Nonabelian $(2,0)$ Tensor Multiplets and 3-algebras

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(with Neil Lambert, arXiv:1007.2982)

Motivation

Over the last two years there has been significant amount of work towards actions for multiple M2-branes.

Progress relied on the introduction of a novel algebraic structure: a 3-algebra. [Bagger-Lambert, Gustavsson]

This is defined through

$$[T^A, T^B, T^C] = f^{ABC}{}_D T^D$$

and satisfies the 'fundamental identity'

$$f^{[ABC}{}_E f^{D]EF}{}_G = 0 \ .$$

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- maximal supersymmetry ($\mathcal{N} = 8$)
- conformal invariance
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But: $\exists$ only one real Euclidean 3-algebra (looks like $\mathrm{SU}(2) \times \mathrm{SU}(2)$ CS-matter), moduli space looks like exotic M-theory orbifold, Lorentzian 3-algebras have negative norm states or look too much like 3d SYM...

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$\Rightarrow$ To date no known string theory interpretation of BLG

These ideas solidified in the ABJM proposal for bifundamental $\mathrm{U}(N) \times \mathrm{U}(N)$ Chern-Simons-matter theory with $\mathcal{N} = 6$, describing N M2-branes on a $\mathbb{C}^4/\mathbb{Z}_k$ M-theory singularity. [Aharony-Bergman-Jafferis-Maldacena]

Important developments in $\mathrm{AdS}_4/\mathrm{CFT}_3$...

3-algebra description not necessary but possible. This is a complex 3-algebra [Bagger-Lambert, Schnabl-Tachikawa]

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But what about the M5-brane??

Low-energy M5-brane dynamics governed by a theory in 6d with: [Strominger, Witten]

- $(2,0)$ supersymmetry
- conformal invariance
- $\mathrm{SO}(5)$ R-symmetry

The $(2,0)$ tensor multiplet contains 5 scalars and a selfdual antisymmetric 3-form field strength + fermions

But it's complicated: cannot even get Lagrangian for single M5 because of selfdual three-form field strength.

Note: Many indirect ways of attacking the abelian problem

- ◇ Sacrificing manifest 6d Lorentz invariance
- ◇ Introducing auxiliary scalar field
- ◇ Not requiring selfduality at Lagrangian but directly at path integral

[Aganagic-Park-Popescu-Schwarz, Pasti-Sorokin-Tonin, Bandos et al., Cederwall-Nilsson-Sundell]

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⇒ Attempt a similar approach to multiple M2-branes for multiple M5-branes.

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- This 6d theory involves 3-algebras
- No manifest evidence of multiple M5-branes
- Theory has some interesting features
- A sector of the theory could be related to lightcone description of M5-branes

Outline

- Set-up of the calculation
- Susy closure
- Spacelike reduction
- Null reduction
- 5d SYM $\Leftrightarrow$ (2,0)

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- Investigate the closure of the susy algebra
- Obtain e.o.m. and constraints
- Interpret the result

The susy transformations for the free 6d $(2,0)$ tensor multiplet are

$$\begin{aligned} \delta X^I &= i\bar{\epsilon}\Gamma^I\Psi \\ \delta\Psi &= \Gamma^\mu\Gamma^I\partial_\mu X^I\epsilon + \frac{1}{3!}\frac{1}{2}\Gamma^{\mu\nu\lambda}H_{\mu\nu\lambda}\epsilon \\ \delta H_{\mu\nu\lambda} &= 3i\bar{\epsilon}\Gamma_{[\mu\nu}\partial_{\lambda]}\Psi \end{aligned}$$

with

$$\Gamma_{012345}\epsilon = \epsilon \qquad \text{and} \qquad \Gamma_{012345}\Psi = -\Psi$$

This algebra closes on-shell up to translations, with e.o.m.

$$\partial_\mu\partial^\mu X^I = \Gamma^\mu\partial_\mu\Psi = \partial_{[\mu}H_{\nu\lambda\rho]} = 0$$

Make this ‘nonabelian’: Assume fields take values in some vector space with basis T^A such that $X^I = X^I_A T^A$.

Promote the derivatives to covariant derivatives

$$D_\mu X^I_A = \partial_\mu X^I_A - \tilde{A}_\mu{}^B{}_A X^I_B$$

with $\tilde{A}_\mu{}^B{}_A$ a new gauge field. $\exists$ an associated gauge symmetry.

Propose a nonabelian ansatz analogous to that of the M2-brane.

Consider:

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\delta X^I_A &= i\bar{\epsilon}\Gamma^I\Psi_A \\
\delta\Psi_A &= \Gamma^\mu\Gamma^I D_\mu X^I_A\epsilon + \frac{1}{3!}\frac{1}{2}\Gamma_{\mu\nu\lambda}H^{\mu\nu\lambda}_A\epsilon \\
&\qquad\qquad -\frac{1}{2}\Gamma_\lambda\Gamma^{IJ}C^\lambda_B X^I_C X^J_D f^{CDB}{}_A\epsilon \\
\delta H_{\mu\nu\lambda\ A} &= 3i\bar{\epsilon}\Gamma_{[\mu\nu}D_{\lambda]}\Psi_A + i\bar{\epsilon}\Gamma^I\Gamma_{\mu\nu\lambda\kappa}C^\kappa_B X^I_C\Psi_D g^{CDB}{}_A \\
\delta\tilde{A}_\mu{}^B{}_A &= i\bar{\epsilon}\Gamma_{\mu\lambda}C^\lambda_C\Psi_D h^{CDB}{}_A \\
\delta C^\mu_A &= 0
\end{aligned}
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Here $f^{CDB}{}_A$, $g^{CDB}{}_A$ and $h^{CDB}{}_A$ are some objects with properties to be determined.

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Consistency of these transformations with respect to their scaling dimensions gives

$$[H] = [X] + 1\ , \qquad [\tilde{A}] = 1\ , \qquad [C] = 1 - [X]$$
$$[\epsilon] = -\tfrac{1}{2}\ , \qquad [\Psi] = [X] + \tfrac{1}{2}\ , \qquad [X]$$

The assignments are all related to the choice of $[X]$. For the canonical choice $[X] = 2$ we have that $[C] = -1$.

Susy closure

We find that the susy algebra closes on-shell up to a translation and a gauge transformation, subject to the constraints:

$$g^{ABC}{}_D = h^{ABC}{}_D = f^{ABC}{}_D = f^{[ABC]}{}_D$$

and

$$f^{[ABC}{}_E f^{D]EF}{}_G = 0$$

This is the fundamental identity for real 3-algebras (the $\mathcal{N} = 8$ 3-algebras in 3d theories).

E.o.m. for X^I_A:

$$D^2 X^I = \frac{i}{2}\bar{\Psi}_C C^\nu_B \Gamma_\nu \Gamma^I \Psi_D f^{CDB}{}_A + C^\nu_B C_{\nu G} X^J_C X^J_E X^I_F f^{EFG}{}_D f^{CDB}{}_A$$

E.o.m. for Ψ_A:

$$\Gamma^\mu D_\mu \Psi_A + X^I_C C^\nu_B \Gamma_\nu \Gamma^I \Psi_D f^{CDB}{}_A = 0$$

E.o.m. for $H_{\mu\nu\lambda\ A}$:

$$D_{[\mu} H_{\nu\lambda\rho]\ A} = -\frac{1}{4}\epsilon_{\mu\nu\lambda\rho\sigma\tau} C^\sigma_B f^{CDB}{}_A \Big(X^I_C D^\tau X^I_D + \frac{i}{2}\bar{\Psi}_C \Gamma^\tau \Psi_D \Big)$$

E.o.m for $\tilde{A}_{\mu}^{A}{}_{B}$:

$$\tilde{F}_{\mu\nu}{}^{B}{}_{A} = C^{\lambda}_{C} H_{\mu\nu\lambda\, D} f^{BDC}{}_{A}$$

$\Rightarrow$ No new d.o.f. are introduced on-shell

Constraints on C^{μ}_{A}:

$$D_{\nu} C^{\mu}_{A} = 0\ , \qquad C^{\lambda}_{B} C^{\rho}_{C} f^{CDB}{}_{A} = 0$$

and

$$\Rightarrow C^{\rho}_{C} D_{\rho} \Big\{ X^{I}_{D}, \Psi_{D}, H_{\mu\nu\lambda\, D} \Big\} f^{CDB}{}_{A} = 0 \Leftarrow$$

Summary thus far

- Wrote ansatz for susy xfms of nonabelian (2,0) theory in 6d
- This involved a new nondynamical gauge field $\tilde{A}_{\mu}{}^{A}{}_{B}$
- The gauge symmetry was associated to a 3-algebra
- Also introduced an auxiliary vector field C_A^{μ}
- Obtained e.o.m and constraints that define the theory
- Proceed to study the interpretation

3-algebra 101

The structure constants are those of a real 3-algebra. Endow it with a metric

$$h^{AB} = \mathrm{Tr}(T^A, T^B)$$

$\exists$ two kinds of real 3-algebras (depending on signature):

- The Euclidean $\mathcal{A}_4$-algebra, with $f^{ABCD} = \epsilon^{ABCD}$ [Papadopoulos, Gauntlett-Gutowski]
- The Lorentzian algebras [Gomis-Milanesi-Russo, Benvenuti-Rodríguez-Gómez-Tonni-Verlinde, Ho-Imamura-Matsuo]

Lorentzian 3-algebras: start with ordinary Lie algebra $\mathcal{G}$ and add two lightlike generators $T^{\pm}$ such that $A = +, -, a, b, ...$. The structure constants are given by

$$f^{ABC}{}_D \to \quad f^{+ab}{}_c = f^{ab}{}_c \;,\; f^{abc}{}_- = f^{abc} \;,$$

The metric is given by

$$h_{AB} = \left(\begin{array}{cc|ccc} 0 & -1 & 0 & \dots & 0 \\ -1 & 0 & 0 & \dots & 0 \\ \hline 0 & 0 & & & \\ \vdots & \vdots & & h_{\mathcal{G}} & \\ 0 & 0 & & & \end{array}\right) \;.$$

Spacelike reduction

Use the Lorentzian 3-algebra and look for vacua of the theory when $\mathcal{G} = \mathfrak{su}(N)$:

$$X_A^I \to X_a^I, X_\pm^I$$

Get two abelian $(2,0)$ tensor multiplets $(X_\pm^I, \Psi_\pm, H_{\mu\nu\lambda\,\pm})$

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Next, look at nonabelian piece:

$\Rightarrow$ Expand around $\langle C_A^\lambda \rangle = g\delta_5^\lambda \delta_A^+$

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  $\Rightarrow$ Nonabelian physics is five-dimensional

- ◇ g is constant and has scaling dimension -1

  $\Rightarrow$ $g^{\frac{1}{2}}$ has correct scaling dimension for g_{YM} in 5d.

Make identifications:

$$g = g_{YM}^2 \ , \qquad H^a_{\alpha\beta 5} = \frac{1}{g_{YM}^2} F^a_{\alpha\beta}$$

...and recover e.o.m., Bianchi identity and susy xfms of five-dimensional $\mathrm{SU}(N)$ SYM theory.

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The off-shell $\mathrm{SO}(5,1)$ Lorentz and conformal symmetries are spontaneously broken to $\mathrm{SO}(4,1)$ Lorentz invariance.

Very similar to what happened for Lorentzian M2-brane theories in relation to D2-branes. [Gomis-Rodríguez-Gómez-Van Raamsdonk-Verlinde, Ezhuthachan-Mukhi-CP]

In that case, the Lorentzian BLG theory in 3d expanded around generic vacua was shown to be equivalent to 3d SYM. The off-shell $\mathrm{SO}(8)$ R-symmetry and conformal invariance are spontaneously broken to $\mathrm{SO}(7)$ R-symmetry.

Aside: Try and introduce a field $B_{\mu\nu\ A}$ such that $H_{\mu\nu\lambda\ A} = 3D_{[\mu}B_{\nu\lambda]\ A}$.

This would lead to the algebraic relation

$$\tilde{F}_{[\mu\nu}{}^{B}{}_{A}B_{\lambda\rho]\ B} = -\frac{1}{6}\epsilon_{\mu\nu\lambda\rho\sigma\tau}C^{\sigma}_{B}(X^{I}_{C}D^{\tau}X^{I}_{D} + \frac{i}{2}\bar{\Psi}_{C}\Gamma^{\tau}\Psi_{D})f^{CDB}{}_{A}$$

This seems to be over-constraining the fields....

Look at this in the context of the reduced theory.

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For the nonabelian fields we have

$$\tilde{F}_{\mu\nu}{}^{B}{}_{A} = C^{\lambda}_{C} H_{\mu\nu\lambda\;D} f^{CDB}{}_{A} \quad \Longrightarrow \quad \tilde{F}_{\alpha\beta}{}^{b}{}_{a} = 2g^{2}_{YM}\tilde{D}_{[\alpha}B_{\beta]5\;c}f^{cb}{}_{a}$$

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and compare with

$$\tilde{F}_{\alpha\beta}{}^{a}{}_{b} = \partial_{\beta}\tilde{A}_{\alpha}{}^{a}{}_{b} - \partial_{\alpha}\tilde{A}_{\beta}{}^{a}{}_{b} - \tilde{A}_{\alpha}{}^{a}{}_{c}\tilde{A}_{\beta}{}^{c}{}_{b} + \tilde{A}_{\beta}{}^{a}{}_{c}\tilde{A}_{\alpha}{}^{c}{}_{b}$$

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and

$$\tilde{D}_{\alpha}B_{\beta 5\,a} = \partial_{\alpha}B_{\beta 5\,a} - \tilde{A}_{\alpha}{}^{b}{}_{a}B_{\beta 5\,b}$$

$\Rightarrow$ It doesn't work and one can't have $H_{\mu\nu\lambda\,A} = 3D_{[\mu}B_{\nu\lambda]\,A}$

Euclidean case

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What about the Euclidean 3-algebra $\mathcal{A}_4$? This was the example that was genuinely different to SYM in 3d and led to ABJM.

$\Rightarrow$ In 6d, Lorentzian and Euclidean cases not dramatically different:

Set $f^{ABCD} = \epsilon^{ABCD} \rightarrow \epsilon^{abc4} \equiv \epsilon^{abc} \in \mathfrak{su}(2)$ and expand theory around $\langle C^\lambda_A \rangle = g\delta^\lambda_5 \delta^4_A$

$$X^I_A \rightarrow X^I_a, X^I_4$$

Get a single free $(2,0)$ tensor multiplet plus $\mathrm{SU}(2)$ 5d SYM

Null Reduction

One could also consider 6d coordinates $x^\mu = (u, v, x^i)$ where $u = \frac{1}{\sqrt{2}}(x^0 - x^5)$, $v = \frac{1}{\sqrt{2}}(x^0 + x^5)$ and $i = 1, 2, 3, 4$.

Expand around

$$\langle C^\mu_A \rangle = g\delta^\mu_v \delta^+_A$$

$\Rightarrow$ Abelian sector again consists of two 6-dimensional $(2, 0)$ tensor multiplets.

$\Rightarrow$ Nonabelian sector is a susy system in effectively 4 space and 1 null dimensions with 16 susies and $\mathrm{SO}(5)$ R-symmetry.

We now have

$$D^2 X^I_A = \frac{i}{2}\bar{\Psi}_C C^\nu_B \Gamma_\nu \Gamma^I \Psi_D f^{CDB}{}_A + C^\nu_B C_{\nu G} X^J_C X^J_E X^I_F f^{EFG}{}_D f^{CDB}{}_A$$

$$\Longrightarrow \qquad D^2 X^I_a = \frac{ig}{2}\bar{\Psi}_c \Gamma_v \Gamma^I \Psi_d f^{cd}{}_a$$

and

$$C^\rho_C D_\rho X^I_D f^{CDB}{}_A = 0 \qquad \Longrightarrow \qquad \partial_v X^I_a = 0$$

⇒ Note that term proportional to scalar potential absent in scalar e.o.m.

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⇒ Tempting to speculate that this may be related to a lightcone formulation for M5-branes

BPS solutions in null reduction

⇒ Abelian solutions: Right-moving ($\partial_v X^I_a = 0$) modes of selfdual strings and their 'neutral string' generalisations [Howe-Lambert-West, Gauntlett-Lambert-West]

Selfdual strings: are $\frac{1}{2}$-BPS solutions describing the M2⊥M5 intersection with

$$H_{uvi} = \partial_i X^6 \ , \qquad \partial^i \partial_i X^6 = 0$$

Neutral strings: are instanton-like configurations on the relative transverse M5-brane directions. They have zero H-charge

$$H_{uij} = \frac{1}{2}\epsilon_{ijkl} H_{ukl}$$

$\Rightarrow$ Nonabelian solutions: We obtain $\frac{1}{4}$-BPS solutions

$$H_{uvi\ a} = D_i X_a^6\ , \qquad H_{uij\ a} = \frac{1}{2}\epsilon_{ijkl} H_{ukl\ a} \qquad D^i D_i X_a^6 = 0$$

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⇒ Right-movers ($\partial_v X^I_a = 0$) of lightlike 'dyonic instanton' strings describing 'W-boson' M2's stretched between multiple M5's in the maximally Higgsed phase...

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In that sense the strong-coupling dynamics of D4-theory should be encoded in M5-theory.

From the gauge theory point of view this looks far from trivial:

- 5d SYM has a UV fixed-point which should correspond to the (2,0) theory
- It is naïvely non-renormalisable and as such new d.o.f. should appear at some scale

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$\Rightarrow$ Technical hurdle in making this precise: instanton zero mode quantisation leads to continuous spectrum...

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Would be interesting to see whether our formulation could shed some light in any of these directions.

Summary

- Starting from abelian M5-brane susy transformations, we constructed a nonabelian $(2,0)$ tensor multiplet
- We recovered the presence of 3-algebras in this 6d theory
- Around $\langle C^\mu_A \rangle = g\delta^\mu_5 \delta^+_A$ physics were 5d SYM plus free 6d abelian $(2,0)$ tensor multiplets
- Around $\langle C^\mu_A \rangle = g\delta^\mu_v \delta^+_A$ physics were 4 space, 1 null direction susy system plus 6d abelian $(2,0)$ tensor multiplets

- We found BPS solutions corresponding to the right-moving sector of lightlike ‘dyonic instanton’ strings and having interpretation as M2-branes suspended between parallel M5-branes
- Although the M-theory interpretation of our $(2, 0)$ tensor multiplet is unclear, interesting to see these solutions arise
- Due to its potential connection with multiple M5-branes and UV finiteness of 5d SYM, this system warrants further investigation