R-Symmetry and Non-Perturbative QFT

Rutgers NHETC Seminar

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Overview

- RG generalities.

- Operator mappings: from conserved currents and the chiral ring to long multiplets.

- R-symmetry, the R-current multiplet, the U multiplet, and the RG flow.

- Examples: SQCD, the Kutasov theory, and aSQCD.

- Correlation functions of the U multiplet and IR phases of gauge theories.

- IR interacting versus IR free.

RG Generalities

• All UV-complete QFTs can be understood as interpolations between UV and IR fixed points (may also be gapped and hence empty in IR).

• Given well-defined operators and correlation functions of the UV theory, can we say something about the corresponding objects in the IR?

• What are the emergent symmetries of the IR fixed points?

• In general, new internal and space-time symmetries. What are they? How do we get a handle on them?

RG Generalities (cont...)

• Non-perturbative dynamics along the RG flow make these questions hard to answer.

• We will specialize to four-dimensional R-symmetric theories.

• As we will see SUSY, and, in particular R-symmetry give us strong handles to use to answer a lot of these questions in controlled settings.

Mapping Operators

- **Question**: Given $\mathcal{O}^{UV}$, what is $\mathcal{O}^{IR}$?

- Easy operators to map: short multiplets, like members of the chiral ring, conserved currents.

- Harder operators to map: long multiplets.

- Sometimes can embed these long multiplets inside short multiplets of higher spin and use these larger multiplets to gain traction.

Mapping Operators (cont...)

• Quantities of interest, real UV bilinears (and their generalizations):

$$c^i_j \Phi^\dagger_i \Phi^j + \tilde{c}^i_j \tilde{\Phi}^\dagger_i \tilde{\Phi}^j \ , \tag{1}$$

Appropriate factors of e^V, etc.

• For generic $c, \tilde{c}$ this defines a long multiplet, i.e.,

$$\bar{D}^2 \left(c^i_j \Phi^\dagger_i \Phi^j + \tilde{c}^i_j \tilde{\Phi}^\dagger_i \tilde{\Phi}^j \right) = c \text{Tr} W^2_\alpha + ... \tag{2}$$

• Can we map such an operator to the IR?

• To do that, we need a short multiplet in which to embed it. Natural candidates: symmetry currents of various kinds. R-symmetry current a good option (if present).

The Role of the R-symmetry Current

• Since $[R,Q] \sim Q$, $\{Q,\bar{Q}\} \sim P$, the R-current transforms in a multiplet with $S_{\mu\alpha}$ and $T_{\mu\nu}$.

$$\bar{D}^{\dot{\alpha}}\mathcal{R}_{\dot{\alpha}\alpha} = \chi_\alpha \ , \quad D^\alpha\chi_\alpha - \bar{D}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}} = \bar{D}_{\dot{\alpha}}\chi_\alpha = 0 \ . \qquad (3)$$

When $\chi_\alpha = 0$, this is the superconformal R-symmetry.

• There is an ambiguity in the above equation under $\mathcal{R}_{\alpha\dot{\alpha}} \to \mathcal{R}_{\alpha\dot{\alpha}} + [D_\alpha, \bar{D}_{\dot{\alpha}}]\, J$ and $\chi_\alpha \to \chi_\alpha + \frac{3}{2}\bar{D}^2 D_\alpha J$ for conserved J, i.e., $\bar{D}^2 J = 0$. This affects the supercurrent and stress tensor through improvements.

The Role of the R-symmetry Current (cont...)

- For the theories we will consider, can write

$$\chi_\alpha = \bar{D}^2 D_\alpha U \ , \tag{4}$$

for a well-defined U.

- Solving the above equations in the UV, we find

$$\begin{aligned} \mathcal{R}^{UV}_{\alpha\dot{\alpha}} &= \sum_i \left(2 D_\alpha \Phi_i \bar{D}_{\dot{\alpha}} \bar{\Phi}^i - r_i [D_\alpha, \bar{D}_{\dot{\alpha}}] \Phi_i \bar{\Phi}^i \right), \\ U^{UV} &= -\frac{3}{2} \sum_i \left(r_i - \frac{2}{3} \right) \bar{\Phi}^i \Phi_i \ . \end{aligned} \tag{5}$$

More generally: $U^{UV}_\mu = \frac{3}{2}\left(R^{UV}_\mu - \tilde{R}^{UV}_\mu \right)$.

- U fixed up to $U \to U + Y + \bar{Y}$. Will see later that such terms may appear in the IR.

The R-symmetry Current and the RG Flow

- **Idea:** Use the $\mathcal{R}$-multiplet to follow U along the flow.

- **Assumption**: The UV and IR fixed points are SCFTs (this can be made rigorous in SQCD-like theories **[1102.2294]**)

- At the IR fixed point, we know what should happen to $\mathcal{R}_{\alpha\dot\alpha}$. Indeed, either this multiplet flows to the superconformal R-multiplet or to an object that can be improved to the superconformal R-multiplet:

$$\tilde{\mathcal{R}}_{\alpha\dot\alpha} = \mathcal{R}^{IR}_{\alpha\dot\alpha} - [D_\alpha, \bar{D}_{\dot\alpha}]J \ , \quad \tilde{U} = U^{IR} - \frac{3}{2}J = 0 \ . \qquad (6)$$

Determine $\tilde{\mathcal{R}}_{\alpha\dot\alpha}$ from duality or a-maximization.

- **Upshot:** Therefore, $U \to \frac{3}{2}J$, where $U^{IR}_\mu = \frac{3}{2}\left(R^{IR}_\mu - \tilde{R}^{IR}_\mu\right)$.

The R-symmetry Current and the RG Flow (cont...)

• J may be a conserved current of the full theory or an accidental symmetry of the IR. We will see an extreme version of this for SQCD in the free magnetic range.

• In the case that $U^{IR} = 0$, we can say a bit more using conformal perturbation theory. If approach is via a marginally irrelevant operator, we have $U \sim \gamma J$. Otherwise, we have $U \sim \Lambda^{2-d}\mathcal{O}$ for $d > 2$ (using unitarity).

• In the case of a free magnetic phase, we have

$$U^{IR} = -\frac{3}{2}\sum_i \left(r_i - \frac{2}{3}\right)\bar{\phi}^i\phi_i \ , \qquad (7)$$

for the “emergent” d.o.f’s.

Example I: SQCD in the Free Magnetic Range

• Consider $SU(N_c)$ with $N_c + 1 < N_f \leq 3N_c/2$: this is a flow between Gaussian fixed points

• The UV (electric) theory:

$$\begin{array}{cccccc} & SU(N_c) & SU(N_f) \times SU(N_f) & U(1)_R & U(1)_B \\ & & & & \\ Q & \mathbf{N_c} & \mathbf{N_f} \times \mathbf{1} & 1 - \frac{N_c}{N_f} & 1 \\ \tilde{Q} & \mathbf{\bar{N}_c} & \mathbf{1} \times \mathbf{\bar{N}_f} & 1 - \frac{N_c}{N_f} & -1 \end{array} \tag{8}$$

• Some bilinears that we can write are $c_i^j Q^i Q_j^\dagger + \tilde{c}_i^j \tilde{Q}^i \tilde{Q}_j^\dagger$. What are they in the IR?

Example I: SQCD in the Free Magnetic Range (cont...)

- We have the following IR (magnetic) theory

$$
\begin{array}{ccccc}
 & SU(N_f - N_c) & SU(N_f)\times SU(N_f) & U(1)_R & U(1)_B \\
q & \mathbf{N_f - N_c} & \mathbf{\bar{N}_f \times 1} & \frac{N_c}{N_f} & \frac{N_c}{N_f - N_c} \\
\tilde{q} & \mathbf{\bar{N}_f - \bar{N}_c} & \mathbf{1 \times \bar{N}_f} & \frac{N_c}{N_f} & -\frac{N_c}{N_f - N_c} \\
M & \mathbf{1} & \mathbf{N_f \times N_f} & 2 - 2\frac{N_c}{N_f} & 0
\end{array}
\tag{9}
$$

- Some objects are trivial to map, e.g. $QQ^\dagger - \tilde{Q}\tilde{Q}^\dagger \longrightarrow \frac{N_c}{N_f - N_c}\left(|q|^2 - |\tilde{q}|^2\right)$.

Example I: SQCD in the Free Magnetic Range (cont...)

• But what about $J_A = QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger$? It is not conserved:

$$\bar{D}^2 J_A = \mathrm{Tr} W_\alpha^2 \ . \tag{10}$$

• **Claim:** We can follow this operator using the $\mathcal{R}$ multiplet. Indeed, using the R-charge assignments in the electric table, we find

$$U^{UV} = \left(-\frac{1}{2} + \frac{3N_c}{2N_f}\right)\left(QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger\right) \tag{11}$$

• Using the R-charge assignments in the IR, we find

$$U^{IR} = \left(1 - \frac{3N_c}{2N_f}\right)\left(qq^\dagger + \tilde{q}\tilde{q}^\dagger\right) - \left(2 - \frac{3N_c}{N_f}\right) MM^\dagger \tag{12}$$

Example I: SQCD in the Free Magnetic Range (cont...)

- Therefore, we find

$$QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger \longrightarrow \frac{2N_f - 3N_c}{3N_c - N_f}\left(qq^\dagger + \tilde{q}\tilde{q}^\dagger - 2MM^\dagger\right) \qquad (13)$$

- Acting with $\bar{D}^2$ on both sides of the above equation, we find

$$W^2_{\alpha,\mathrm{el}} \longrightarrow \frac{2N_f - 3N_c}{3N_c - N_f} W^2_{\alpha,\mathrm{mag}} \qquad (14)$$

Example II: The Deformed Moduli Space

• Consider $SQCD$ with $N_f = N_c > 2$

• The IR is described by M and B satisfying

$$\det M - B\tilde{B} = \Lambda^{2N_c} \ . \tag{15}$$

Therefore some of the short distance symmetries are spontaneously broken.

• Will find some ambiguities in following U. In some vacua we will have enough (broken) symmetry to fix U. In others we won't, but we won't discuss these cases here.

Example II: The Deformed Moduli Space (cont...)

- Consider first the following vacuum

$$M = 0 \ , \qquad B = \tilde{B} = \Lambda^{N_c} \ . \tag{16}$$

- In this vacuum the symmetry is broken as follows

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_R \hookrightarrow SU(N_f)_L \times SU(N_f)_R \times U(1)_R \tag{17}$$

- We can use our previous techniques to fix U as follows:

$$U = \delta M \delta M^\dagger + \delta b \delta b^\dagger \ , \tag{18}$$

where δb is the Goldstone superfield for the $U(1)_B$ breaking.

Example II: The Deformed Moduli Space (cont...)

• Demanding invariance under the (non-linearly realized) $U(1)_B$ symmetry requires

$$QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger \longrightarrow \text{Tr}\left(\delta M \delta M^\dagger\right) + \frac{1}{2}(\delta b + \delta b^\dagger)^2 \ . \tag{19}$$

• Note that this fixes the holomorphic + anti-holomorphic ambiguity.

Example III: The Kutasov Theory

- We consider the following electric theory with $\frac{N_c}{k} < N_f < \frac{2N_c}{2k-1}$

$$
\begin{array}{ccccc}
 & SU(N_c) & SU(N_f)\times SU(N_f) & U(1)_R & U(1)_B \\
 & & & & \\
Q & \mathbf{N_c} & \mathbf{N_f}\times\mathbf{1} & 1-\frac{2}{k+1}\frac{N_c}{N_f} & 1 \\
\tilde{Q} & \mathbf{\bar{N}_c} & \mathbf{1}\times\mathbf{\bar{N}_f} & 1-\frac{2}{k+1}\frac{N_c}{N_f} & -1 \\
X & \mathbf{N_c^2}-\mathbf{1} & \mathbf{1}\times\mathbf{1} & \frac{2}{k+1} & 0
\end{array}
\tag{20}
$$

and the following superpotential

$$W = s_0 \mathrm{Tr}(X^{k+1}) \ . \tag{21}$$

Example III: The Kutasov Theory (cont...)

- And the following magnetic theory

	$SU(kN_f - N_c)$	$SU(N_f) \times SU(N_f)$	$U(1)_R$
q	$\mathbf{kN_f - N_c}$	$\mathbf{\bar{N}_f \times 1}$	$1 - \frac{2}{k+1}\frac{kN_f - N_c}{N_f}$
$\tilde{q}$	$\overline{kN_f - N_c}$	$\mathbf{1 \times N_f}$	$1 - \frac{2}{k+1}\frac{kN_f - N_c}{N_f}$
Y	$\mathbf{(kN_f - N_c)^2 - 1}$	$\mathbf{1 \times 1}$	$\frac{2}{k+1}$
M_j	$\mathbf{1}$	$\mathbf{N_f \times \bar{N}_f}$	$2 - \frac{4}{k+1}\frac{N_c}{N_f} + \frac{2}{k+1}(j-1)$

(22)

and the following superpotential

$$W_{\rm mag} = -\frac{s_0}{k+1}{\rm Tr}\ Y^{k+1} + \frac{s_0}{\mu^2}\sum_{j=1}^{k} M_j \tilde{q} Y^{k-j} q\ . \qquad (23)$$

Example III: The Kutasov Theory (cont...)

• The UV superpotential breaks the symmetry associated with the current

$$J_X = \frac{N_c}{N_f}\left(QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger\right) - XX^\dagger \ . \tag{24}$$

• Using baryon matching we can fix the coefficient of $YY^\dagger$ in the IR.

• This operator cannot be followed using the $\mathcal{R}$-multiplet

• But, using our previous tricks, there is another interesting long multiplet that we can follow

$$U^{UV} = \left(-\frac{1}{2} + \frac{3}{k+1}\frac{N_c}{N_f}\right)\left(QQ^\dagger + \tilde{Q}\tilde{Q}^\dagger\right) + \left(1 - \frac{3}{k+1}\right)XX^\dagger. \tag{25}$$

Example III: The Kutasov Theory (cont...)

- Using the $\mathcal{R}$ multiplet we find that in the IR

$$\begin{aligned} U^{IR} \;=\;& \left(-\frac{1}{2}+\frac{3}{k+1}\frac{kN_f-N_c}{N_f}\right)\left(qq^\dagger+\tilde{q}\tilde{q}^\dagger\right)+\left(1-\frac{3}{k+1}\right)YY^\dagger \\ +\;& \sum_j\left(-2+\frac{6}{k+1}\frac{N_c}{N_f}-\frac{3(j-1)}{k+1}\right)M_jM_j^\dagger\ . \end{aligned} \tag{26}$$

- Therefore:

$$\left(-\frac{1}{2}+\frac{3}{k+1}\frac{N_c}{N_f}\right)\left(QQ^\dagger+\tilde{Q}\tilde{Q}^\dagger\right)+\left(1-\frac{3}{k+1}\right)XX^\dagger \to$$
$$\left(-\frac{1}{2}+\frac{3}{k+1}\frac{kN_f-N_c}{N_f}\right)\left(qq^\dagger+\tilde{q}\tilde{q}^\dagger\right)+\left(1-\frac{3}{k+1}\right)YY^\dagger$$

$$+ \sum_j \left(-2 + \frac{6}{k+1}\frac{N_c}{N_f} - \frac{3(j-1)}{k+1} \right) M_j M_j^\dagger \qquad (27)$$

Example IV: Adjoint SQCD

• We have focused mostly on theories with a free IR description. Here we will discuss adjoint SQCD. It is believed to flow to an interacting IR SCFT.

	$SU(N_c)$	$SU(N_f) \times SU(N_f)$	$U(1)_R$	$U(1)'$	$U(1)_B$
Q	$\mathbf{N_c}$	$\mathbf{N_f} \times \mathbf{1}$	$1 - \frac{2N_c}{3N_f}$	1	1
$\tilde{Q}$	$\bar{\mathbf{N}}_\mathbf{c}$	$\mathbf{1} \times \bar{\mathbf{N}}_\mathbf{f}$	$1 - \frac{2N_c}{3N_f}$	1	-1
X	$\mathbf{N_c^2} - \mathbf{1}$	$\mathbf{1} \times \mathbf{1}$	2/3	-1	0

(28)

• Don't know much about the IR, but we can infer when some fields $M^i = QX^i\tilde{Q}$ become free—e.g. for $N_f/N_c < (3+\sqrt{7})^{-1}$, $M_0 = Q\tilde{Q}$ becomes free.

Example IV: Adjoint SQCD (cont...)

- Using out techniques, we can map

$$\left(-\frac{1}{2}+\frac{N_c}{N_f}\right)\left(QQ^\dagger+\tilde{Q}\tilde{Q}^\dagger\right) \longrightarrow \sum_{j=0}^{P(N_f/N_c)}\left(1-\frac{3R(M_j)}{2}\right)M_jM_j^\dagger+\ldots$$
$$= -\sum_{i=0}^{P(N_f/N_c)}\left(j+2-2\frac{N_c}{N_f}\right)M_jM_j^\dagger+\ldots \quad (29)$$

The R-current Multiplet and IR Phases of Gauge Theories

• We have seen that the R-current multiplet gives us a handle on a particular long (spin zero) multiplet, U.

• **Question:** Does it also contain some global information? Encodes the phase of the IR theory? Is the deep IR an interacting or a free SCFT (perhaps below some confining scale, Λ)?

• **Claim:** There is strong evidence that suggests the answer is yes!

The R-current Multiplet and IR Phases of Gauge Theories (cont...)

• To support this claim, we will study $\langle U(x)U(0)\rangle$.

• It turns out that we will be able to make more explicit statements, with less information, about (strongly) interacting theories than we could when studying the mapping of the full U operator along the RG flow.

• But which U (and R_μ)? This is ambiguous.

• We will study the one defined (up to some exceptions) by a-maximization in the deformed UV theory, $(\mathcal{R}^{UV}_{\mu,\text{vis}}, U^{UV}_{\text{vis}})$.

The R-current Multiplet and IR Phases of Gauge Theories (cont...)

- We will study τ_U:

$$\langle U^{UV,IR}_{\mu,\text{vis}}(x) U^{UV,IR}_{\nu,\text{vis}}(0)\rangle = \frac{\tau_U^{UV,IR}}{(2\pi)^4}\left(\partial^2\eta_{\mu\nu} - \partial_\mu\partial_\nu\right)\frac{1}{x^4} \ . \tag{30}$$

- Note that in theories without accidental symmetries, $\tau_U^{UV} > 0 = \tau_U^{IR}$.

- **Conjecture:** $\tau_U^{UV} > \tau_U^{IR}$ more generally (new information not contained in $a_{UV} > a_{IR}$).

The R-current Multiplet and IR Phases of Gauge Theories (cont...)

• We will provide strong evidence (although not a conclusive proof) in support of the above conjecture.

•This conjecture implies a UV bound on accidental symmetries.

• If true, this conjecture will resolve a longstanding problem: the IR phase of the ISS theory.

• τ_U^{UV} is a quantity in the UV SCFT, although it is not intrinsically defined in it (only defined once have in mind an R-symmetric relevant deformation and/or R-symmetry-preserving vev).

Defining τ_U

• We start by using a-maximization to find the UV superconformal R-current; consider $\mathcal{R}^{t*}_{\mu,UV} = \mathcal{R}^{(0)*}_{\mu,UV} + \sum_i t^i J^{UV*}_{\mu,i}$, where $J^{UV*}_{\mu,i}$ are the full set of non-R symmetries of the UV SCFT.

• Taking $\tilde{a}^t_{UV} = 3\text{Tr}\ \left(\mathcal{R}^{t*}_{UV}\right)^3 - \text{Tr}\ \mathcal{R}^{t*}_{UV}$, solve $\partial_{t^i}\tilde{a}^t_{UV}|_{t^i=t^i_*} = 0, \quad \partial^2_{t^i t^j}\tilde{a}^t_{UV}|_{t^{i,j}=t^{i,j}_*} < 0$. This defines $\tilde{R}^{UV}_\mu$.

• Deform the theory by turning on an R-symmetry-preserving relevant deformation and/or an R-symmetry-preserving vev. Now only $\left\{\hat{J}^{UV*}_{\mu,a}\right\} \subset \left\{J^{UV*}_{\mu,i}\right\}$ are still conserved currents that respect the vacuum.

• Maximizing $\tilde{a}$ over this subset yields $\mathcal{R}^{UV}_\mu = \mathcal{R}^{(0),UV}_\mu + \sum_a \hat{t}^a_* \hat{J}^{UV}_{\mu,a}$. This operator and U^{UV} partner descend from a corresponding pair in the undeformed UV SCFT, $(\mathcal{R}^{UV}_{\mu,\text{vis}}, U^{UV}_{\text{vis}})$.

Defining τ_U (cont...)

• Sometimes this procedure is not sufficient to fix some of the $\hat{t}_*^A$. This may happen in the presence of massive particles or more generally.

• In such a case, we can then fix the corresponding mixing with $\left\{\tilde{J}_{\mu,A}^{UV}\right\} \subset \left\{\hat{J}_{\mu,a}^{UV}\right\}$ by demanding

$$\langle U_{\mu,\text{vis}}^{UV}(x)\tilde{J}_{\nu,A}^{UV\,*}(0)\rangle = 0 \ . \tag{31}$$

• For free UV (IR) theories, we find

$$\tau_U^{UV,IR} = \text{Tr}\ \left(U_{\text{vis}}^{UV,IR}\right)^2 \ . \tag{32}$$

Defining τ_U (cont...)

• More generally, can sometimes use

$$\tau_U^{UV,IR} = -3\mathrm{Tr}\tilde{\mathcal{R}}_p^{UV,IR} U_{\mathrm{vis,p}}^{UV,IR} U_{\mathrm{vis,p}}^{UV,IR} \, , \tag{33}$$

and 't Hooft anomaly matching to obtain explicit expressions.

• In the IR (and also in the UV), sometimes one needs more complicated generalizations (for example, when the IR is an interacting SCFT with some decoupled fields and accidental symmetries).

• Won't discuss these cases in the talk (see **[1109.3279]** for further discussion).

Two Simple Examples

• As a simple sanity check (and important result), consider a free chiral multiplet, Φ and the deformation $W = m\Phi^2$.

• There is a unique R-symmetry; therefore, $\mathcal{R}^{UV}_{\text{vis}}(\Phi) = 1$ and $U^{UV}(\Phi) = 1/2$. As a result, $\tau^{UV}_U = 1/4$. The theory is trivial in the IR and so

$$\tau^{UV}_U = 1/4 > 0 = \tau^{IR}_U \ . \tag{34}$$

• Slight complication with two free chiral superfields, $\Phi_{1,2}$ and $W = m\Phi_1\Phi_2$. This preserves a non-R symmetry, J, under which the Φ_i transform with opposite charges. Need to impose $\langle U^{UV}_{\mu,\text{vis}}(x) J^{UV*}_\nu(0)\rangle = 0$. Find $\tau^{UV}_U = 1/2 > 0 = \tau^{IR}_U$.

SQCD

• Our procedure fixes $\mathcal{R}^{UV}_{\text{vis}}(Q) = \mathcal{R}^{UV}_{\text{vis}}(\tilde{Q}) = 1 - \frac{N_c}{N_f}$ and $U^{UV}_{\text{vis}}(Q) = U^{UV}_{\text{vis}}(\tilde{Q}) = \frac{1}{2} - \frac{3N_c}{2N_f}$.

• Consider $N_f < 3N_c$, and start from the free UV theory.

• Begin with $N_f = N_c$ and work our way up. All the subtleties we have discussed in this talk are present in this class of theories (accidental symmetries, Goldstone bosons, interacting fixed points etc.).

SQCD (cont...)

• $N_f = N_c$; $\tau_U^{UV} = 2N_c^2$; in the IR have a deformed moduli space $\det M + B\tilde{B} = \Lambda^{2N_c}$ with $< N_c^2 + 2$ mesons, M, and baryons, $B, \tilde{B}$.

• Since $\mathcal{R}^{IR}_{\text{vis}}(M) = \mathcal{R}^{IR}_{\text{vis}}(B) = \mathcal{R}^{IR}_{\text{vis}}(\tilde{B}) = 0$ and $U^{IR}_{\text{vis}}(M) = U^{IR}_{\text{vis}}(B) = U^{IR}_{\text{vis}}(\tilde{B}) = -1$, we have $\tau_U^{IR} < N_c^2 + 2$.

$$\tau_U^{UV} = 2N_c^2 > N_c^2 + 2 > \tau_U^{IR}. \tag{35}$$

• $N_f = N_c + 1$; $\tau_U^{UV} = \frac{N_c(1-2N_c)^2}{2(1+N_c)}$; confinement without chiral symmetry breaking, $(N_c+1)^2$ mesons, M, and $2(N_c+1)$ baryons B and $\tilde{B}$.

SQCD (cont...)

• Have $\mathcal{R}_{\text{vis}}(M) = \frac{1-2N_c}{1+N_c}, \mathcal{R}_{\text{vis}}(B) = \mathcal{R}_{\text{vis}}(\tilde{B}) = \frac{N_c}{2}\frac{1-2N_c}{1+N_c}$, $U(M) = -1+\frac{3}{N_c+1}$, $U(B) = U(\tilde{B}) = \frac{N_c-2}{2(N_c+1)}$. Therefore, $\tau_U^{IR} = \frac{(N_c-2)^2(3+2N_c)}{2(1+N_c)}$ and

$$\tau_U^{UV} = \frac{N_c(1-2N_c)^2}{2(1+N_c)} > \frac{(N_c-2)^2(3+2N_c)}{2(1+N_c)} = \tau_U^{IR}\ . \qquad (36)$$

• Can see that fully conserved current two-point functions have no definite behavior along the RG flow. Therefore, a-maximization picks out a current, U, that has nice properties.

SQCD (cont...)

• $N_f = N_c{+}2$, confining description breaks down; $\tau_U^{UV} = \frac{2N_c(N_c-1)^2}{N_c+2}$ while $\tau_U^{\text{conf}} = \frac{5N_c^3-10N_c^2-4N_c+36}{N_c+2}$, and so conjecture would be violated in a hypothetical confining phase.

• Luckily, correct description is free magnetic with $\mathcal{R}_{\text{vis}}^{IR}(M) = 2\left(1 - \frac{N_c}{N_f}\right), \mathcal{R}_{\text{vis}}^{IR}(q) = \mathcal{R}_{\text{vis}}^{IR}(\tilde{q}) = \frac{N_c}{N_f}$ and $U_{\text{vis}}^{IR}(M) = 2-\frac{3N_c}{N_f}$, $U_{\text{vis}}^{IR}(q) = U_{\text{vis}}^{IR}(\tilde{q}) = -1 + \frac{3N_c}{2N_f}$. Therefore:

$$\tau_U^{UV} = \frac{N_c(N_f - 3N_c)^2}{2N_f} > \frac{(3N_f - N_c)(3N_c - 2N_f)^2}{2N_f} = \tau_U^{IR} \ . \quad (37)$$

SQCD (cont...)

• The above expressions are valid for $N_c + 1 < N_f \leq 3N_c/2$. The inequality holds up to $N_f \sim 1.79N_c$ (where the theory flows to an interacting conformal fixed point, and the above expressions don't apply). Comes close to predicting onset of conformal window.

• In conformal window, $3N_c/2 < N_f < 3N_c$, trivially have (from assumed lack of accidental symmetries)

$$\tau_U^{UV} > 0 = \tau_U^{IR} \ . \tag{38}$$

• Can do some more complicated tests of conformal window.

SQCD (cont...)

• Start from the interacting fixed point and turn on $W = \lambda Q_a \tilde{Q}^a$, $a = 1, \cdots, k$. Need to use $\langle U^{UV}_{\mu,\text{vis}}(x) \tilde{J}^{UV*}_{\nu,A}(0)\rangle = 0$ for conserved currents that act non-trivially on the $Q_a, \tilde{Q}^a$.

• If $k < N_f - \frac{3}{2}N_c$, the theory flows to another theory in the conformal window. Trivially satisfy $\tau_U^{UV} > 0 = \tau_U^{IR}$. Same for $k = N_f - \frac{3}{2}N_c$.

• For $N_f - N_c - 1 > k > N_f - \frac{3}{2}N_c$, the IR phase is free magnetic. We find

$$\tau_U^{UV} = \frac{27kN_c^4}{2(N_f - k)N_f^2} > \frac{(3(N_f - k) - N_c)(3N_c - 2(N_f - k))^2}{2(N_f - k)} = \tau_U^{IR}\,. \tag{39}$$

• Can also verify the inequality for $k = N_f - N_c - 1, N_f - N_c$.

SQCD (cont...)

• Can also consider RG flows with Higgsing. Take $\langle Q^a_a\rangle = \langle \tilde{Q}^a_a\rangle = v_a$, for $a = 1, \cdots k$. Suppose all v_a distinct. Find $SU(N_c - k)$ SQCD with $N_f - k$ flavors, $\mathcal{Q}^i_A$, $\tilde{\mathcal{Q}}^A_i$, k^2 singlets, S_I, k gauge singlets Φ_a and k gauge gauge singlets, $\tilde{\Phi}^a$, transforming under $\mathbf{N_f - k}$ and $\overline{N_f - k}$ of $SU(N_f - k)_{L,R}$ respectively.

• We find $\mathcal{R}_{\text{vis}}(\mathcal{Q}) = \mathcal{R}_{\text{vis}}(\tilde{\mathcal{Q}}) = \mathcal{R}_{\text{vis}}(\Phi) = \mathcal{R}_{\text{vis}}(\tilde{\Phi}) = 1 - \frac{N_c - k}{N_f - k}$, $\mathcal{R}_{\text{vis}}(S) = 0$ (note that for the case $k = N_c$ we use $\langle U^{UV}_{\mu,\text{vis}}(x) \tilde{J}^{UV*}_{\nu,A}(0)\rangle = 0$).

• Trivially true that $\tau^{UV}_U > \tau^{IR}_U$ for flows starting from the free UV fixed point. Consider now flows starting from an interacting fixed point.

SQCD (cont...)

• If $k < \min\Big((3N_c - N_f)/2, N_c - 1\Big)$, theory flows to a more weakly coupled interacting fixed point. Find that

$$\begin{aligned}
\tau_U^{UV} &= \frac{27kN_c^2(N_c - N_f)^2}{2(N_f - k)N_f^2} \\
&> \frac{k(2k^2 + N_f^2(1 - 3N_c/N_f)^2 + 6kN_f(1 - 2N_c/N_f))}{2(N_f - k)} \\
&= \tau_U^{IR} \qquad (40)
\end{aligned}$$

• If $(3N_c - N_f)/2 \leq k \leq N_c$, the IR endpoint is free. We find

$$\begin{aligned}
\tau_U^{IR} &= -\frac{1}{2(N_f - l)} \cdot (2k^3 - N_f^3(1 - 3N_c/N_f)^2(N_c/N_f) \\
&+ 4kN_fN_c(3N_c/N_f - 1) - 2k^2N_f(1 + 2N_c/N_f)) \ , \qquad (41)
\end{aligned}$$

which still satisfies $\tau_U^{UV} > \tau_U^{IR}$.

SQCD (cont...)

• Easy to generalize the above discussion to $SO(N_c)$ and $Sp(N_c)$ gauge groups

• Also other more exotic s-confining theories; SCFTs with accidental symmetries; $\mathcal{N} = 2$ SYM; Kutasov and Brodie theories; See **[1109.3279]** for details.

The IR Phase of ISS

• Intriligator, Seiberg, and Shenker consider an $SU(2)$ gauge theory with a single field, Q, in the isospin 3/2 representation.

• They conjectured that the IR theory at the origin is described by a confined $u = Q^4$ field (classically, the Kähler potential is singular at the origin); indeed, since $\mathcal{R}_{\text{vis}}^{UV}(Q) = 3/5$ and $\mathcal{R}_{\text{vis}}^{UV}(u) = 12/5$, the $U(1)_R$ and $U(1)_R^3$ anomalies match.

• If the confining description is correct, then, upon deforming the theory by $W = \lambda u$, we would find a simple model of (dynamical) SUSY breaking. In this vacuum, there would be a preserved R-symmetry that is a mixture of the accidental non-R symmetry under which u transforms and $\mathcal{R}_{\text{vis}}$.

The IR Phase of ISS (cont...)

• Subsequently, other techniques have pointed to the opposite conclusion—namely, that the IR is interacting conformal.

• Our criterion also suggests this is the case. Indeed, $U^{UV}_{\text{vis}}(Q) = -\frac{1}{10}$, $U^{IR}_{\text{vis}}(u) = \frac{13}{5}$ and so

$$\tau_U^{UV} = \frac{1}{25}, \quad \tau_U^{IR,\text{confining}} = \frac{169}{25} , \tag{42}$$

and so $\tau_U^{UV} < \tau_U^{IR,\text{confining}}$. This conflicts with our conjecture.

• Conjecture formalizes the intuition that the theory is too weak to produce confined d.o.f's (the 1-loop beta fn is $b = 6 - 5 = 1$).

• Can also check that our procedure is consistent with better understood misleading anomaly matchings.

Conclusions

• We have seen that the $(\mathcal{R}_\mu, U)$ multiplets contain a great deal of physics.

• We can use this pair to learn things about operator mappings, accidental symmetries, and IR phases.

• Can we extend duality mapping to other non-conserved quantities (as in the case of a UV superpotential)?

• Can we prove that $\tau_U^{UV} > \tau_U^{IR}$? Can define another τ'_U using a minimization procedure. Does this quantity also decrease?