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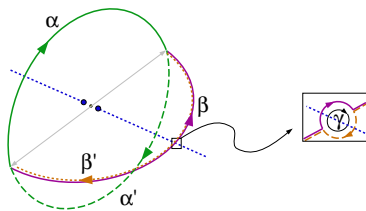
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- Corollary, by comparing to microscopic picture:

angular momentum of pair = open string Witten index

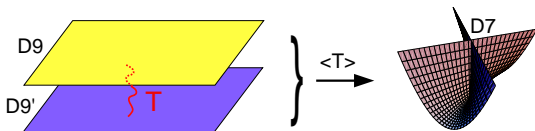
## $\mathbb{Z}_2$ torsion charge



- For charge odd under  $\tau$  (e.g. D6 in O4/O0 case): Aharonov Bohm experiment can distinguish between odd and even number of dipoles.
- Related to subtle anomalies on IIB side.

## Application to counting basic D7 vacua

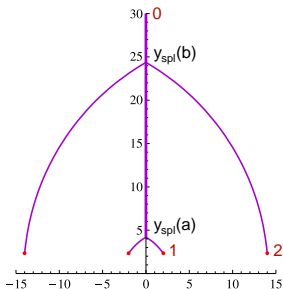
- Example: pure D7 branes in degree 8 hypersurface in  $CP^4_{4,1,1,1,1}$  O3/O7 orientifolded by reflection of first coordinate.
- Complicated story (cf. previous talk at Rutgers)



D7 (possibly with flux) obtained from two D9-D9' pairs with  $a$ ,  $b$  units of flux.

## Application to counting basic D7 vacua

Orientihole split flow:



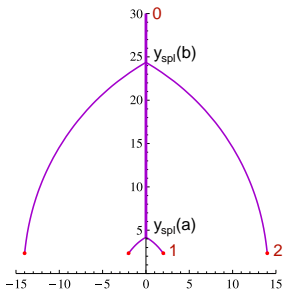
Predicts Euler characteristics moduli spaces:

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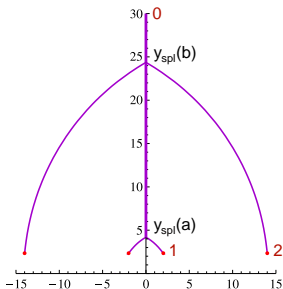
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Note: decays at quite large vol.!

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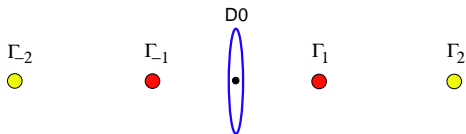
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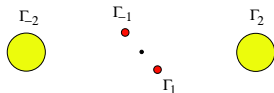
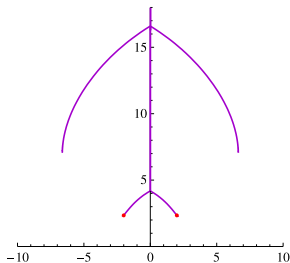
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Physical argument: scaling solutions exist (for  $a = 2$ ).



## General tadpole canceling D7 vacua

Fat multicentered solutions:



$S = 1540$  in large vol. approx. (ok  $\text{Im } t_* \approx 7$ )

$\Rightarrow$  In this sector  $N_{\text{vac}} \approx 10^{668}$ .

## Directions for future work

- lift to M-theory
- solutions in  $T^3$ .  $MS = ??$
- corrections to  $Z$
- map different landscape sectors to different kinds of BH configurations
- implications stability issues for phenomenology
- nonprimitive wall crossing
- OSV, modular forms
- bulk fluxes
- nonsusy