Anomaly Inflow and topological mass terms

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- Geometric Engineering of QFTs is a powerful tool for exploring Strongly Coupled Systems
- The Landscape of SCFTs can be explored by studying the low-energy dynamics of various brane systems
- Reduction of SCFTs on compact manifolds, X – Lower D SCFT defined by X
- Typical SCFT is strongly coupled and may not admit Lagrangian descriptions [Gaiotto '09; Gaiotto, Moore, Neitzke '09]
- Many of such SCFTs can admit an arbitrarily large flavor symmetry – For example: Compactification of 6D SCFTs on punctured Riemann surfaces
- Physical observables of SCFTs from the geometric definitions

Compute 't Hooft anomalies of SCFTs from geometric setup

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- Quantum Anomalies: Partition function not invariant under gauge transformation in presence of background gauge fields

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  - $\alpha(A,\epsilon)$ vanish in the limit $A \to 0$ $\Longrightarrow$ **Can imply higher groups otherwise**

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  - **Non-renormalized under RG flows** – must be reproduced by any effective description
  - Constrain IR phases of quantum systems
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't Hooft anomalies provide a measure for degrees of freedom for QFTs – Defining data for non-Lagrangian theories

Anomaly Polynomials

- Anomalies for **continuous** global symmetries
- The anomaly for a QFT on W_d is given by an integral of a local density

$$\alpha(A, \epsilon) = \delta_\epsilon \mathcal{W}_{QFT}[A] = 2\pi \int_{W_d} I_d^{(1)}$$

- Wess-Zumino consistency conditions imply descent relations for anomaly

$$dI_d^{(1)} = \delta I_{d+1}^{(0)}, \qquad dI_{d+1}^{(0)} = I_{d+2}$$

  - $I_{d+1}^{(0)}$ is a Chern-Simons form in W_{d+1} with boundary W_d
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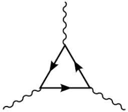
- I_{d+2} is a polynomial in curvatures of the background fields whose coefficients encode the 't Hooft anomaly of the global symmetry – **Anomaly Polynomial**
- Example in 4d: a_{IJK} and a_I are anomaly coefficients from triangle diagram

$$I_6 = a_{IJK} F^I \wedge F^J \wedge F^K + a_I F^I \wedge \text{tr}(R \wedge R),$$



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- I_{d+2} is a polynomial in curvatures of the background fields whose coefficients encode the 't Hooft anomaly of the global symmetry – **Anomaly Polynomial**
- Captures anomalies for Discrete Symmetries when embedded in continuous symmetries
- Quantization conditions on background fields and anomaly polynomial – **Global anomalies** – Anomaly form as differential co-cycle

Outline

1. Anomalies of SCFTs in M-theory

2. Topological mass terms and discrete symmetry

Outline

Setup with M5-branes

- Consider a stack of N M5-branes in M-theory
  - Flat branes: $(2,0)$ A_{N-1} SCFTs in 6D
  - Probing $\mathbb{C}^2/\Gamma$ singularity: $(1,0)$ SCFTs in 6D
  - Wrapped on a surface X: SCFTs in 4D, SCFTs in 2D
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- The 4-form flux of M-theory admits a singular magnetic source and the M-theory background has an internal boundary

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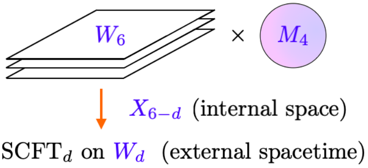
- M_{10} is the boundary of a tubular neighborhood of the source:

$$M_{10-d} \hookrightarrow M_{10} \rightarrow W_d, \quad M_4 \hookrightarrow M_{10-d} \rightarrow X_{6-d}$$

- M_{10-d}: defines the SCFT in M-theory, can have orbifold fixed points
- M_4: The angular directions that surround the branes
- M_4 fibration fixed by topological twist



- Reducing M-theory on M_{10-d} can lead to interesting gauge symmetry, G
- Components of G: the isometry group of M_{10-d}, massless fluctuations of the C_3 potential – Expanded on $H^*(M_{10-d}, \mathbb{Z})_{free}$

$$\delta C_3 = c_3 + b_2^u \lambda_u^1 + a_1^\alpha \omega_\alpha^2 + t_0^x \Lambda_x^3$$

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- **Anomaly Inflow**: The quantum anomalies for the boundary degrees of freedom on the M5-branes must **cancel** the classical bulk anomaly

The bulk supergravity action can be used to obtain the anomalies for SCFTs from M5-branes

Flux Boundary Condition

- The anomalous variation of the M-theory action depends on the boundary condition of G_4 corresponding to the singular source [Freed, Harvey, Minasian, Moore '98]

$$G_4 = 2\pi\rho(r)\bar{G}_4 + \cdots \quad \text{with} \quad \int_{M_4} \bar{G}_4 = N$$

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- $\bar{G}_4$ can be extended to a closed, gauge invariant and globally defined four-form, E_4, on the space M_{10} by gauging the action of the group G

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On W_d, the gauging corresponds to turning on background fields for the global symmetry

Background fields $\Longleftrightarrow$ Boundary value of bulk gauge fields

Anomalous Variation of M-theory [IB, Bonetti, Minasian, Nardoni '18, '19]

- The variation of the M-theory action localizes on the boundary

$$\frac{\delta S_M}{2\pi} = \int_{M_{10}} \mathcal{I}_{10}^{(1)}, \qquad d\mathcal{I}_{10}^{(1)} = \delta \mathcal{I}_{11}^{(0)}, \quad d\mathcal{I}_{11}^{(0)} = \mathcal{I}_{12}$$

- The 12-form anomaly polynomial is completely characterized by E_4 and the M-theory action

$$\mathcal{I}_{12} = -\frac{1}{6} E_4 \wedge E_4 \wedge E_4 - E_4 \wedge X_8$$

the 8-form, $X_8 = \frac{1}{192}\left[p_1(TM_{11})^2 - 4p_2(TM_{11})\right] \sim R^4$, decomposed on $M_{11} = \mathbb{R}^+ \times M_{10}$ – Gravitational anomalies

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Anomaly inflow statement:

$$I_{d+2}^{\text{inf}} + I_{d+2}^{\text{CFT}} + I_{d+2}^{\text{decoupled}} = 0, \qquad I_{d+2}^{\text{inf}} = \int_{M_{10-d}} \mathcal{I}_{12}$$

Anomaly for 6D $(2,0)$ Theory – Flat branes

- M_{10} and boundary condition for G_4 are

$$M_{10} = W_6 \times S^4, \qquad \bar{G}_4 = N\, d\Omega_4$$

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- The extension of $\bar{G}_4$: global angular form of the 4-sphere

$$E_4 = \frac{N}{64\pi^2}\epsilon_{a_1\cdots a_5} y^{a_5}\left[Dy^{a_1}\cdots Dy^{a_4} + 2F^{a_1 a_2} Dy^{a_3} Dy^{a_4} + F^{a_1 a_2} F^{a_3 a_4}\right]$$
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- Integrating $\mathcal{I}_{12}$ on S^4: [Freed, Harvey, Minasian, Moore '98; Harvey, Minasian, Moore '98]

$$I_8^{\text{inf}} + I_8[(2,0)\text{ SCFT}] + I_8[\text{Free (2,0) tensor}] = 0$$

General Properties of E_4 [IB, Bonetti, Minasian, Nardoni '19]

- The extension E_4 has different components

$$E_4 = \sum_p E_4^p$$

- E_4^p: expansion along a basis of $H^p(M_{10-d}, \mathbb{Z})_{free}$

$$\bar{G}_4 = N^a \Omega_a^4 \quad \rightarrow \quad E_4^4 = N^a \left[\Omega_a^{4,g} + F^I \omega_{a,I}^g + F^I F^J \sigma_{a,IJ} \right]$$

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Choices for E_4 labeled by G_{isom}-equivariant cohomology of M_{10-d}

Compute anomaly by considering local ansatz for metric and p-forms on M_{10-d} consistent with symmetry and topology

- Impose regularity conditions on E_4
- Regularity conditions related to integrals of internal forms $(\Omega^4, \omega^2, \cdots)$
- The Inflow anomaly depends on background fields and on flux parameters of M_{10-d}

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- Consider an $AdS_{d+1} \times \mathcal{M}_{10-d}$ solution in M-theory supported by a G_4^{ads} flux
- We can identify $\mathcal{M}_{10-d} = M_{10-d}$ and $G_4^{ads} = \bar{G}_4$
- The 4-form E_4 can be constructed and $\mathcal{I}_{12}$ yields the anomaly for the dual SCFT
- The X_8 term in $\mathcal{I}_{12}$ yields the $\frac{1}{N^2}$ corrections to the anomaly polynomial
- Extremization principles [Intriligator, Wecht '03; Benini, Bobev '15]
- We expect the anomaly to be exact up to $\mathcal{O}(1)$ corrections due to decoupled center-of-mass degrees of freedom

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$$\mathcal{L} = \frac{1}{2\pi}\Omega_{uv}\ b_2^u \wedge db_2^v + \frac{N_\alpha}{2\pi}\ a_1^\alpha \wedge dc_3 + \cdots$$

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Topological mass terms have important consequences for anomaly inflow results!

Discrete symmetries

- Consider the 5d topological action [Banks, Seiberg '11]

$$S = \frac{M}{2\pi}\int_{\mathcal{M}_5} b_2 \wedge d\tilde{b}_2 + \frac{k}{2\pi}\int_{\mathcal{M}_5} c_3 \wedge da_1$$

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$$W_b(\mathcal{C}_2, n, \tilde{n}) = \exp\left(i\int_{\mathcal{C}_2}[\tilde{n}\ b_2 - n\ \tilde{b}_2]\right)$$

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$$W_b(\mathcal{C}_2, n, \tilde{n}) = \exp\left(i \int_{\mathcal{C}_2} [\tilde{n}\ b_2 - n\ \tilde{b}_2]\right)$$

- Correlation functions of "Wilson lines" implies that
  - c_3, a_1 are flat connections with holonomies in $\mathbb{Z}_k \in U(1)$
  - b_2, $\tilde{b}_2$ are flat connections with holonomies in $\mathbb{Z}_M \in U(1)$
- Topological mass terms are dual to the **Stückelberg action** – Discrete symmetry left over from spontaneous breaking of $U(1)$ symmetries

$$\mathcal{L} = \frac{\Omega_{uv}}{2\pi}\ b_2^u \wedge db_2^v + \frac{N_\alpha}{2\pi}\ a_1^\alpha \wedge dc_3 + \cdots$$

- In suitable normalization of gauge fields, and due to flux quantization, (Ω_{uv}, N_α) are quantized
- The topological mass terms describe discrete gauge symmetries in the 5D supergravity
- For $\Omega_{12} = M$, and $k = gcd(N_\alpha)$ the discrete gauge symmetries are

$\mathbb{Z}_k$	2-form with	c_3	
$\mathbb{Z}_k$	0-form with	$a_1 = m_\alpha a_1^\alpha,$	$N_\alpha = k\ m_\alpha$
$\mathbb{Z}_M \times \mathbb{Z}_M$	1-form with	(b_2^1, b_2^2)	

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(a)	c_3: free a_1: Dirichlet	$\mathbb{Z}_k$ 0-form symmetry
(b)	a_1: free c_3: Dirichlet	$\mathbb{Z}_k$ 2-form symmetry
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- Dirichlet boundary conditions fix a source for discrete symmetry in the dual theory [Gaiotto, Kapustin, Seiberg, Willett '14; Hofman, Iqbal, '18]
- Mixed boundary conditions between the fields lead to a larger class of possible choices of boundary discrete symmetry [Gaiotto, Kapustin, Seiberg, Willett '14]
- Similar choices exist for the 1-form discrete symmetry from $(b_2, \tilde{b}_2)$

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$$I_6 \supset k \frac{dA_1}{2\pi} \wedge \frac{d\widetilde{C}_3}{2\pi} + \Omega_{uv} \frac{dB_2^u}{2\pi} \wedge \frac{dB_2^v}{2\pi}$$

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- $(A_1, \widetilde{C}_3, B_2^u, B_2^v)$ are the boundary values of the gauge field (a_1, c_3, b_2^u, b_2^v)
- These anomalies determine the surface and line operators that can exist for the gauge theory
- From the bulk, the choice of boundary condition determines which bulk "Wilson lines" can end on the boundary

Mixed 't Hooft Anomalies

- In general, the anomaly polynomial includes terms

$$I_6 \supset N_\alpha \frac{F^\alpha}{2\pi} \wedge \frac{d\widetilde{C}_3}{2\pi} + \mathcal{K}_{\alpha\bullet} \frac{F^\alpha}{2\pi} \wedge Q_4^\bullet + \mathcal{K}_\bullet \frac{d\widetilde{C}_3}{2\pi} \wedge \widetilde{Q}_2^\bullet$$
$$+ \mathcal{K}_{\alpha\beta\bullet} \frac{F^\alpha}{2\pi} \wedge \frac{F^\beta}{2\pi} \wedge Q_2^\bullet + \mathcal{K}_{\alpha\beta\gamma} \frac{F^\alpha}{2\pi} \wedge \frac{F^\beta}{2\pi} \wedge \frac{F^\gamma}{2\pi}$$

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- A basis transformation $(A^\alpha) \longrightarrow (A_1, \mathcal{A}^{\alpha'})$ that is consistent with quantization of flux is necessary
- When successful mixed 't Hooft anomalies between discrete and continuous symmetries can be read off from the anomaly polynomial

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- T_0^x – background dependent coupling parameters!
- Constraints on background fields translate to constraints on symmetry generators J^α:

$$N_\alpha F^\alpha \longrightarrow M_\alpha J^\alpha = 0$$

a-maximization for CFT is sensitive to constraints over U(1) symmetries that can mix with the R-symmetry

Green-Schwarz terms [IB, Bonetti, Minasian, Nardoni '19]

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- Anomaly inflow for 6D $(1,0)$ SCFTs from $M5$ branes at orbifolds [Ohmori, Shimizu, Tachikawa, Yonekura, '14]
- Interpreted as a Green-Schwarz term associated to the decoupled center of mass mode of the stack in Ohmori et al.

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- Bulk equation of motion fix Green-Schwarz term!

Singletons and Decoupled modes [IB, Bonetti, Minasian: 2007.15003]

- In presence of a boundary, BF theories admit singleton modes [Witten '99; Maldacena, Moore, Seiberg '01]
- Singletons: Pure gauge modes in the bulk and dynamical in the boundary

$$\frac{M}{2\pi} b_p \wedge da_{d-p-1} \qquad \rightarrow \qquad \text{(p-1)-form gauge field singleton}$$

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- Singletons account for all decoupling modes in SUSY compactifications of M5-branes on punctured Riemann surfaces! (not including orbifold theories)

The symmetry and topology of M_{10-d} completely fix the anomaly of SCFTs from M5-branes and its compactifications

Applications to orbifold theories

- Consider a stack of N $M5$-branes wrapped on a Riemann surface Σ_g and probing a $\mathbb{C}^2/\mathbb{Z}_k$ singularity
- The linking space $M_4 = S^4/\mathbb{Z}_k$, there are two $\mathbb{Z}_k$ orbifold fixed points at the poles
- Space that define the QFT is $M_6 = M_4 \times \Sigma_g$ with a topological twist to preserve SUSY

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Symmetry of system

One-cycles:

$[\lambda^u, \tilde{\lambda}^u]$ on the Σ_g, $b^1(M_6) = 2g$

- $U(1)^{2g}$ 1-form gauge symmetry with $Sp(2g, \mathbb{Z})$ $\mathcal{S}$-duality group
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Three-cycles:

$(\lambda^u \times \mathcal{C}_2^i, \tilde{\lambda}^u \times \mathcal{C}_2^i)$ on the Σ_g, $b^3(M_6) = 4g(k-1)$

- $4g(k-1)$ bulk axions, Boundary value of axions correspond to marginal coupling parameters
- Anomaly involving the axions correspond to anomalies in the space of couplings

[Córdova, Freed, Lam, Seiberg, '19]

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- Conformal blocks relating to Singleton physics and anomalies relating to $Sp(2g, \mathbb{Z})$ duality group (Similar to [Belov, Moore '04])
- Since the analysis relies less on SUSY, we hope to be able to study more general classes of compactifications with punctures and defects

Summary

- Topological mass terms in 5d supergravity encode discrete global symmetries of the dual field theory
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Summary

- Topological mass terms in 5d supergravity encode discrete global symmetries of the dual field theory
- The same bulk theory with different topological boundary conditions gives field theories with different discrete global symmetries
- We can capture 't Hooft anomalies with a 6-form inflow anomaly polynomial
- There is a rich interplay between all p-forms fields from expansion of M-theory C_3 potential
  - Higher-form symmetries
  - Discrete symmetries
  - Anomalies in the space of coupling constants, or "(–1)-form" symmetries

THANK YOU!

IIB and D3 branes [IB, Bonetti, Minasian, Weck '20]

- One can also consider brane systems in type II string theories
- The polynomials that encode the anomalies are 11-forms, $\mathcal{I}_{11}$ constructed from gauge invariant boundary conditions of various flux
- The anomaly polynomial of IIA is related to the M-theory $\mathcal{I}_{12}$ by a reduction, It is similarly characterized by IIA Chern-Simons terms
- The anomaly polynomial for IIB receives a contribution from the kinetic term of the self-dual five-form flux
- If we consider a stack of D3-branes supported by the five-form flux, F_5

$$F_5 = 2\pi(1+\star)\rho(r)\bar{F}_5 + \cdots \quad \text{on} \quad M_{10} = \mathbb{R}^+ \times W_d \times M_{9-2d}$$

  The boundary term $\bar{F}_5$ on M_{9-2d} can be extended to E_5 on $W_d \times M_{9-2d}$
- The 11-form and the inflow anomaly polynomial are given as

$$\mathcal{I}_{11} = \frac{1}{2} E_5 \wedge dE_5 - E_5 \wedge H_3 \wedge F_3, \qquad I_{2d+2}^{\text{inf}} = \int_{M_{9-2d}} \mathcal{I}_{11}$$

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- For $\mathcal{N} = 4$ SYM, E_5 is the global angular form of the 5-sphere, e_5! Integrating $\mathcal{I}_{11}$ yields the anomaly for the $SO(6)$ R-symmetry group

$$E_5 = N\, e_5, \qquad dE_5 = -N\, \pi^* \chi\,(SO(6))\,,$$
$$I^{inf}_6 = \frac{1}{2} N^2 \chi\,(SO(6)) = \frac{1}{2} N^2 c_3\,(SU(4))$$

- For more general $\mathcal{N} = 1$, E_5 is the volume of SE_5 gauged over the world volume theory! Consistent with holographic analysis by [Benvenuti, Pando Zayas, Tachikawa 06]
- Anomaly of $\mathcal{N} = 4$ SYM on punctured Riemann surface
- This anomaly formula can be used to study compactifications of 4D SCFTs to 2D QFTs

F-theory [IB, Bonetti, Minasian, Weck '20]

- Generalize type IIB with non-trivial axio-dilaton profile
- Consider an elliptic fibration over the IIB background

$$\mathbb{E}_\tau \hookrightarrow M_{12} \to M_{10}$$

- The anomaly polynomial is

$$\mathcal{I}_{11} = \frac{1}{2} E_5 \wedge dE_5 - E_5 \wedge \pi_* \left[X_8(TM_{12}) + \frac{1}{2} \mathcal{E}_4 \wedge \mathcal{E}_4 \right]$$

- F_3 and H_3 are encoded in $\mathcal{E}_4$, for trivial elliptic fiber

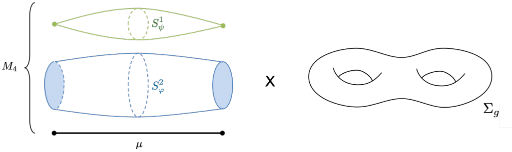
$$\mathcal{E}_4 = F_3 \wedge dx + H_3 \wedge dy$$

- Anomalies of $\mathcal{N} = 4$ with varying coupling, τ_{YM}, can be studied with this generalization [Lawrie, Martelli, Schäfer-Nameki '18]

Things to do

- Compute the anomalies for $\mathcal{N} = 2$ Class $\mathcal{S}$ of A_N type with arbitrary punctures [IB, Nardoni, '18; IB, Bonetti, Minasian, Nardoni '19]
- The possible choices of E_4 from $M_6 = S^4 \times \Sigma_{g,n}$ is in one-to-one correspondence with the classification from Hitchin equations
- Choices come from different resolutions of punctures on $\Sigma_{g,n}$ in M_6
- This provides an alternate derivation of punctures and the data associated with them from bulk SUGRA
- Explore punctures for $\mathcal{N} = 1$ Class $\mathcal{S}$ [IB, Beem, Bobev, Wecht '12] and from Class $\mathcal{S}_k$ [Gaiotto, Razamat, '15; Hanany, Maruyoshi '15 and $\mathcal{S}_\Gamma$ [Heckmann, Jefferson, Rudelius, Vafa, '16]
- Study Class $\mathcal{S}$ from the D-series (Inflow for 6D SCFT from [Yi, '00]) and E-string theories
- Example – Class $\mathcal{S}_2$

A look at class $\mathcal{S}_2$ [IB, Bonetti '19]



- Consider a stack of N M5-branes on Σ_g and probing a $\mathbb{Z}_2$ orbifold fixed point
- Here $M_6 = M_4 \times \Sigma_g$ and M_4 is $S^4/\mathbb{Z}_2$ with resolution two cycles
- The resolution is supported by threading flux (N^N, N^S) on 4-cycles made from the resolution 2-cycles combined with the Riemann surface
- There are a total of three 4-cycles with three flux parameters (N, N^N, N^S), Associated to them are three closed 2-forms by Poincare duality
- The isometry group is $U(1)_R \times SU(2)_F$ and the naive symmetry from C_3 is $U(1)^3$
- From the 6d $(1,0)$ theory, only $U(1)_N \times U(1)_S$ is visible, the third $U(1)_C$ is an accidental symmetry from the compactification!

- A combination of the three U(1)s is broken by a topological mass – Spontaneous symmetry break of a U(1) global symmetry for the field theory
- The symmetry of low-energy theory is then $U(1)'_N \times U(1)'_S \times U(1)_R \times SU(2)_L$
- The generators of the 2 $U(1)$s visible from the 6d SCFT are shifted as

$$T'_N = T_N - \frac{N^N}{N} T_C, \qquad T'_S = T_S - \frac{N^S}{N} T_C$$

- After obtaining anomaly polynomial, compute large N central charge by a-maximization [Intriligator, Wecht '03]
- Inflow data can be matched with a family of $AdS_5 \times \mathcal{M}_6$ obtained in [Gauntlett, Martelli, Sparks, Waldram '04]

- 5d SUGRA theory admits a rich discrete gauge symmetry! Thus complex network of discrete symmetry in SCFT which is acted upon by $Sp(2g, \mathbb{Z})$

multiplicity	fields	top. mass terms	bulk gauge symm.
$b^2(M_6) = 3$	a_1^a	$\frac{1}{2\pi} N_a\, a_1^a \wedge dc_3$	$U(1)^2$ 0-form symm. $\mathbb{Z}_k$ 0-form symm.
1	c_3		$\mathbb{Z}_k$ 2-form symm.
$b^1(M_6) = 2g$	$b_2^i,\ \tilde{b}_2^i$	$\frac{1}{2\pi} M\, \tilde{b}_2^i \wedge db_2^i$	$(\mathbb{Z}_M \times \mathbb{Z}_M)^g$ 1-form symm.
$b^3(M_6) = 4g$	$a_0^{i\pm},\ \tilde{a}_0^{i\pm}$	—	5D axions

- There are $4g$ background 1-forms in the anomaly polynomial associated to the axions – Anomaly for background dependent couplings and "(-1)-form symmetry"? [Córdova, Freed, Lam, Seiberg, '19]

Outlook

- Origin of decoupled modes from M_{10-d}

$$I^{inf} + I^{QFT} + I^{decoupled} = 0$$

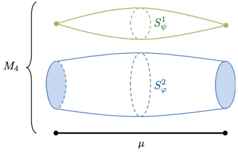
- Discrete symmetries and higher form symmetries – role of torsion in Cohomology group
- Anomalies related to large gauge transformations and duality groups of QFTs – Global anomalies
- Defects and extended operators – higher form discrete symmetry
- Explore general compactifications of 6D theories in IIB/F-theory (Inflow polynomial in [IB, Bonetti, Minasian, Weck '20]), massive IIA
- Since the analysis relies less on SUSY, we hope to be able to study more general classes of compactifications with punctures and defects

THANK YOU!

M5-branes probing $\mathbb{C}^2/\mathbb{Z}_2$ fixed point

- When the stack of M5-branes is probing a $\mathbb{C}^2/\mathbb{Z}_k$ fixed point, $M_4 \cong S^4/\mathbb{Z}_k$
- $\mathbb{Z}_k \subset SU(2)_L$ from $SU(2)_L \times SU(2_R) \subset SO(5)$ of the isometry group
- When $k = 2$, the orbifold action preserves the $SU(2)_L \times SU(2)_R$ subgroup
- On the branes, $SU(2)_L$ is a flavor symmetry and $SU(2)_R$ is an R-symmetry for the worldvolume $(1, 0)$ SCFT
- There are two $\mathbb{R}^4/\mathbb{Z}_2$ fixed points on the sphere at the north and south poles
- The fluctuations of the C_3 potential leads to an additional $SU(2)_N \times SU(2)_S$ flavor symmetry for the worldvolume theory
- For the purpose of the SUGRA analysis, we consider a resolution of the orbifold fixed points by blowing up two-cycles at the poles of the sphere
- Symmetry breaks:
$SU(2)_N \times SU(2)_S \times SU(2)_R \times SU(2)_L \to U(1)_N \times U(1)_S \times U(1)_R \times SU(2)_L$

M_4 Space



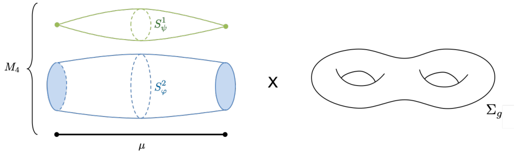
- The space M_4 is a circle fibration, S^1_ψ, over a cylinder $[\mu] \times S^2_\varphi$. The isometries of $S^1_\psi \times S^2_\varphi$ correspond to $U(1)_R \times SU_L(2)$
- $S^1_\psi \times S^2_\varphi$ have a topology of $S^3/\mathbb{Z}_2$
- The circle S^1_ψ shrinks at the end points of the μ-interval while the two sphere S^2_φ never shrinks
- The non-shrinking sphere at the end of the μ-interval correspond to the blowup two-cycles of the orbifold fixed points

Adding a Riemann Surface, Σ_g

- Now we consider the case when the branes wrap a Riemann surface Σ_g while probing the singularity
- This is equivalent to taking the 6D $(1,0)$ theory on a Riemann surface with a topological twist to preserve supersymmetry
- By anomaly matching, the anomaly of the 4D theories can be computed as

$$I_6 = \int_{\Sigma_g} I_8$$

- Anomaly polynomial does not yield correct central charge for "potential" dual holographic solution
- Possible accidental symmetry and interesting decoupled modes!



- In this case, $M_6 = M_4 \times \Sigma_g$. the R-symmetry circle, S^1_ψ, is twisted over the Riemann Surface with curvature $2(g-1)$
- M_6 has three 4-cycles, two of them correspond to taking the product of the polar two-cycles of M_4 with Σ_g. The third is the embedding of M_4 in M_6
- Threading flux on these cycles yields three quantum number (N, N^N, N^S)
- there are three closed 2-forms dual to the 4-cycles. The vector fluctuations of C_3 along these forms implies three $U(1)$ gauge fields in the bulk supergravity
- This suggests a $U(1)^3$ flavor symmetry for the 4d theory
- Compactification of the 6D $(1,0)$ theory only sees $U(1)_N \times U(1)_S$; the third $U(1)_C$ is an accidental symmetry!

Counting symmetries again

- In the reduction of M-theory on M_6, a combination of the vectors from C_3 acquires a topological mass term from M-theory CS term

$$S_{5d} \supset N^\alpha \int \gamma_3 \wedge da_\alpha, \qquad C_3 \supset a_\alpha \wedge \omega^\alpha + \gamma_3$$

- This topological mass term can dualized to Stückelberg kinetic term with $N^\alpha a_\alpha$ eating the axion dual to γ_3
- In the bulk supergravity this is spontaneous breaking of a $U(1)$ gauge symmetry and on the boundary, it corresponds to spontaneous breaking of a $U(1)$ global symmetry!
- The symmetry of low-energy theory is then $U(1)'_N \times U(1)'_S \times U(1)_R \times SU(2)_L$
- The generators are shifted as

$$T'_N = T_N - \frac{N^N}{N} T_C, \qquad T'_S = T_S - \frac{N^S}{N} T_C$$

E_4 for Class $\mathcal{S}_2$ theories

- We write the 4-form as

$$E_4 = N\,(\mathcal{V}_0^g + \cdots) + N^N\,(\mathcal{V}_N^g + \cdots) + N^S\,(\mathcal{V}_S^g + \cdots)$$
$$+ F^0(\omega_0^g + \cdots) + F_{4d}^N(\omega_N^g + \cdots) + F_{4d}^S(\omega_S^g + \cdots)$$

- The field strength for the vector fluctuations of C_3 are $(F^0, F_{4d}^N, F_{4d}^S)$, one of them is removed by the constraint

$$NF^0 + N^N F_{4d}^N + N^S F_{4d}^S = 0$$

This constraint also follows from the tadpole condition

- The 4d curvatures are related to the 6d curvatures as

$$F^N = N^N V_\Sigma + F_{4d}^N, \qquad F^S = N^S V_\Sigma + F_{4d}^S$$

The flux (N^N, N^S) are background flux for the 6D flavor symmetry on the Riemann surface

$$I_{6,\text{ large } N}^{\text{infl}} = \frac{1}{(2\pi)^3}\Big[\tfrac{1}{2}\,N\,(\chi\,N - N^N + N^S)\,F_R^2\,(F_{\rm N} + F_{\rm S}) - \tfrac{1}{2}\,(N^N - N^S)\,F_R\,(F_{\rm N} + F_{\rm S})^2$$
$$+ N^{-1}\,(N^N\,F_{\rm N} + N^S\,F_{\rm S})\,(F_{\rm N}^2 - F_{\rm S}^2) - \tfrac{2}{3}\,\chi\,(F_{\rm N}^3 + F_{\rm S}^3)\Big]$$

Holographic dual

- To check for the existence of a SCFT fixed point, we look for an *AdS* solution of the form

$$ds^2 = e^{2\lambda}\left[ds^2(AdS_5) + e^{-6\lambda} ds^2(\widetilde{M}_6)\right]$$

- The solutions were already found by Gauntlett, Martelli, Sparks and Waldram in 2004!
- By construction, symmetries and topology match
- From our anomaly computation we can match the large N central charge with a-maximization!
- Class $\mathcal{S}_2$ with a torus is dual to the $AdS_5 \times Y^{p,q}$ solutions in IIB supergravity