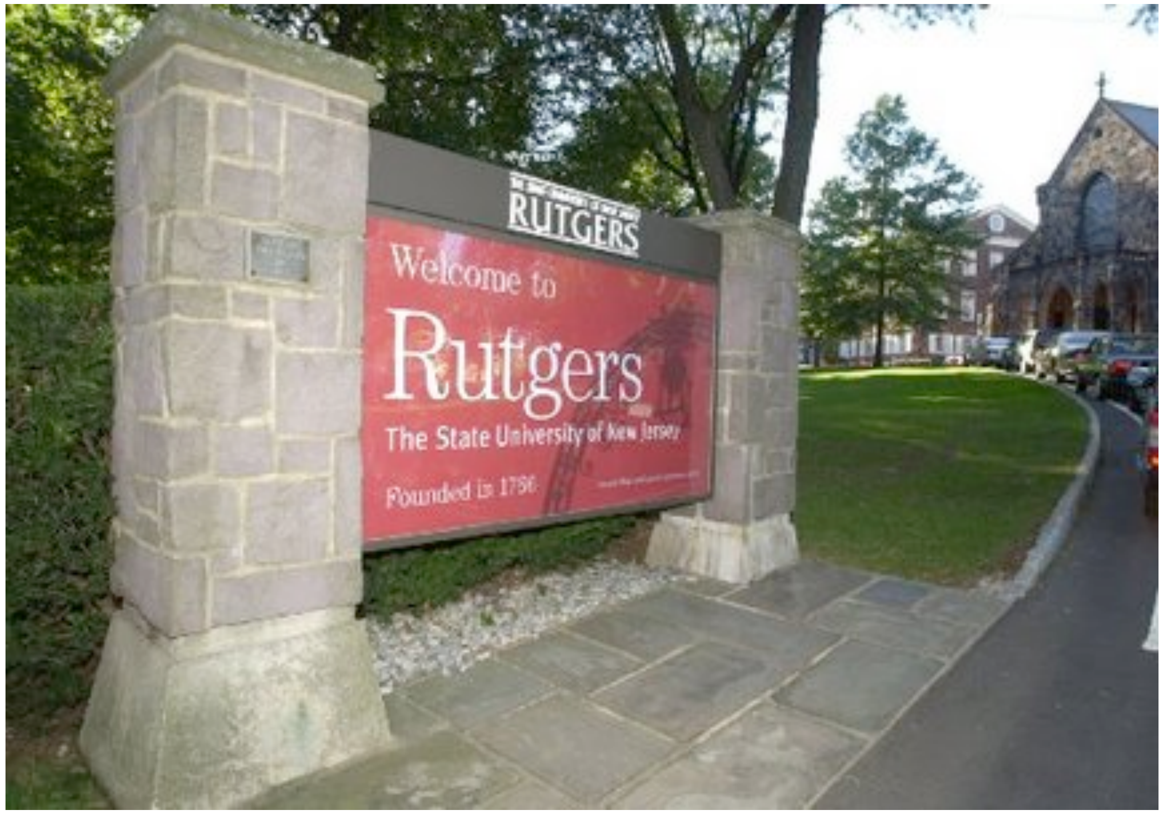
Maximal R-symmetry violating amplitudes in type IIB superstring theory

(based on arXiv:1204.4208 and work in progress)

Rutger Boels
University of Hamburg

A seminar at my university



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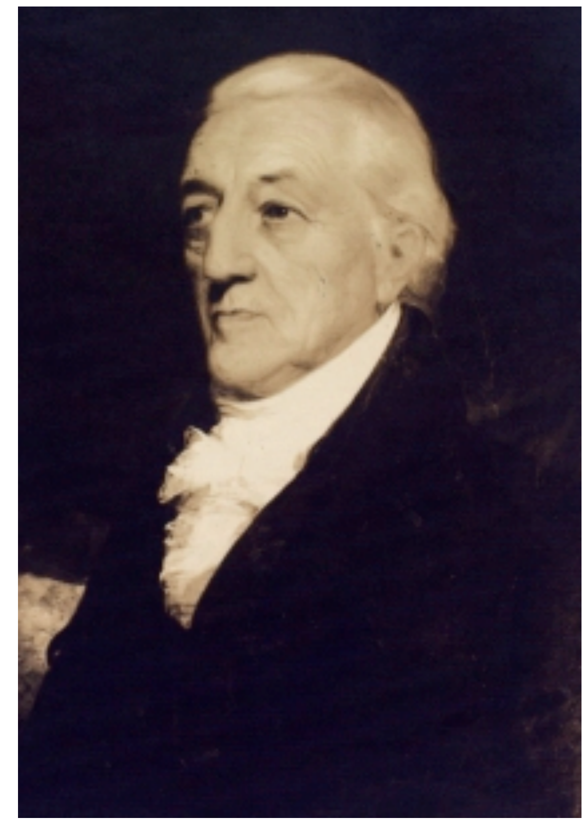

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(1745–1830, Dutch parents)

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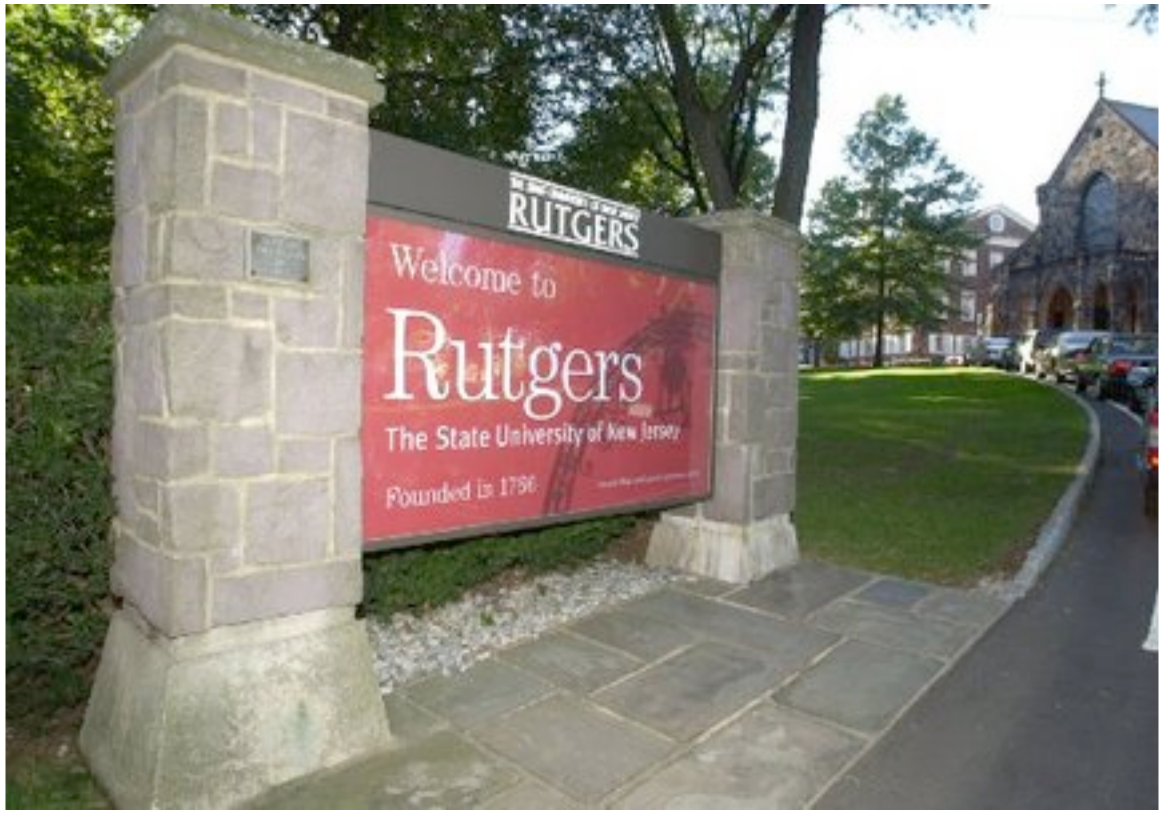

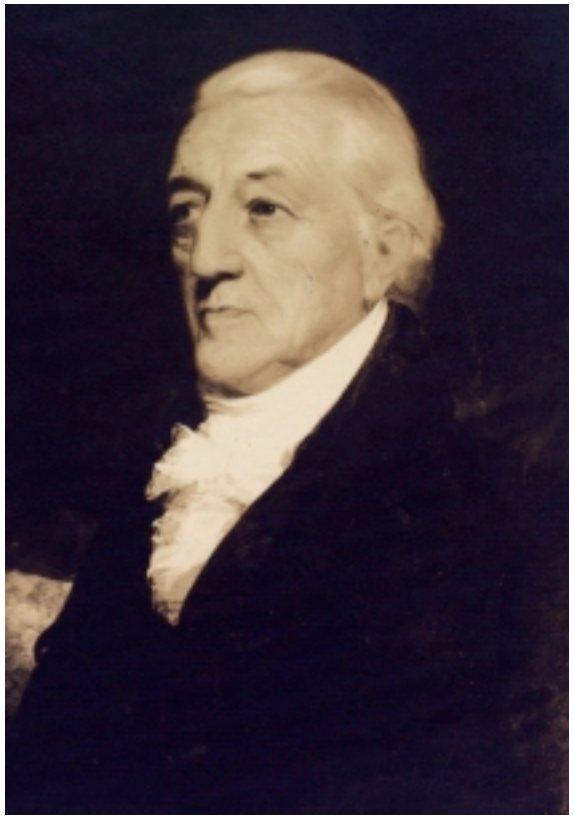

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- Rutger is a Germanic name, related to Rodger
- means something like "famous with the spear"

Why pay attention?

...string scattering amplitudes in a flat background...

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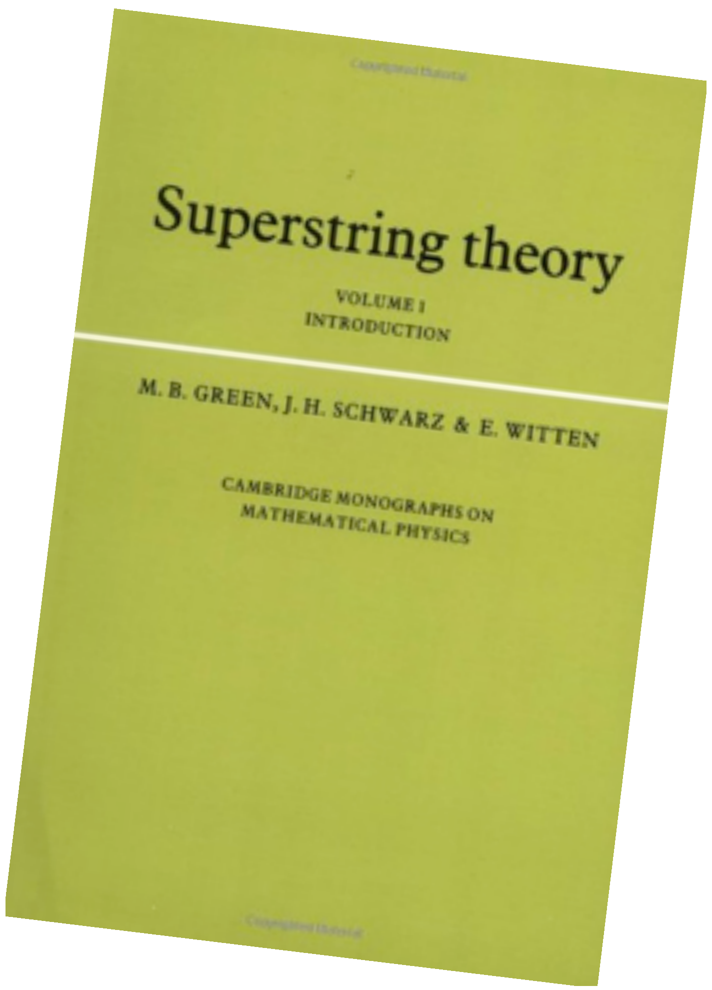

- derive vertex operators
- calculate correlation functions
- integrate over moduli

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→ very complicated above four points!

Two motivations for general study of amplitudes

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Experimental

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⇒ massive progress in four dimensional field theory

- especially with maximal supersymmetry
- many authors

Goal: simple answers

- symmetry vs simplicity → most (manifestly) symmetric answers are the simplest

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- golden standard of simple scattering amplitudes: 4D MHV amplitudes in tree level Yang-Mills

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$$A_n(\mathrm{MHV}) = \frac{\langle i,j\rangle^4}{\langle 12\rangle\langle 23\rangle\ldots\langle n1\rangle}$$

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- similar insight into perturbative string theory?
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D=4 vs D>4?

Right language: spinor helicity

- Poincare quantum numbers for multiple plane waves → covariance

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e.g. in Yang-Mills [Parke-Taylor, 87]:

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  - see also D=10, [Caron-Huot-O'Connell, 10], D=6 [Dennen-Huang-Siegel, 09]

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$1, 2, \ldots, \mathcal{D}$

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symmetries: from $\{\,[\psi\lambda]\,,\,[\lambda\psi]\,,\,\langle\lambda\psi\rangle\,,\,\langle\psi\lambda\rangle\,\}$ 2 independent

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- complete dictionairy between vectors and spinors
- little group basis choice through a set of fixed spinors:

$$\lambda^{A,a} \propto k^{AA'} \xi^a_{A'} \qquad \lambda_{A',a'} \propto k_{A'A} \xi^A_{a'} \,,$$

(leads to complete basis, numerical convenience)

Superpoincare → on-shell superspaces

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identify massless field content:

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- other states antisymmetrized tensor products of ϕ_0 with chiral spinor of SO(D-2)
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Massless on-shell superspace in D=10

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IIB supergravity

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IIB supergravity

U(1) R-charge assignments

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D=10: 65.536 states in the fundamental multiplet

$$\phi(\eta,\iota) = \phi_0 + \phi^a \eta_a + \phi_{a'} \iota^{a'} + \ldots + \bar{\phi}\left((\eta)^8 \iota^8\right)$$

can calculate it's SO(9) Dynkin labels:

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0	$(0,0,0,0)_1$
1	$(0,0,0,1)_{16}$
2	$(0,1,0,0)_{36} + (0,0,1,0)_{84}$
3	$(1,0,0,1)_{128} + (0,1,0,1)_{432}$
4	$(2,0,0,0)_{44} + (0,0,0,2)_{126} + (1,1,0,0)_{231} + (0,2,0,0)_{495} + (1,0,0,2)_{924}$
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can calculate it's SO(16) Dynkin labels:

0	$(0,0,0,0,0,0,0,0)_1$
1	$(1,0,0,0,0,0,0,0)_{16}$
2	$(0,1,0,0,0,0,0,0)_{120}$
3	$(0,0,1,0,0,0,0,0)_{560}$
4	$(0,0,0,1,0,0,0,0)_{1820}$
5	$(0,0,0,0,1,0,0,0)_{4368}$
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various other groups in paper (SO(32), SO(10), etc.)

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- 3 massless particle exception:

$$\delta^{\frac{3}{4}\mathcal{D}}(Q) \sim \epsilon_{A_1 \ldots A_{\mathcal{D}}} \left(Q^{A_1} \ldots Q^{A_{\frac{3}{4}\mathcal{D}}} \xi_{a_1}^{A_{\frac{3}{4}\mathcal{D}+1}} \xi_{a_{\frac{1}{4}\mathcal{D}}}^{A_{\mathcal{D}}}\right)$$

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- also three points, five points, on-shell recursion in paper

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- U(1)R in D=8 → rotations in 9-10 plane → superamplitudes here have weight 2n — conserved
- U(1)R in D=10 → part of SL(2,R)/U(1) of IIB → simple superamplitudes? — not conserved

Massless on-shell superspace in D=10, type IIB

D=10: 256 states in the fundamental multiplet

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graviton → $\underline{35}_0 + \underline{35}'_0$

dilatino → $\underline{8}_3$

conjugate scalar → $\underline{1}_{-4}$

holomorphic scalar → $\underline{1}_4$

Structure of IIB superamplitudes $A_n = \tilde{A}_n \delta^{16}(Q)$

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  - follow from one component amplitude
  - completely Bose symmetric
  - only massive particle poles (n>4)
  - → no poles in field theory limit

MRV amplitudes in field theory limit $A_n = \tilde{A}_n \delta^{16}(Q)$

exceptional case at four points

$$A_4^{D=10} = \frac{\delta^{10}(K)\delta^{16}(Q)}{s\,t\,u}\left[\frac{\Gamma\left(\alpha' s+1\right)\Gamma\left(\alpha' t+1\right)\Gamma\left(\alpha' u+1\right)}{\Gamma\left(1-\frac{(\alpha' s)}{2}\right)\Gamma\left(1-\frac{(\alpha' t)}{2}\right)\Gamma\left(1-\frac{(\alpha' u)}{2}\right)}\right]$$

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general story?

Intermezzo: fun with counting

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by computing the generating function (“Molien series”)

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string theory selects one combination out of these polynomials

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- see [Stieberger, 09], [Schlotterer-Stieberger, 12] for more

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- differential operator annihilates gravitational coupling

→ relates c_i for various multiplicities, up to degeneracy

Return to intermezzo: fun with counting

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conjecture: dimension stabilizes at $n = 2i$

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more stringy symmetry in IIB: SL(2,Z)

[Green-Gutperle, 97], [Green et.al., 97-12]

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- "beta" ↔ alpha' order, "k" ↔ $U(1)_R$ non-conservation

MRV amplitudes in field theory limit $A_n = \tilde{A}_n \delta^{16}(Q)$

more stringy symmetry in IIB: SL(2,Z)

[Green-Gutperle, 97], [Green et.al., 97-12]

- results for effective action, $R^4, D^4R^4, D^6R^4, \lambda^{16}$ couplings
- coefficients as functions of background fields $\tau_b = a_b + \mathrm{i}\, e^{-\phi_b}$

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- weak string coupling expansion:

$$\lim_{\tau_b \to i\infty} f^k_{\frac{3}{2}}(\tau_b, \overline{\tau}_b) \propto \zeta(3) + (\text{1-loop}) + \text{instanton}$$

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"exact" amplitude conjecture:

$$A_n^{\mathrm{MRV}} \propto \delta^{16}(Q)(\alpha'^2 g)^{n-2}\left(\alpha'^3 f^{n-4}_{\frac{3}{2}} + \alpha'^5 f^{n-4}_{\frac{5}{2}}\left([s^2_{12}]_n\right) + \mathcal{O}\left(\alpha'^6\right)\right)$$

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- analytic part of amplitude: the "no logs"-part
- guess for next order exists
- much work: relation to effective action, better normalization...

Summary, outlook

shown examples of applications of 'analytic S-matrix' insight:

scattering amplitudes are functions
- of the quantum numbers
- with physical singularities

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more examples of applications / more explicit amplitudes?
- how deep does MHV -- MRV analogy go?
- worldsheet picture? (pure spinor?)
- IIA? D=11? open strings? → constrained superspaces

Your Question Here?