Searching for light dark matter with magnetic materials

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Fermilab

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Outline

Introduction to light dark matter

Axion induced currents: ϵ and μ engineering

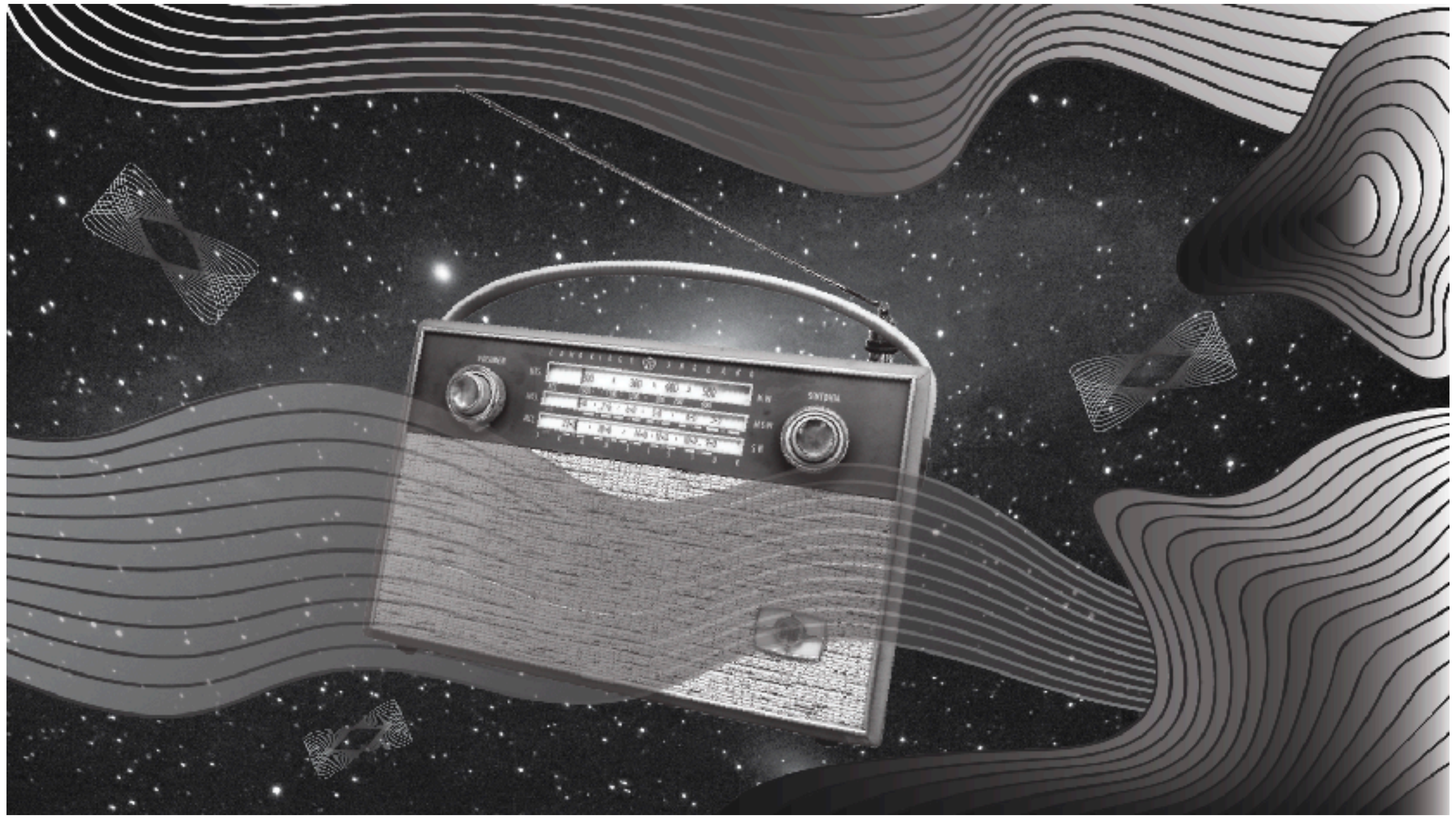
Spin dependent scattering: μ characterization

Roadmap to the future

Dark Matter Candidates

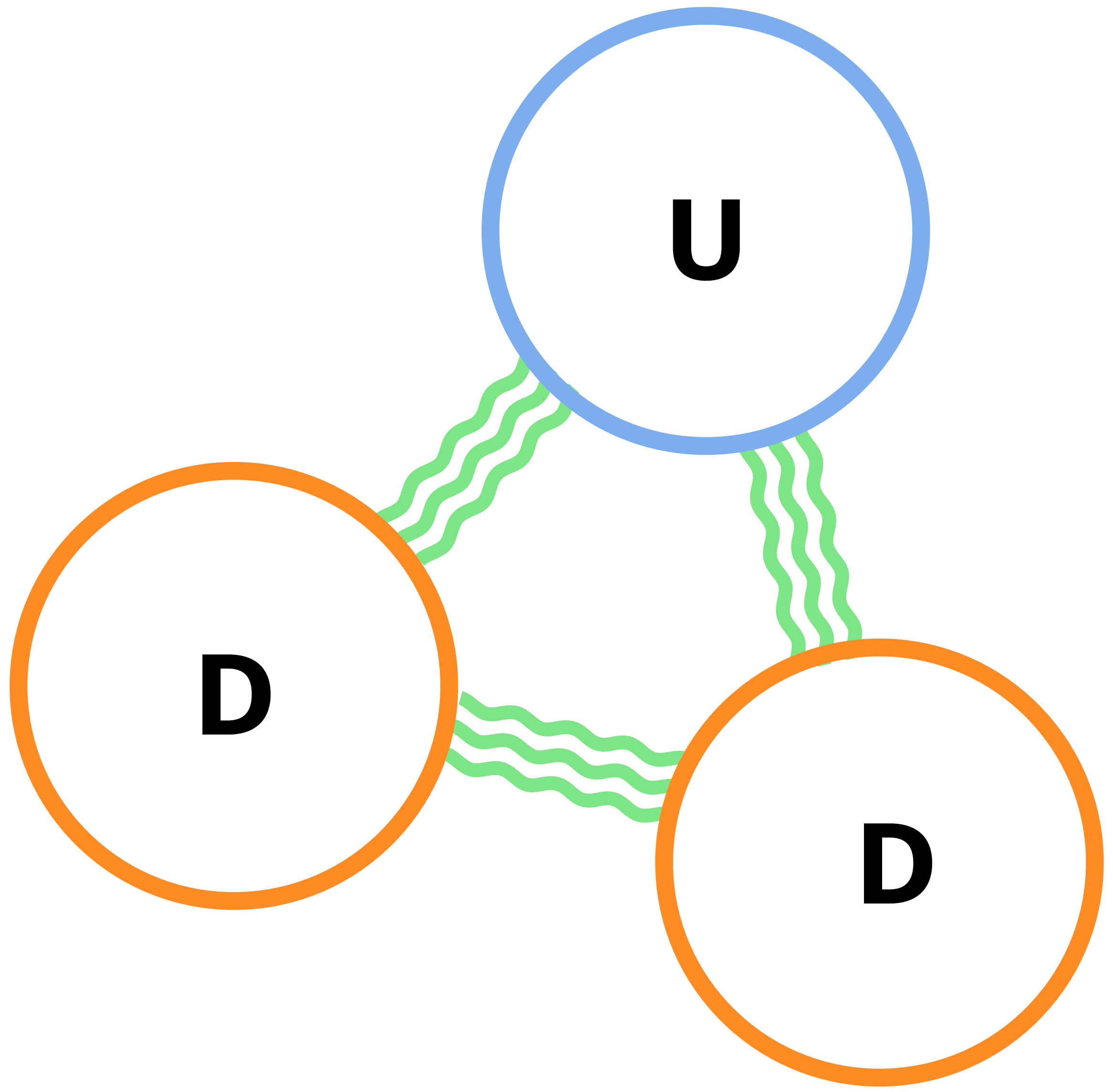
Wavelike Dark Matter

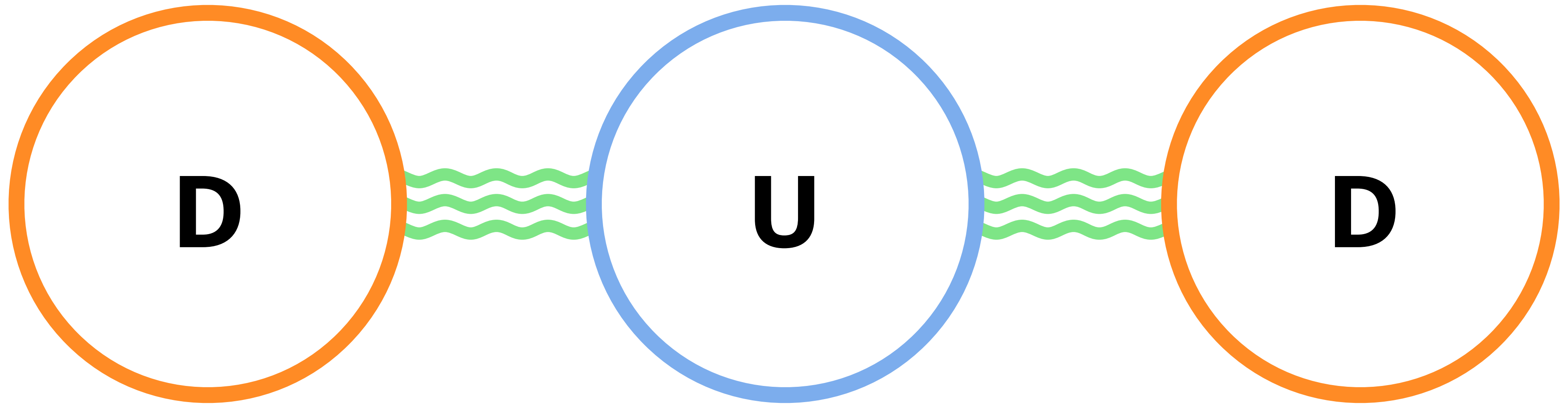
- Much lighter than WIMPs: ~μeV (GHz)
- Looking for dark matter is like tuning a radio to find the right station (dark matter mass)
- $\omega = m_a + \frac{1}{2} m_a v^2$
- Axions, ALPs, dark photons, scalars...



The Strong CP Problem

- The Strong force should have a CP violating term $\mathscr{L} \supset \bar{\theta}\frac{g^2}{32\pi^2}G\tilde{G}$
- In principle can be $\bar{\theta} \in [0,2\pi]$, initial condition from vacuum topology
- Limit from neutron EDM is $\bar{\theta} \lesssim 10^{-10}$



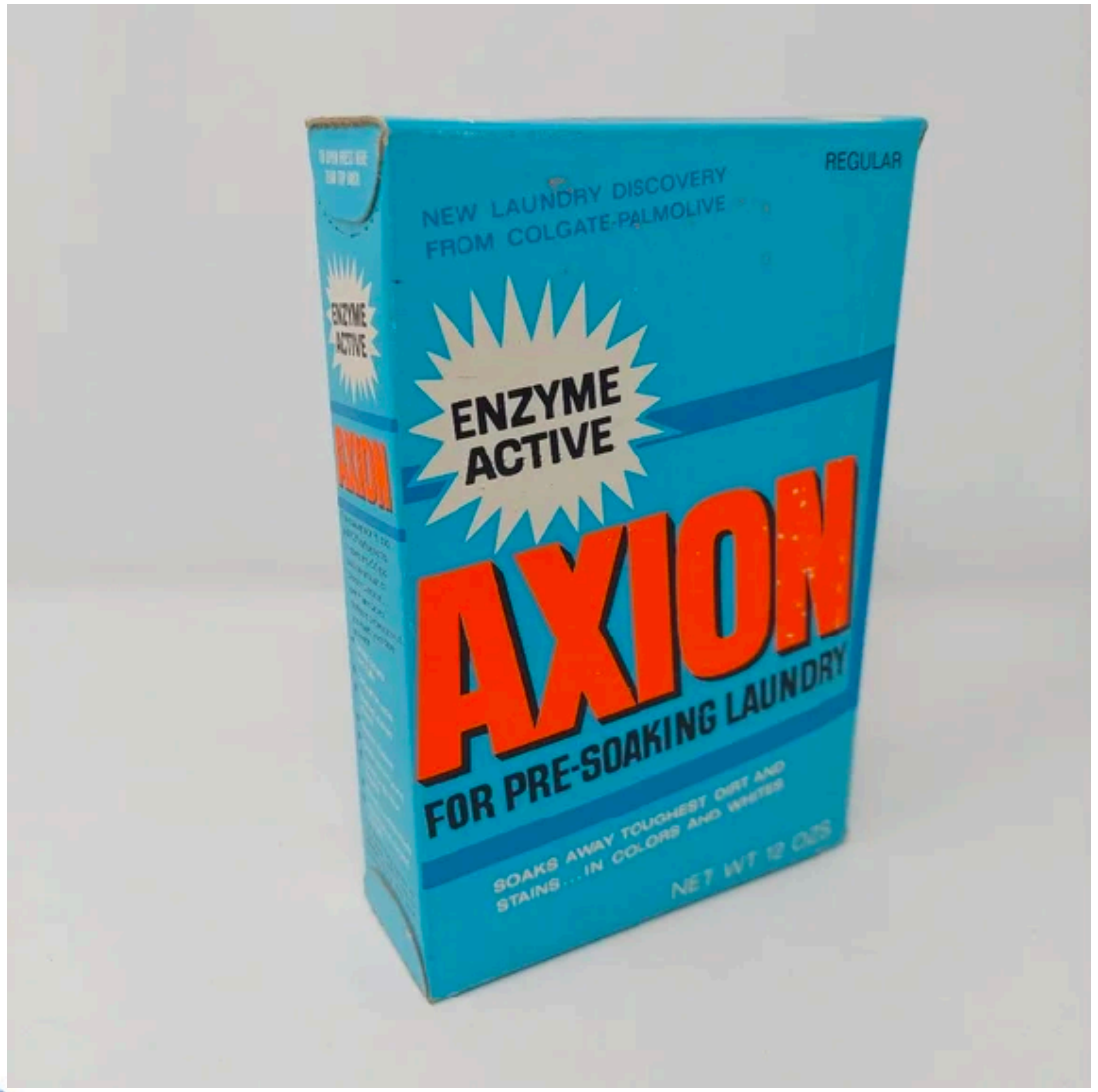


Axions

- New U(1) chiral symmetry (PQ) spontaneously broken at some scale f_a and anomalous under QCD

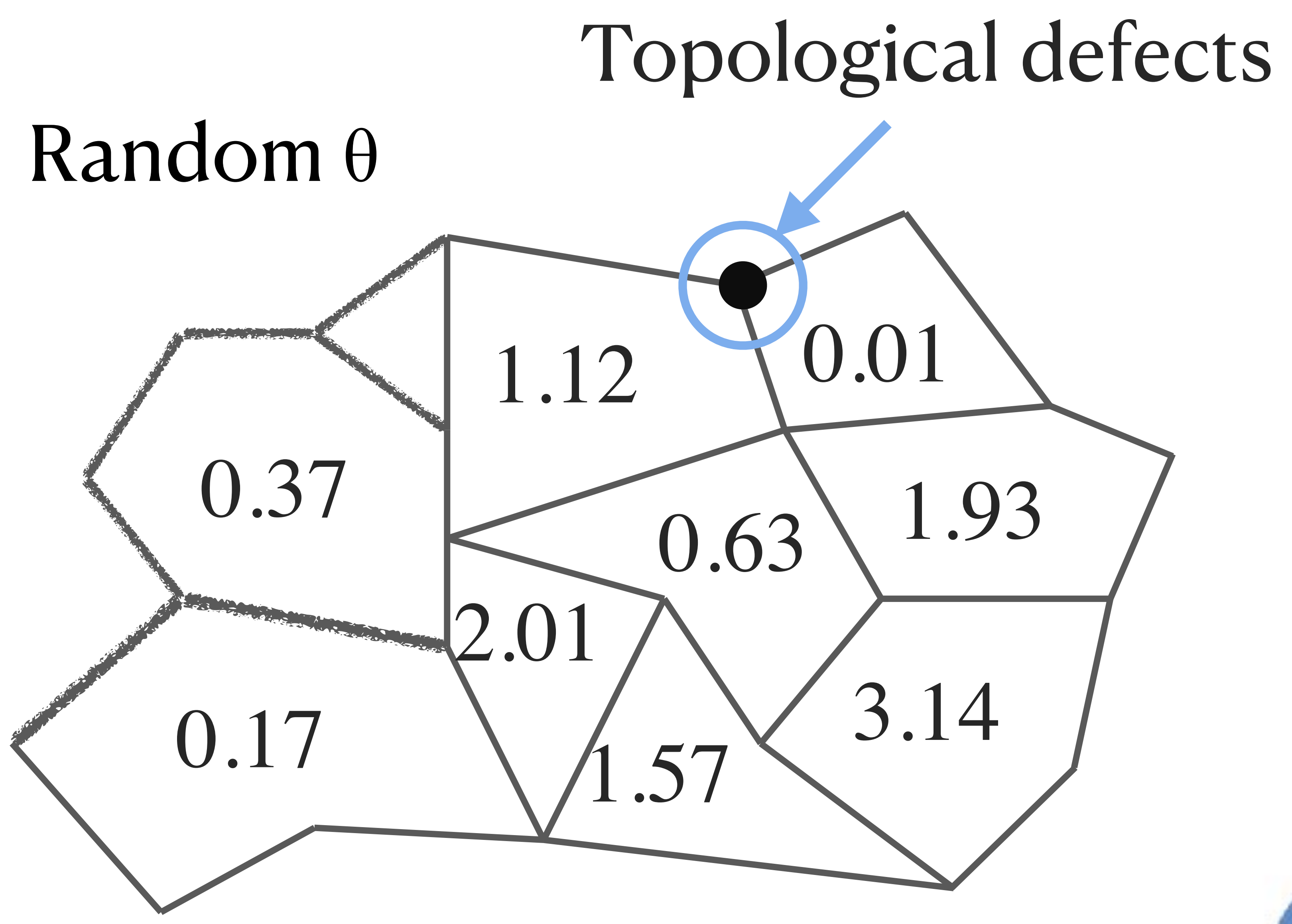
$$\mathscr{L} \supset \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{a}{f_a}\frac{g^2}{32\pi^2}G\tilde{G}$$

- Absorb the angle into a new field, the axion $a/f_a \to a/f_a - \bar{\theta}$
- New pseudoscalar (parity odd) degree of freedom
- Can be produced early in the universe as coherent waves
- Also couple to photons and matter

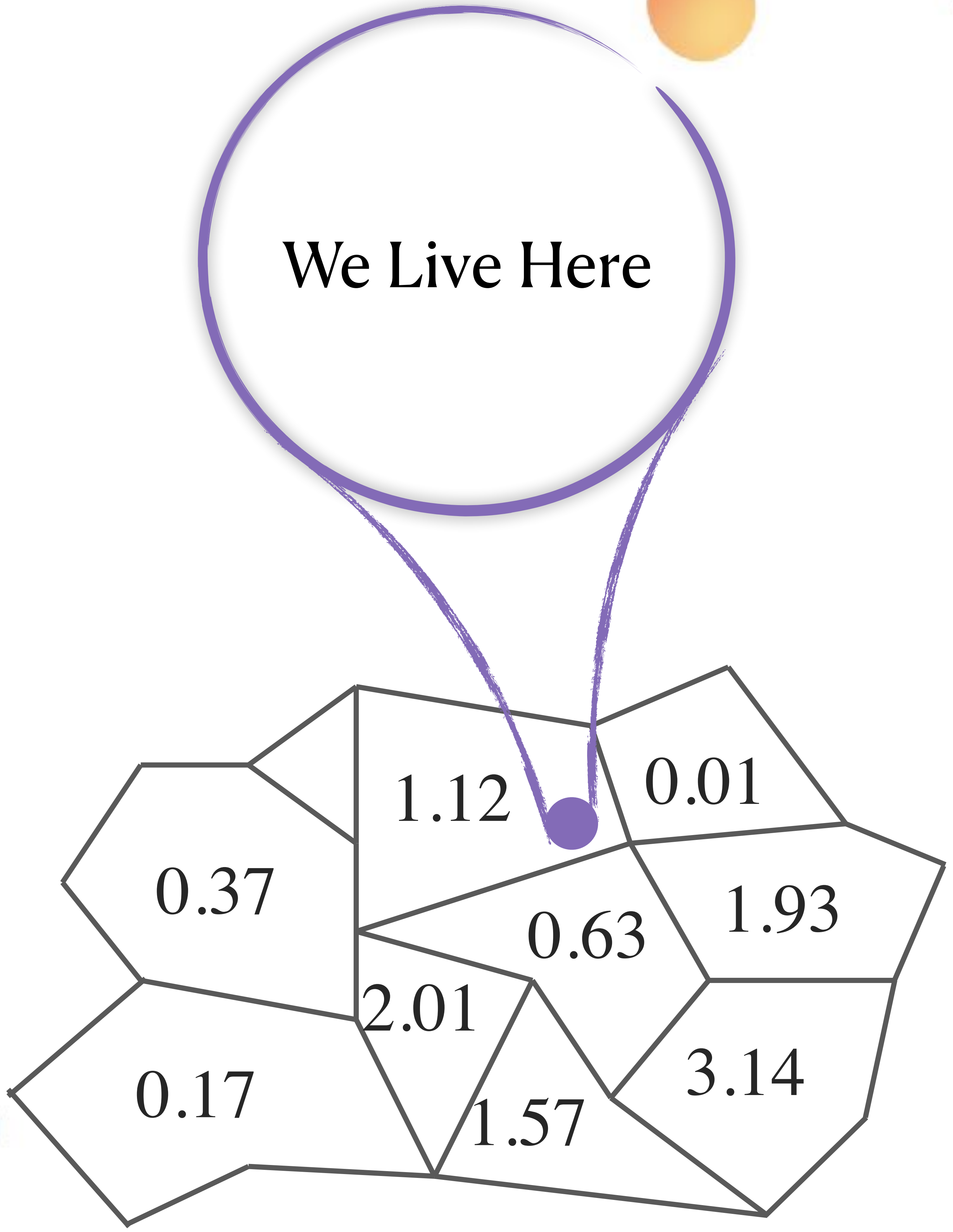


Where Should We Look?

Is PQ symmetry is broken before or after inflation?

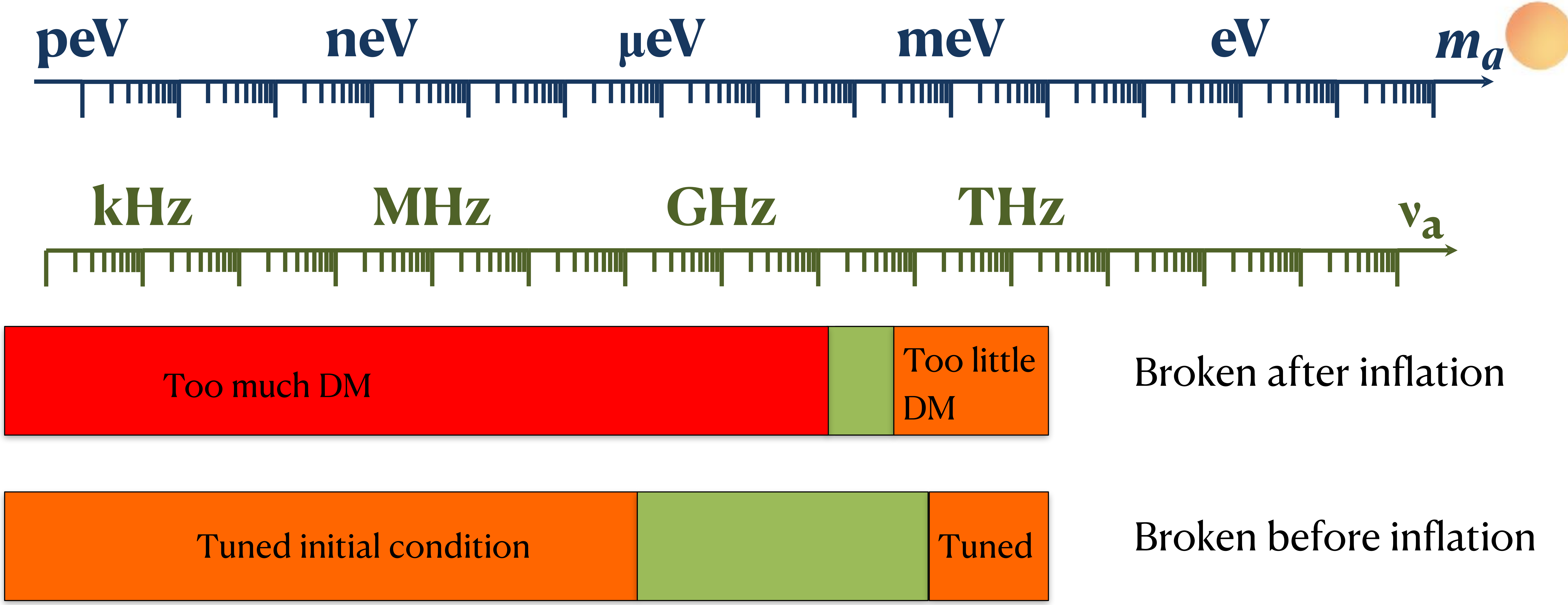


After inflation



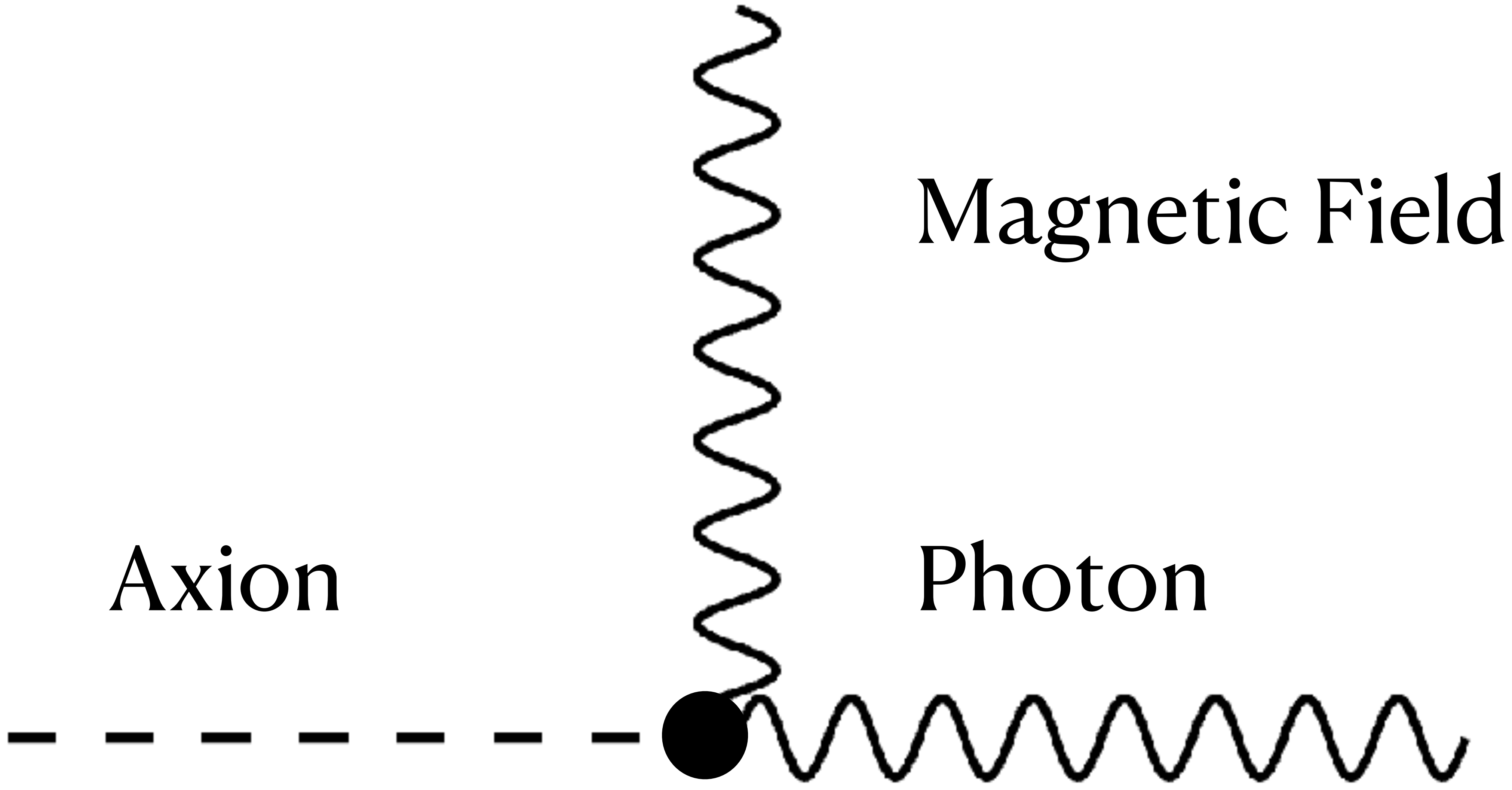
Before inflation

Where Should We Look?

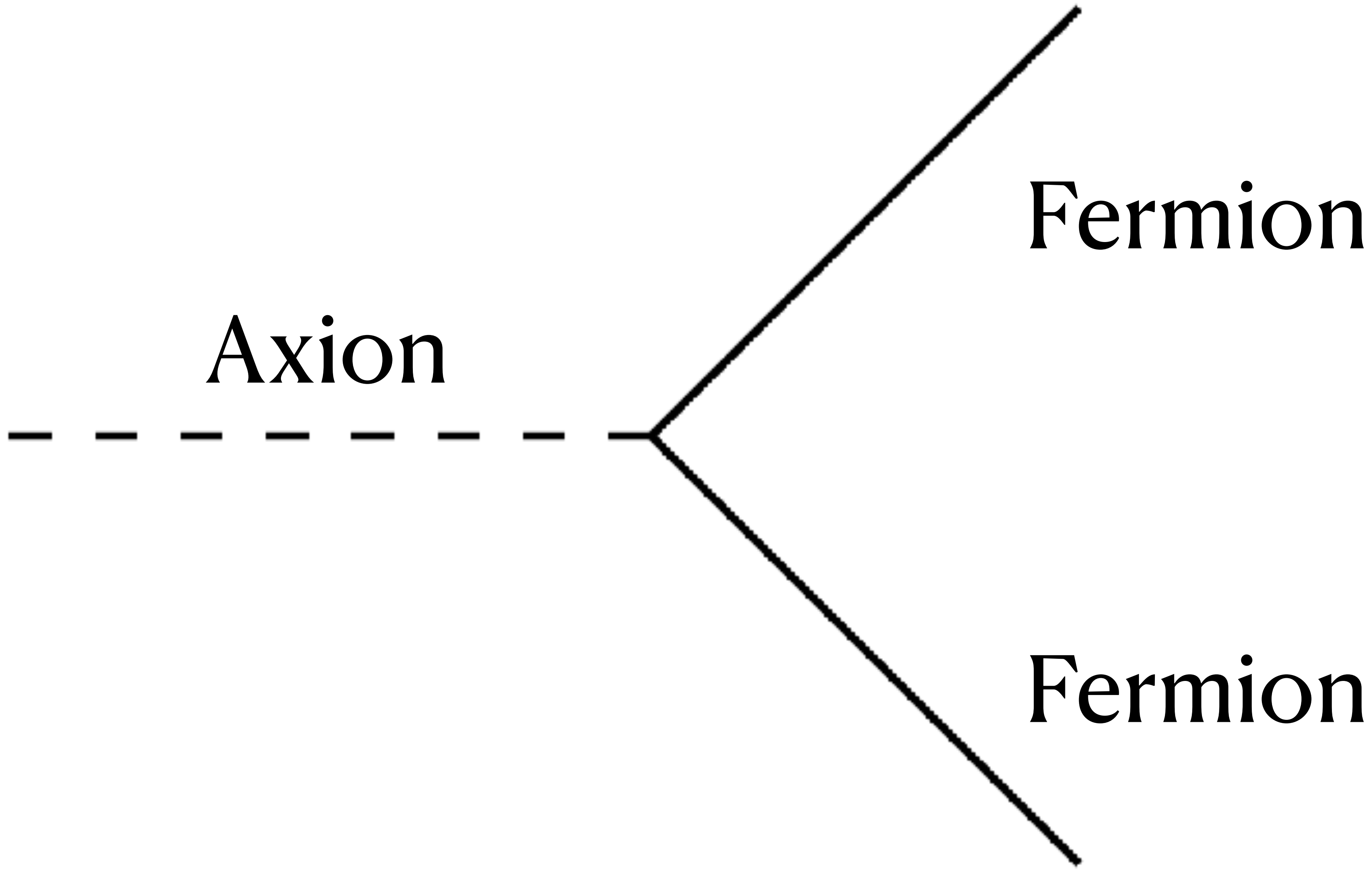


How Should We Look?

Lots of details depend on the model but we will only focus on two interactions



Coupling to electromagnetism



Coupling to matter (mostly spin)

How Should We Look?

- Lots of details depend on the model but we will only focus on two interactions

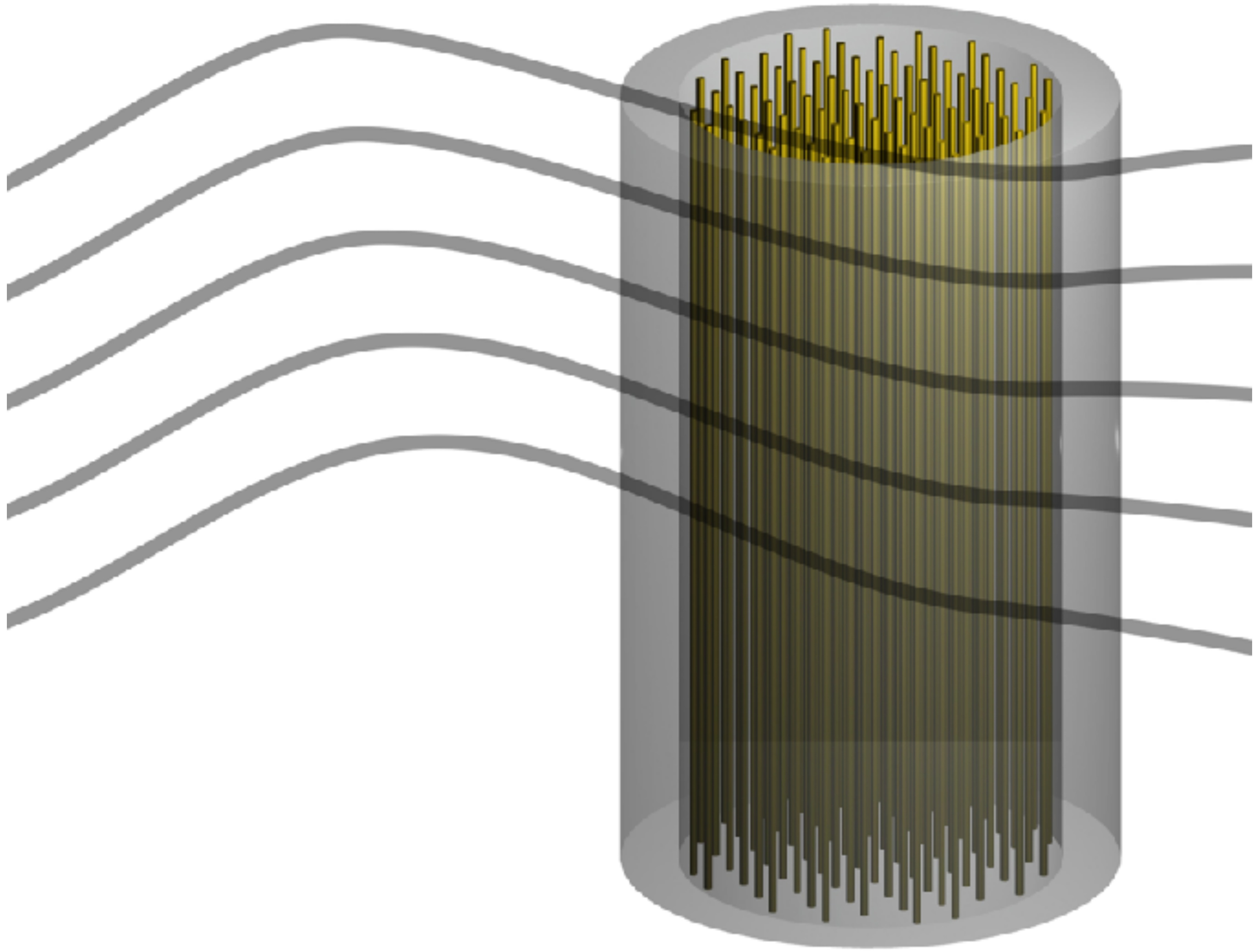
Coupling to matter (mostly spin)

$$L_{\text{int}} \supset \boxed{g_{a\gamma} a \mathbf{E} \cdot \mathbf{B}} + \boxed{g_{af} \left(\partial_\mu a\right) \bar{\Psi} \gamma^\mu \gamma^5 \Psi}$$

Coupling to electromagnetism

How Do You Find a Wave?

- Can't just look for scatterings
- Exploit the coherence of the field to increase the signal
- Analogue: finding the right radio station



Axion Induced Currents

Taking the Non-Relativistic Limit

- Matter tends to be slow: reduce to a non-relativistic description
- Lowest order terms (σ is electron spin)

$$g_{af}\,(\partial_\mu a)\,\bar{\Psi}\gamma^\mu\gamma^5\Psi\ ,$$

$$\Big\downarrow$$

$$H \supset -g_{af}\,(\nabla a)\cdot\boldsymbol{\sigma} - \frac{g_{af}}{m_f}\,\dot{a}\,\boldsymbol{\sigma}\cdot\boldsymbol{\pi}\ ,$$

Wind

Axio-electric

$$\boldsymbol{\pi} \equiv \mathbf{p} - q_f\mathbf{A}$$

Conjugate momentum

Axion-Induced Torques

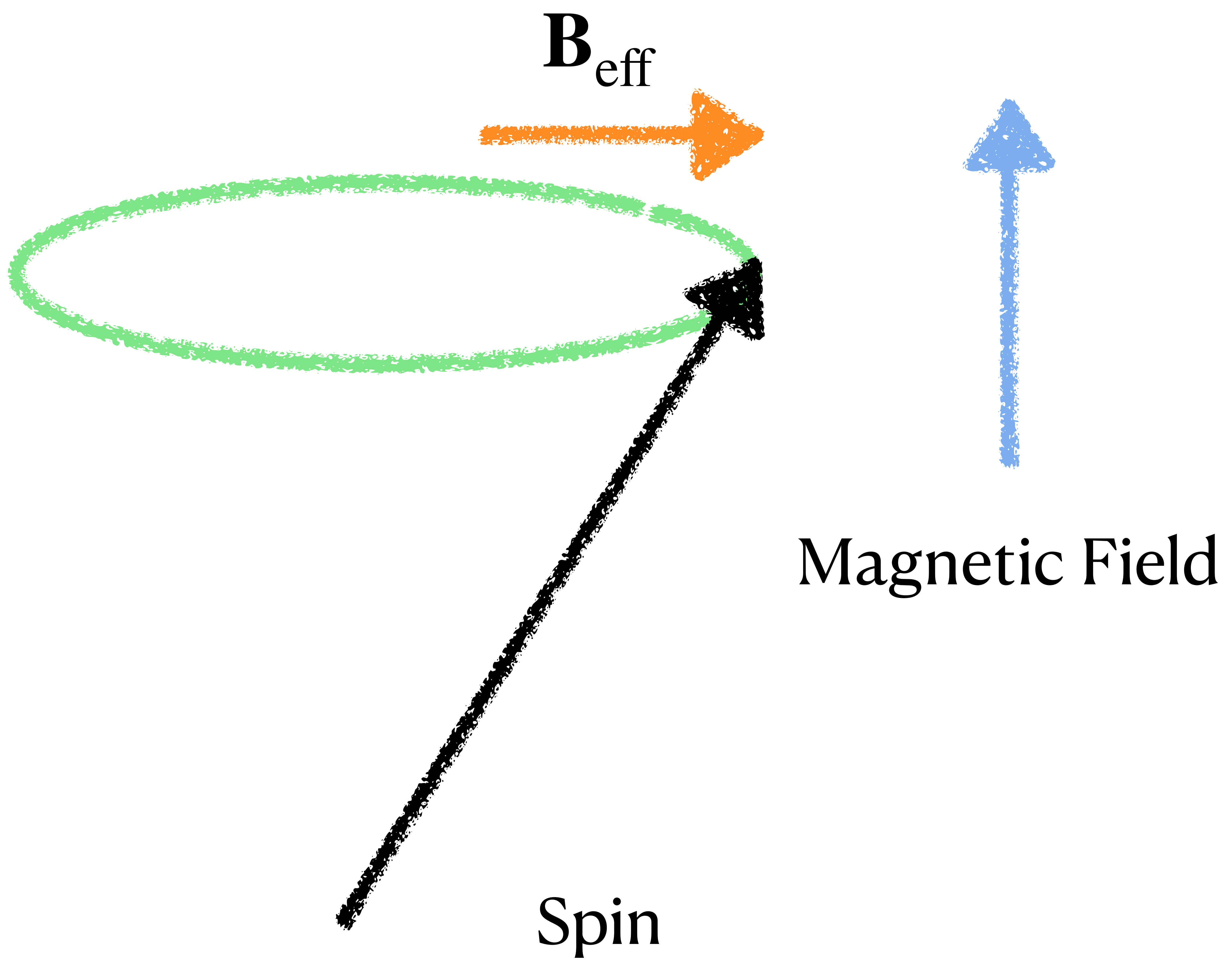
- Most well known effect of axion-fermion couplings
- Acts on spins similarly to a B-field

$$\frac{d}{dt}\langle \mathbf{S} \rangle = \langle\, 2\,\mu_f\, \mathbf{S} \times \mathbf{B} + \boxed{2 g_{af}\, \mathbf{S} \times \nabla a} \rangle$$

$$\mathbf{B}_{\text{eff}} = (g_{af}/\mu_f)\, \nabla a$$

Axion-Induced Torques

- Most exploited fermion coupling
- Can use nuclear magnetic resonance techniques
- Includes CASPER WIND and ferromagnet haloscopes like QUAX
- Tends to be most important for low axion masses



Axion-Induced Forces

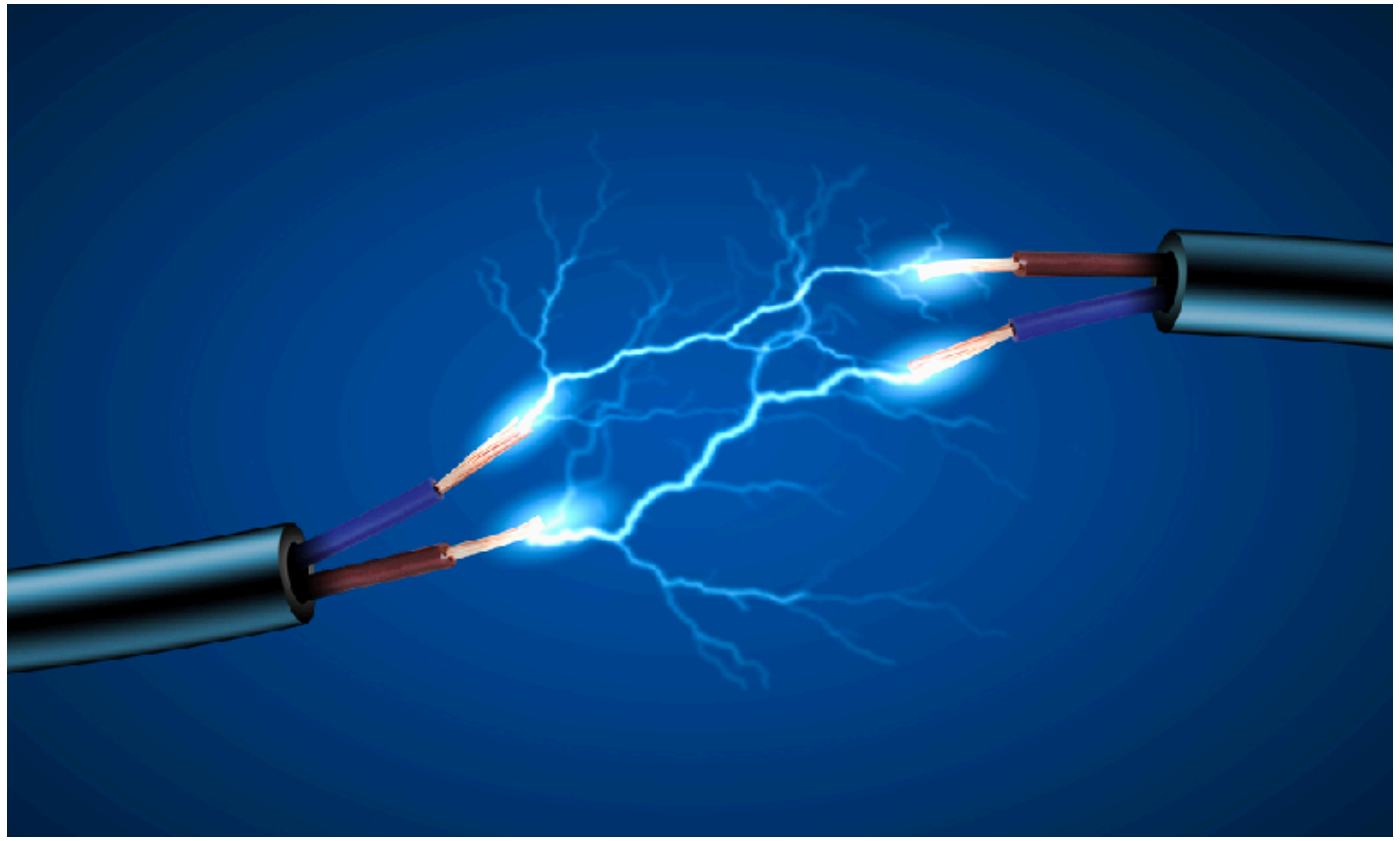
- How does the axio-electric term act on the electron?
- We generalized the Lorentz force law, ***JHEP* 05 (2024)**

$$\mathbf{F} \equiv m_f \frac{d\mathbf{v}}{dt} \simeq q\,\mathbf{E} + q\,(\mathbf{v}\times\mathbf{B}) + \mu_f\,(\boldsymbol{\sigma}\cdot\mathbf{B}) - \boxed{g_{af}\,\frac{d}{dt}\,(\dot{a}\,\boldsymbol{\sigma})}$$

$$\mathbf{E}_{\text{eff}} \simeq -(g_{af}/q)\,\frac{d}{dt}\,(\dot{a}\boldsymbol{\sigma})$$

Axion-Induced Forces

- This looks like an E-field, but it couples to spin rather than to charge
- Not well studied in the literature
- How can we exploit an effective electric field?
- Turn it into a real electric field



https://www.shutterstock.com/

Axion-Induced Currents

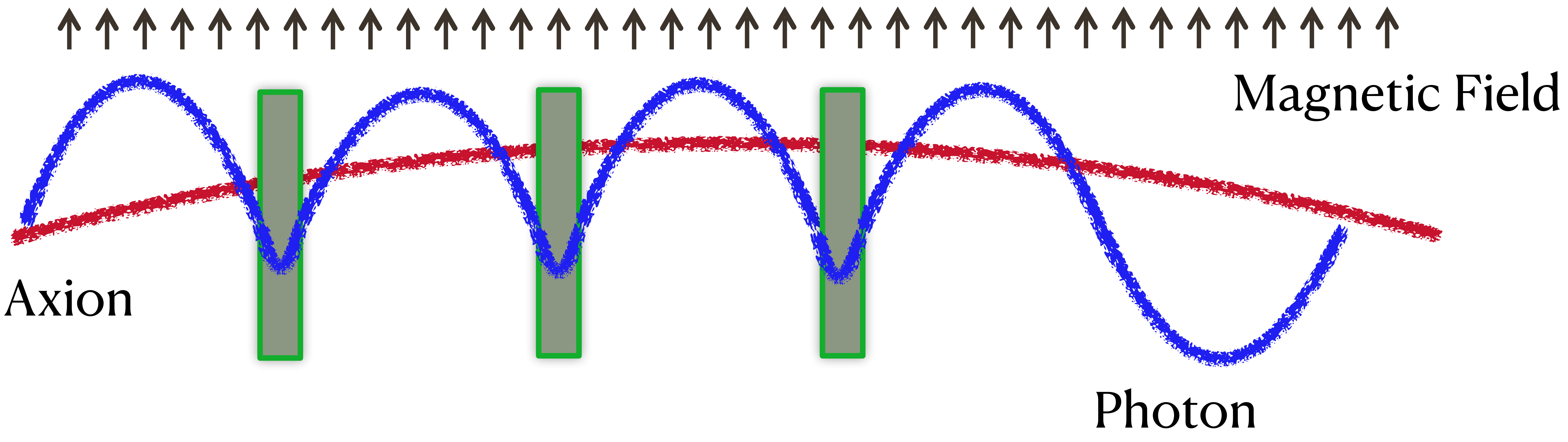
- New REAL currents to source Maxwell equations ***JHEP* 05 (2024)**

$$\mathbf{J}_a = \mathbf{J}_a^P + \mathbf{J}_a^M = (\varepsilon_{\sigma e} - 1)\,\partial_t \mathbf{E}_{\text{eff}} + \nabla \times \left(\left(1 - \mu^{-1}\right) \mathbf{B}_{\text{eff}}\right)$$

- $\epsilon_{\sigma e}$ is spin version of dielectric constant
- Many ways to exploit currents
- Two main approaches: quasiparticle resonances and breaking translation invariance

Dielectric Haloscopes

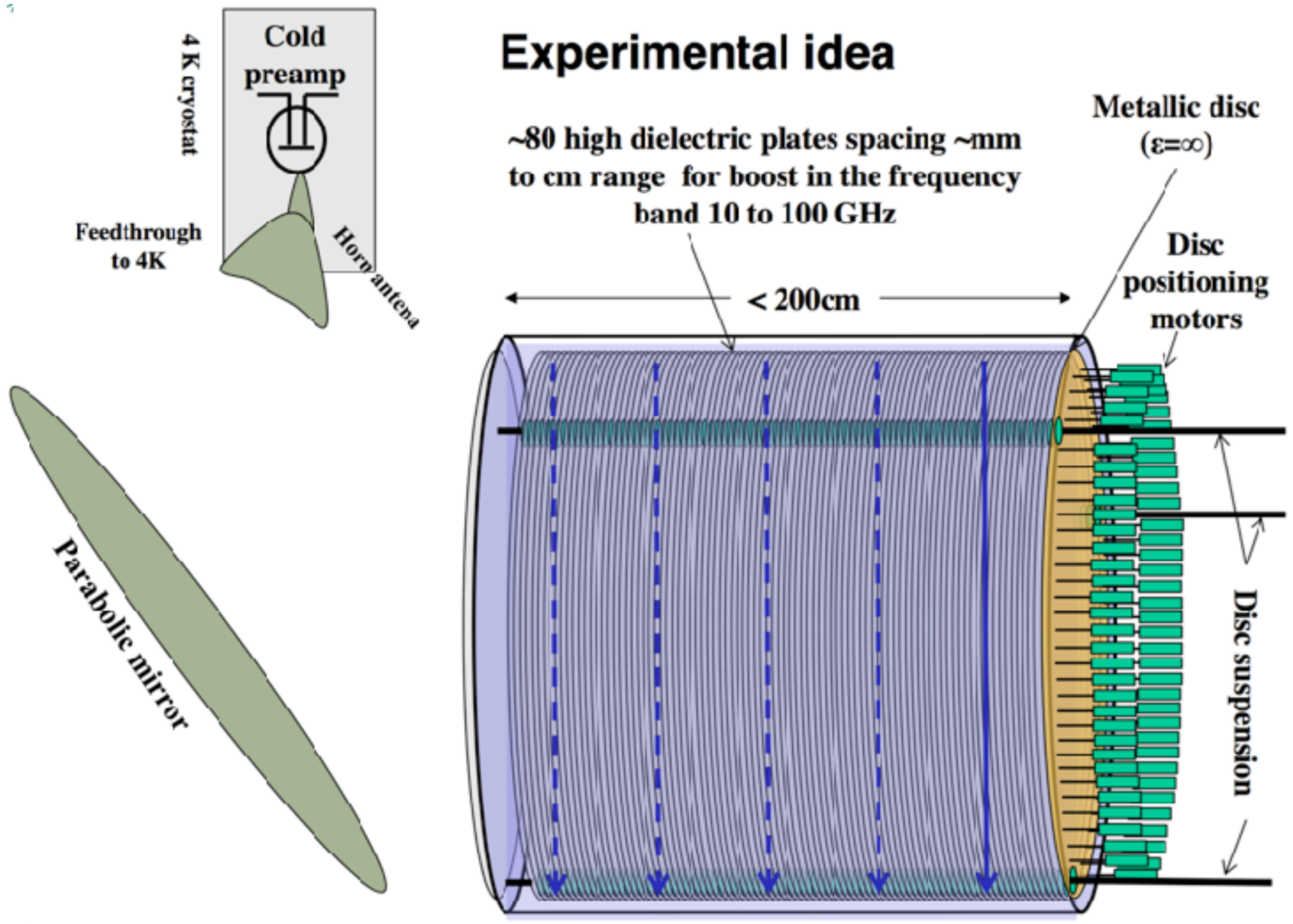
- Introduce a series of dielectric layers



- Breaking translation invariance provides momentum

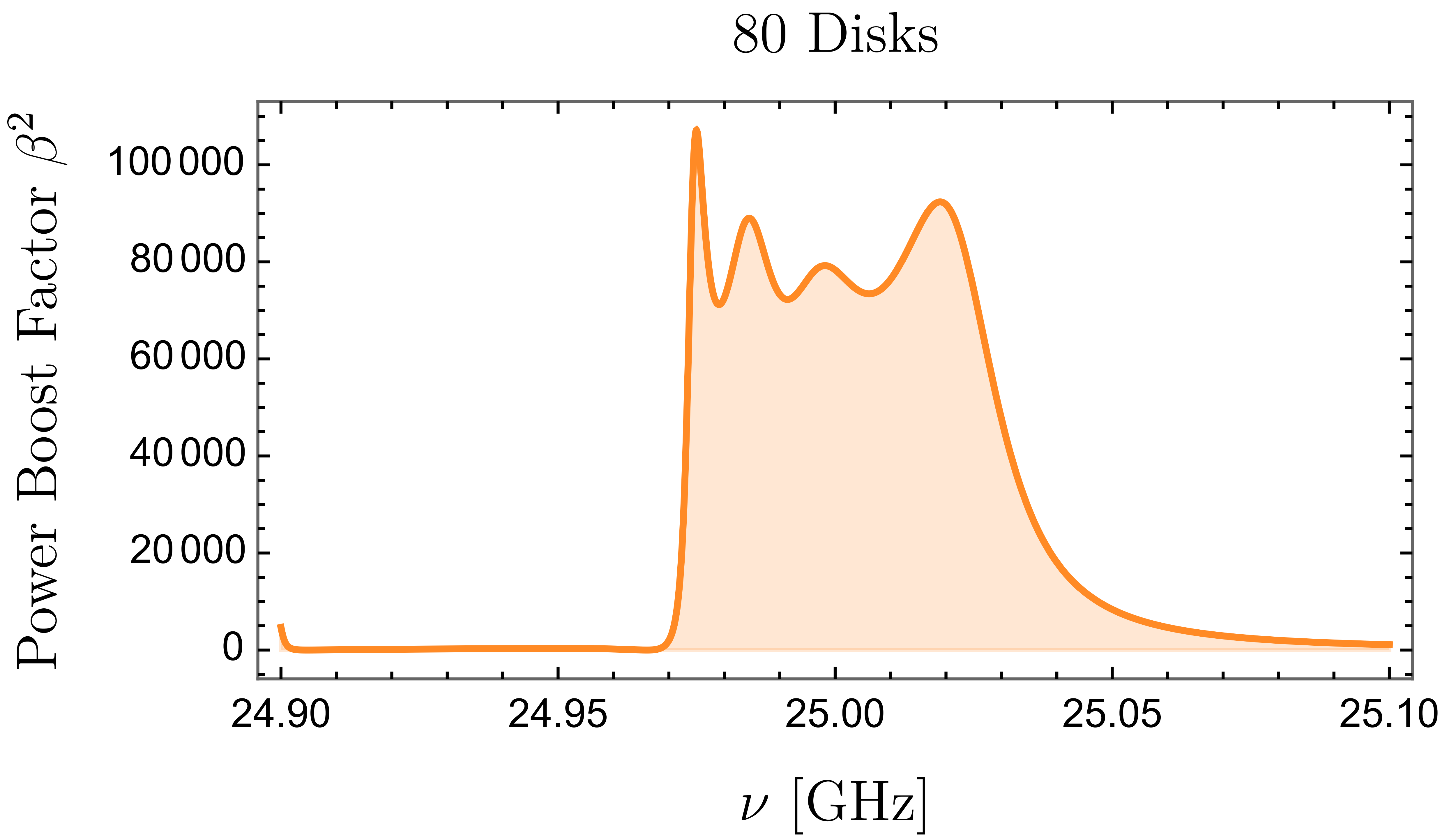
Dielectric Haloscopes

- Idea from the photon coupling
  ***Phys. Rev. Lett.* 118 (2017)**
- Arrange layers for constructive interference
- Tune frequencies by controlling disk spacings
- Many disks = strong signals



Constructing Construction

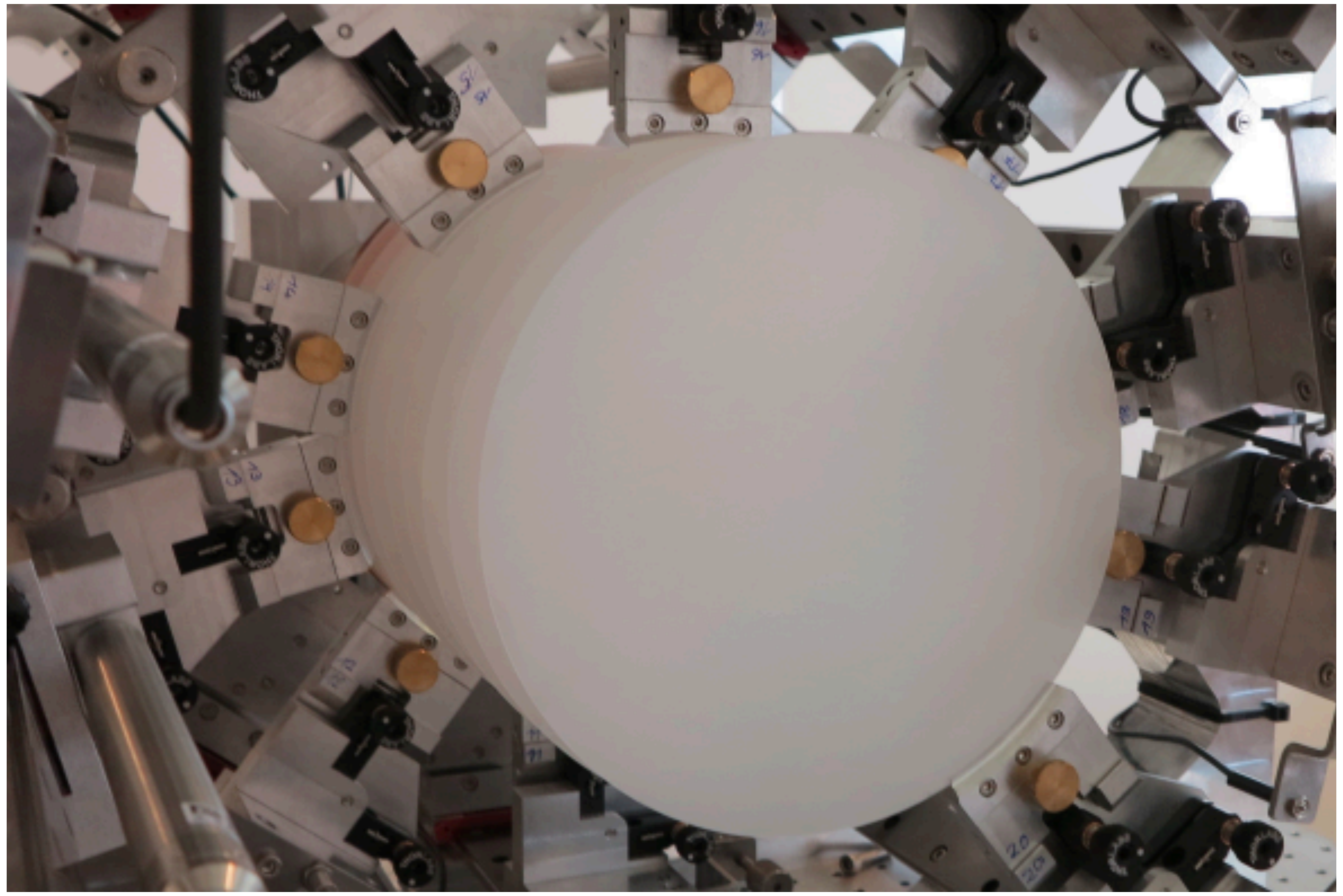
- Can use classical transfer matrices or QFT overlap integral
  ***JCAP* 01 (2017)**,
  ***JCAP* 09 (2017)**
- Simple for one or two disks, or transparent disks
- Numerical optimization for many layers and 3D effects
  ***JCAP* 01 (2017)**,
  ***JCAP* 08 (2019)**

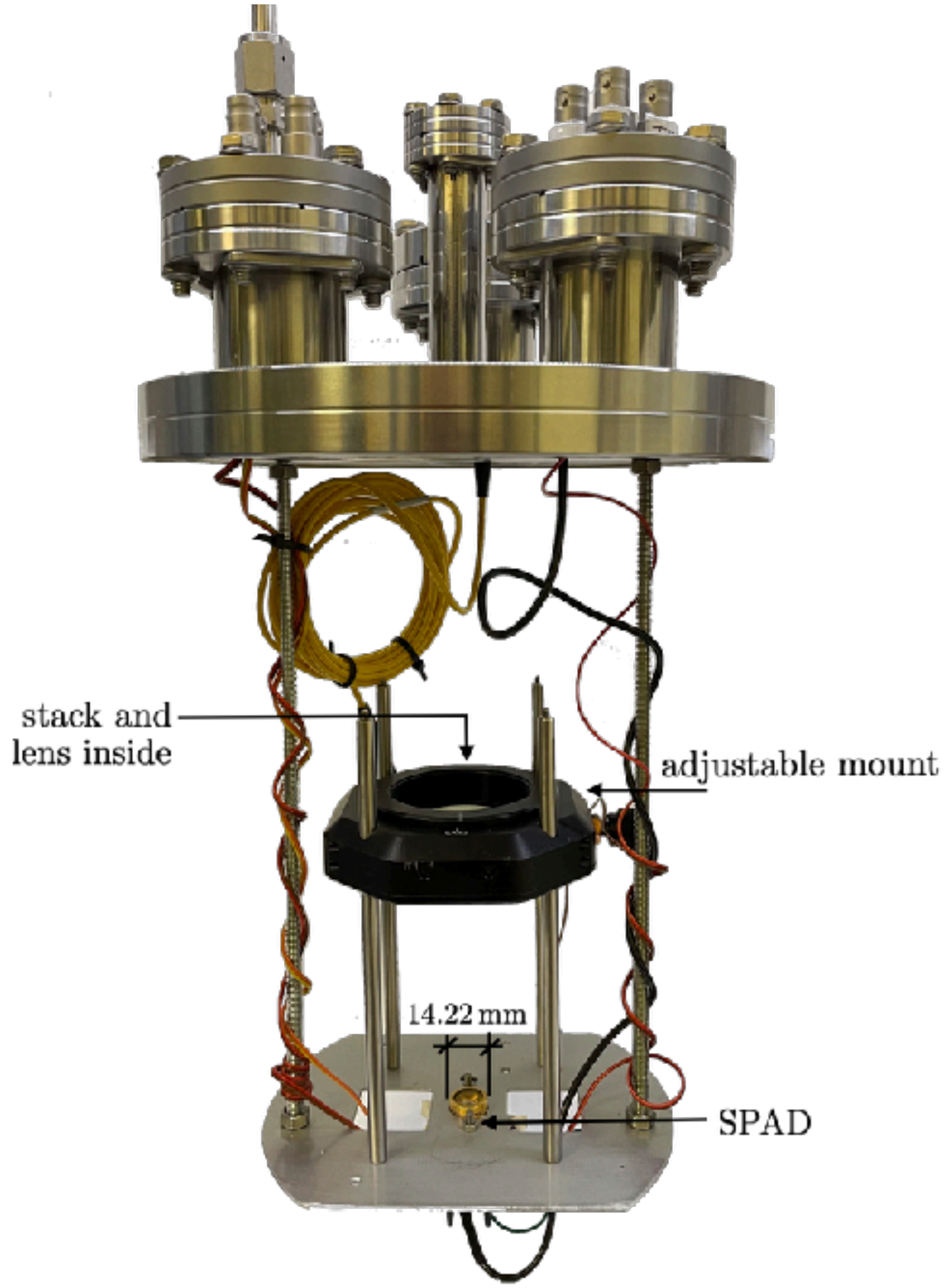


AJM+, *JCAP* 10 (2017)

Dielectric Haloscopes

- Two versions being pursued: movable disks, GHz version (MADMAX, DALI)
- Thin film optical version (MuDHI, LAMPOST)





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Stefan Knirck

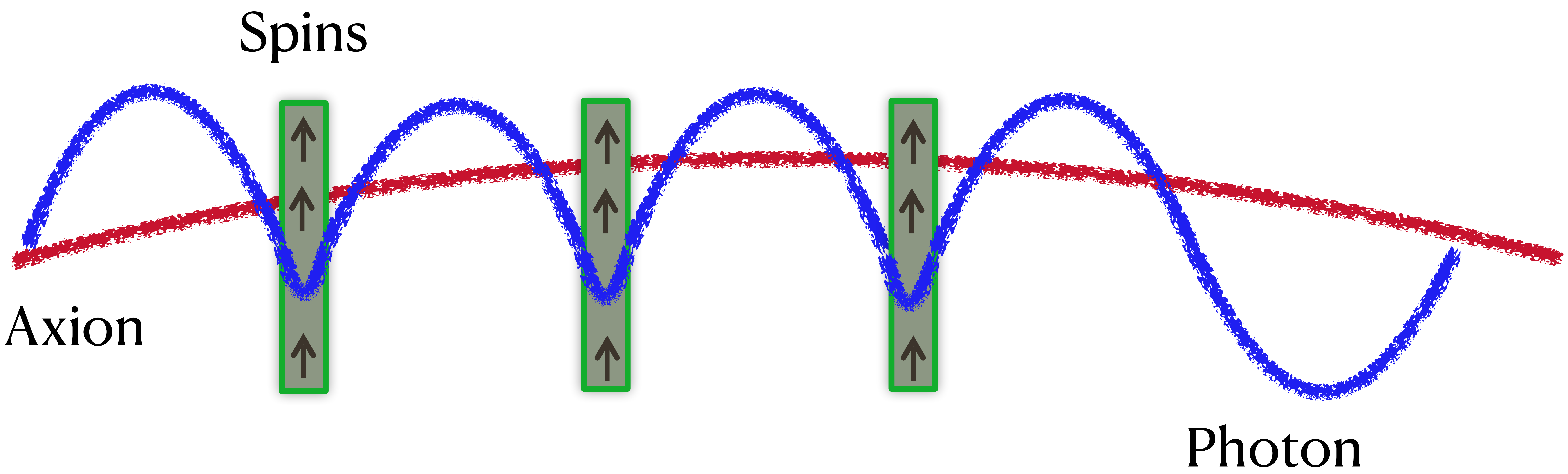
***Phys. Rev. D* 105 (2022)**

Axio-Electric Effect

- Spin polarized slab emits propagating radiation *JHEP* 05 (2024)
- Can directly map from the photon case
- Tends to be best for optical frequencies

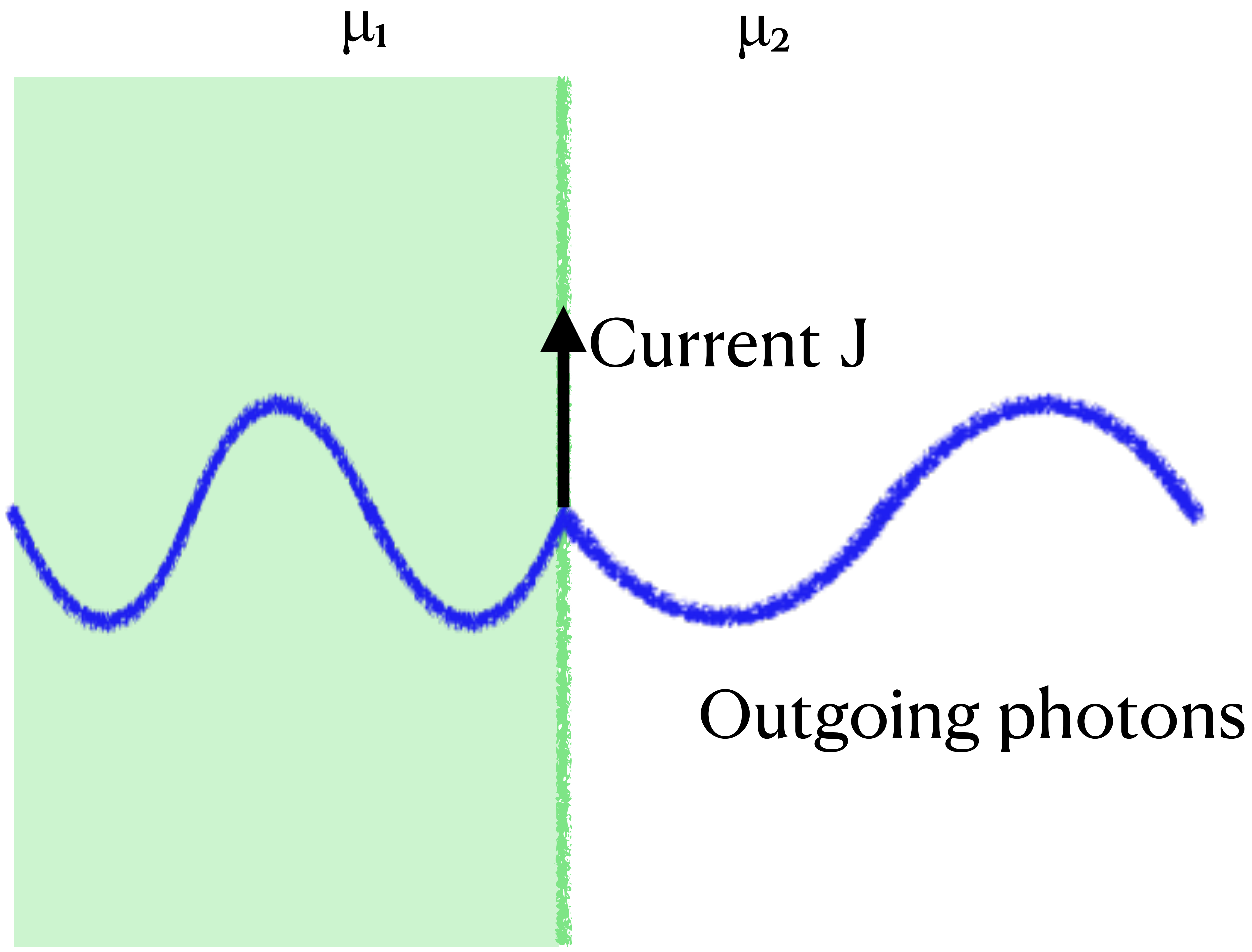
$$\mathbf{E} = \frac{1-\varepsilon_{\sigma e}}{\varepsilon}\,\mathbf{E}_{\rm eff}$$

$$g_{ae} \;\leftrightarrow\; g_{a\gamma\gamma}\,(e\,B_0/m_a^2)$$



What About the Axion Wind?

- No bulk currents! ***JHEP* 05 (2024)**
- $\nabla \times \mathbf{B}_{\rm eff} \propto \nabla \times (\nabla a/\mu)$
- Discontinuity in μ leads to boundary currents
- Doesn't directly map onto the photon coupling

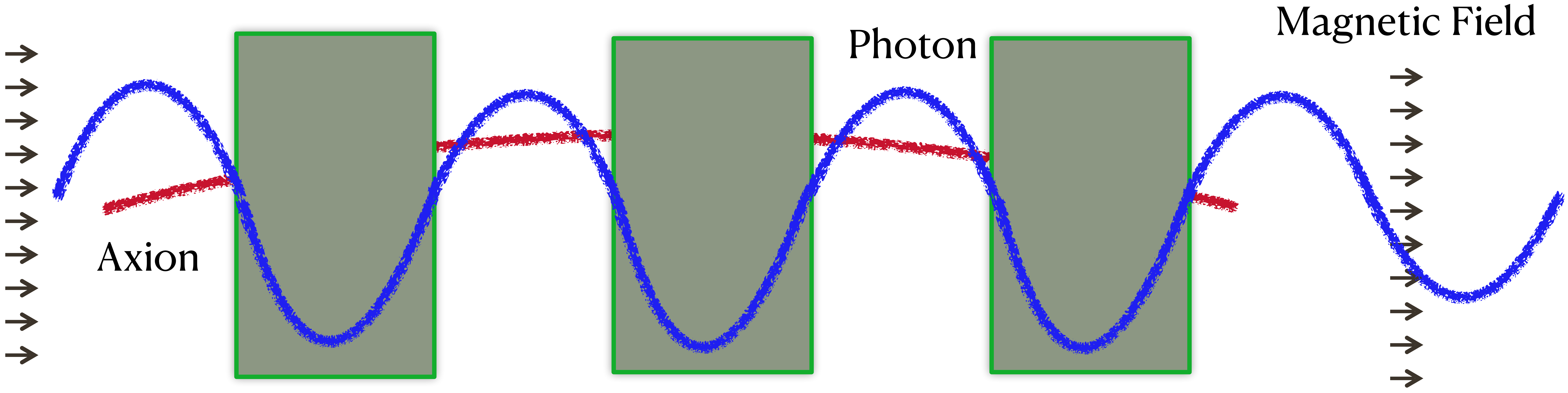


Axion Wind

- High frequency μ needs an applied B-field
- Can use larger size, lower Q materials than NMR
- Ferrites ideal ***JHEP* 05 (2024)**
- Magnon resonance tunable with B-field
- Can also be used on resonance similar to the quasiparticle haloscopes

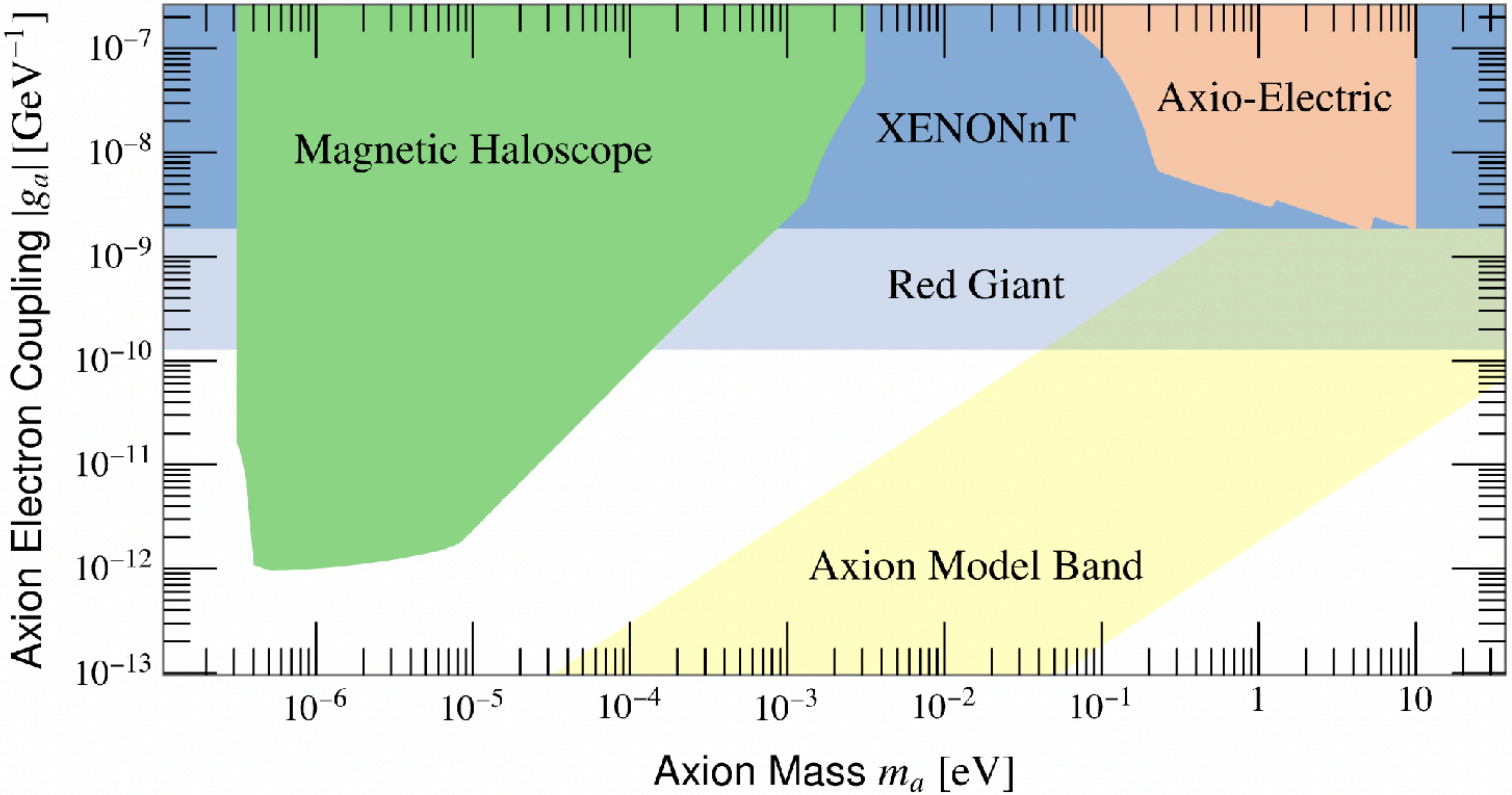
Magnetic Haloscope

- Introduce a series of magnetic layers



- Boundary radiation emitted from each slab

Sensitivity



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Absorption

- More generally one can consider the absorption of an axion
- What if the system is polarized or magnetic?
- Can just consider the total energy dissipated by the axion into a material
- Easy to calculate from the axion equation of motion

$$(\partial^2 + m_a^2)\, a = -g_{ae}\left(\partial_t j_\sigma + \nabla \cdot \mathbf{n}_\sigma\right)$$

$$e\, j_\sigma = (\varepsilon - 1)\, \partial_t E_{\text{eff}} + (\varepsilon_{\sigma e} - 1)\, \partial_t \langle \mathbf{E} \cdot \hat{\mathbf{s}} \rangle$$

Axio-electric Wind

Absorption: Axio-Electric

$$R \simeq \frac{g_{ae}^2 m_a^2}{e^2} \, \frac{\rho_{\rm DM}}{\rho_{\rm det}} \times \begin{cases} 3\,\mathrm{Im}\left[\,\varepsilon(m_a)\right] & \text{(unpolarized target)} \\ \mathrm{Im}\left[\dfrac{-1}{\varepsilon(m_a)}\right] & \text{(polarized target)}\ , \end{cases}$$

- Polarized targets haven't been considered before!
- Two advantages
- Can spin polarize a system to remove background
- Absorption higher on resonances

Absorption: Wind

- Axion absorption onto magnons is not new (arXiv:2005.10256)
- Only been done from first principles calculations
- More generally one can just consider an arbitrary magnetized medium
- Magnetic equivalent of the "energy loss function"

$$R \simeq \left(\frac{g_{ae}\, v_{\rm DM}}{\mu_B}\right)^2 \frac{\rho_{\rm DM}}{\rho_{\rm det}}\, {\rm Im}\left[\frac{-1}{\mu}\right] ,$$

- Anything with μ close to zero may be an interesting detector!

Takeaways from Induced Currents

Media effects contain all the microphysics

Can just measure the materials

Axion induced currents provide viable new detection strategies

Spin-Dependent Scattering

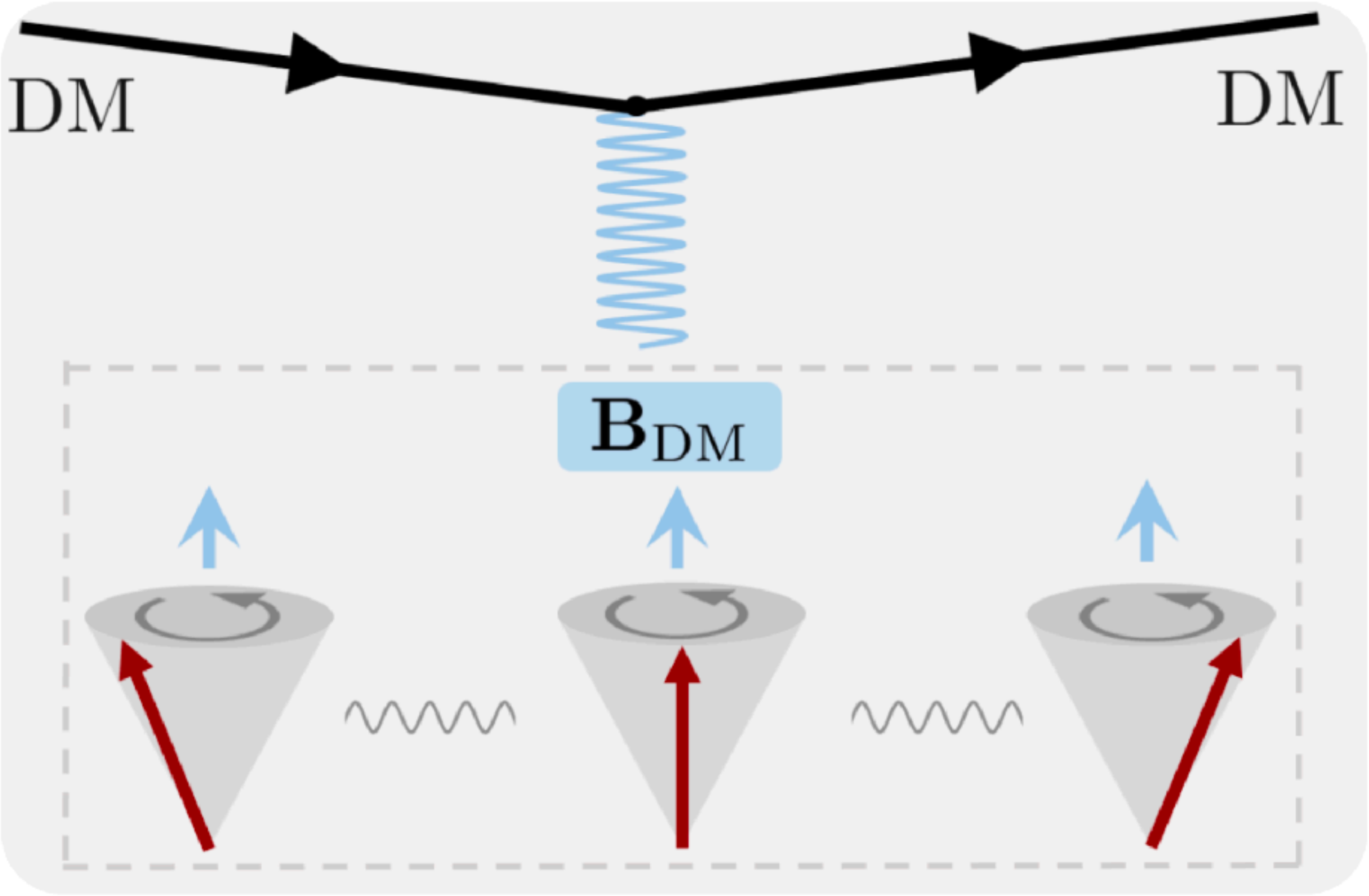
Light Dark Matter

Can be boson or fermion

Typically scattering kinematics

Model	Lagrangian	form factor $\mathcal{F}^{ij}$
Magnetic dipole DM	$\frac{g_\chi}{4\,m_\chi}\,\bar{\Psi}_\chi\sigma^{\mu\nu}\Psi_\chi\,F'_{\mu\nu} + g_e\,\bar{\Psi}_e\gamma^\mu\Psi_e A'_\mu$	$2\left(\frac{g_\chi g_e}{\lvert\mathbf{q}\rvert^2+m_{\text{med}}^2}\right)^2\left(\frac{\lvert\mathbf{q}\rvert^2}{4\,m_\chi m_e}\right)^2(\delta^{ij}-\hat{\mathbf{q}}^i\hat{\mathbf{q}}^j)$
Anapole DM	$\frac{g_\chi}{4\,m_\chi^2}\,\bar{\Psi}_\chi\gamma^\mu\gamma^5\Psi_\chi\,\partial^\nu F'_{\mu\nu} + g_e\,\bar{\Psi}_e\gamma^\mu\Psi_e A'_\mu$	$2\left(\frac{g_\chi g_e}{\lvert\mathbf{q}\rvert^2+m_{\text{med}}^2}\right)^2\left(\frac{\lvert\mathbf{q}\rvert^3}{8\,m_\chi^2 m_e}\right)^2(\delta^{ij}-\hat{\mathbf{q}}^i\hat{\mathbf{q}}^j)$
Axial vector mediator V_μ	$V_\mu\,(g_\chi\,\bar{\Psi}_\chi\gamma^\mu\gamma^5\Psi_\chi + g_e\,\bar{\Psi}_e\gamma^\mu\gamma^5\Psi_e)$	$2\left(\frac{g_\chi g_e}{\lvert\mathbf{q}\rvert^2+m_{\text{med}}^2}\right)^2\delta^{ij}$
Pseudoscalar mediator ϕ	$\phi\,(g_\chi\,\bar{\Psi}_\chi i\gamma^5\Psi_\chi + g_e\,\bar{\Psi}_e i\gamma^5\Psi_e)$	$2\left(\frac{g_\chi g_e}{\lvert\mathbf{q}\rvert^2+m_{\text{med}}^2}\right)^2\left(\frac{\lvert\mathbf{q}\rvert^2}{4\,m_\chi m_e}\right)^2\hat{\mathbf{q}}^i\hat{\mathbf{q}}^j$
CP violating scalar mediator ϕ	$\phi\,(g_\chi\,\bar{\Psi}_\chi\Psi_\chi + g_e\,\bar{\Psi}_e i\gamma^5\Psi_e)$	$2\left(\frac{g_\chi g_e}{\lvert\mathbf{q}\rvert^2+m_{\text{med}}^2}\right)^2\left(\frac{\lvert\mathbf{q}\rvert}{2\,m_e}\right)^2\hat{\mathbf{q}}^i\hat{\mathbf{q}}^j$
Dark electron EDM	$g_\chi\,\bar{\Psi}_\chi\gamma^\mu\Psi_\chi A'_\mu + \frac{g_e}{4\,m_e}\,i\bar{\Psi}_e\sigma^{\mu\nu}\gamma^5\Psi_e\,F'_{\mu\nu}$	$2\left(\frac{g_\chi g_e}{\lvert\mathbf{q}\rvert^2+m_{\text{med}}^2}\right)^2\left(\frac{\lvert\mathbf{q}\rvert}{2\,m_e}\right)^2\hat{\mathbf{q}}^i\hat{\mathbf{q}}^j$

Magnon Scattering



Susceptibility

- We can consider some general interaction via

$$\mathcal{H}_e(\mathbf{x}, t) = -\mathbf{\Phi}(\mathbf{x}, t) \cdot \mathbf{s}_e(\mathbf{x}, t)$$

- Where the potential is given by

$$\mathbf{\Phi}(\mathbf{x}, t) = \boldsymbol{\mathcal{O}}(\mathbf{q}, \omega_\mathbf{q})\, e^{-iq\cdot x} \times \begin{cases} 1/V & \text{(scatter)} \\ 1/\sqrt{2m_\chi V} & \text{(absorb)} \end{cases}$$

Susceptibility

- The electron spin simply pulls out the magnetic susceptibility

$$\chi \approx \frac{1}{\mu_B^2} \times \left((\mu - 1)^{-1} + \hat{\mathbf{q}}\hat{\mathbf{q}}\right)^{-1}$$

$$\Gamma_{\mathrm{s}}(\mathbf{v}) = \int \frac{\mathrm{d}^3\mathbf{q}}{(2\pi)^3} \, \mathcal{F}^{ij}(\mathbf{q}, \omega_{\mathbf{q}}) \, \chi''_{ij}(\mathbf{q}, \omega_{\mathbf{q}})$$

$$\mathcal{F}^{ij}(\mathbf{q}, \omega_{\mathbf{q}}) \equiv \frac{2}{2S_{\mathrm{DM}} + 1} \sum_{ss'} \mathcal{O}^j(\mathbf{q}, \omega_{\mathbf{q}}) \, \mathcal{O}^{*\,i}(\mathbf{q}, \omega_{\mathbf{q}})$$

- How do we measure χ?

Neutron Scattering

- Neutron scattering can be dominated by the magnetic dipole

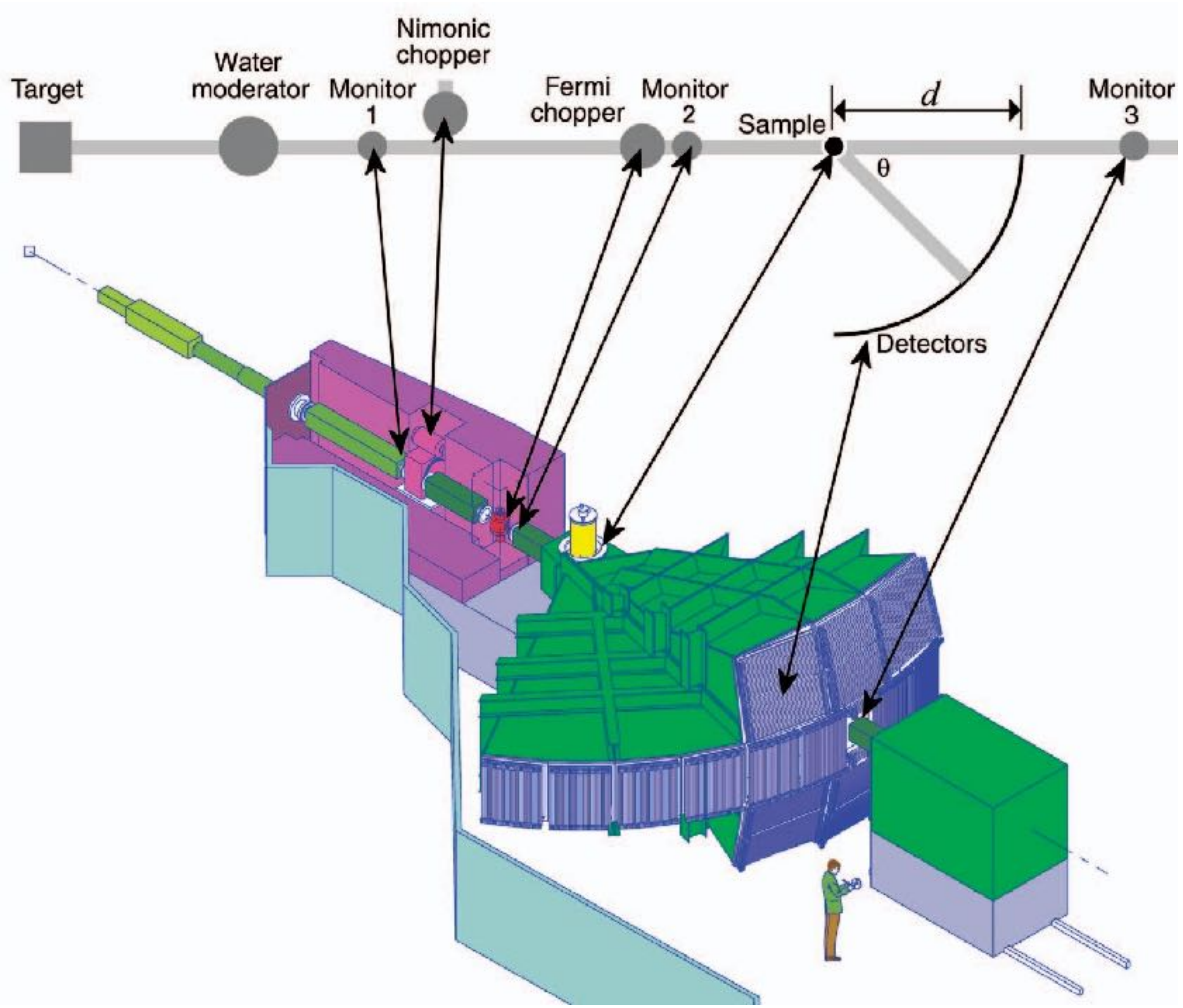
$$\mathcal{L} \supset \frac{\gamma_n e}{4m_n} \bar{n}\sigma^{\mu\nu} n \, F_{\mu\nu} - eA_\mu \, \bar{e}\gamma^\mu e$$

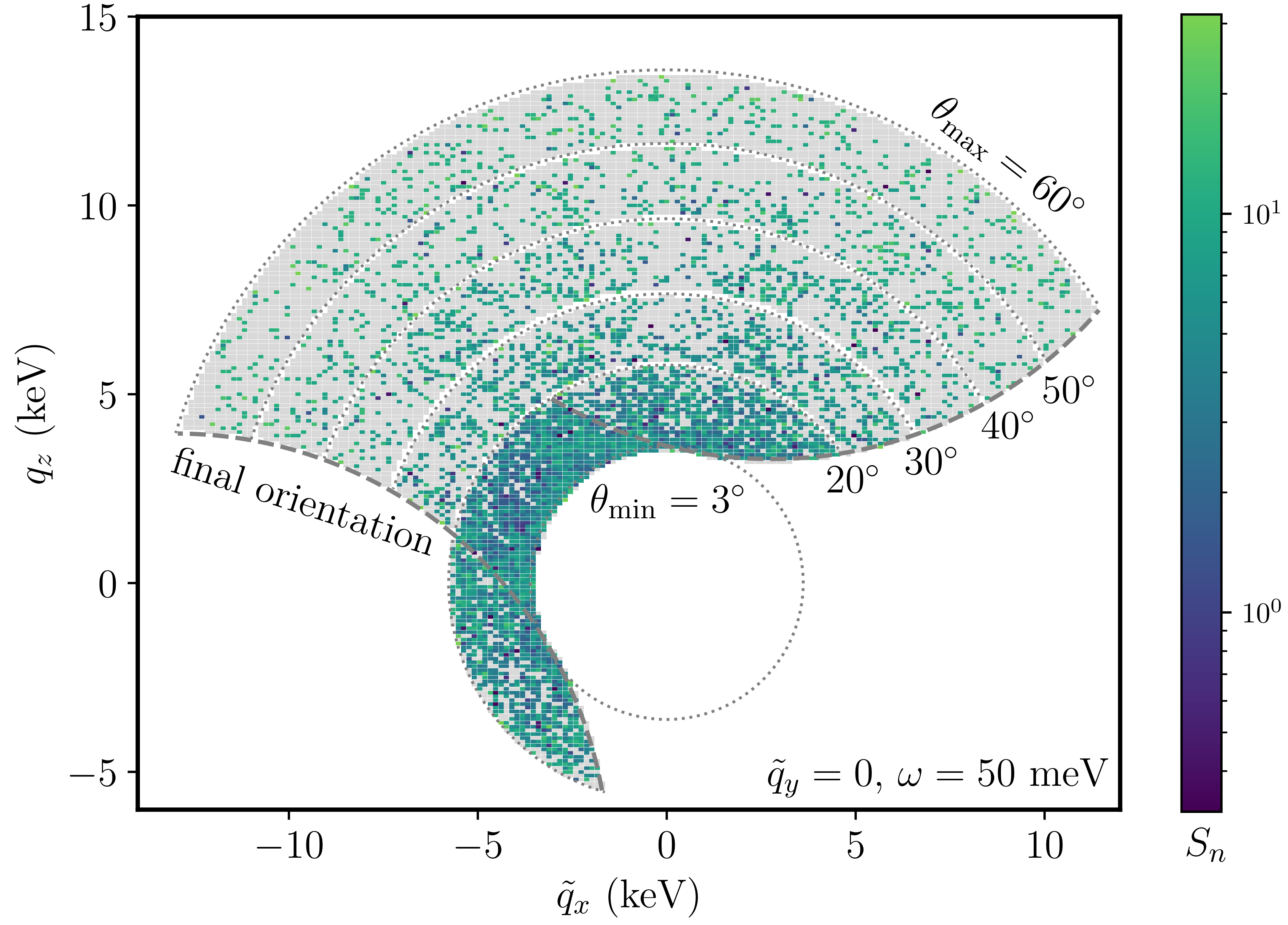
- Can measure the 4D cross section information

$$\frac{d\sigma_n(\mathbf{v})}{d\Omega dE'} = \frac{|\mathbf{p}'|}{|\mathbf{p}|} \boxed{S_n(\mathbf{q}, \omega_{\mathbf{q}})}$$

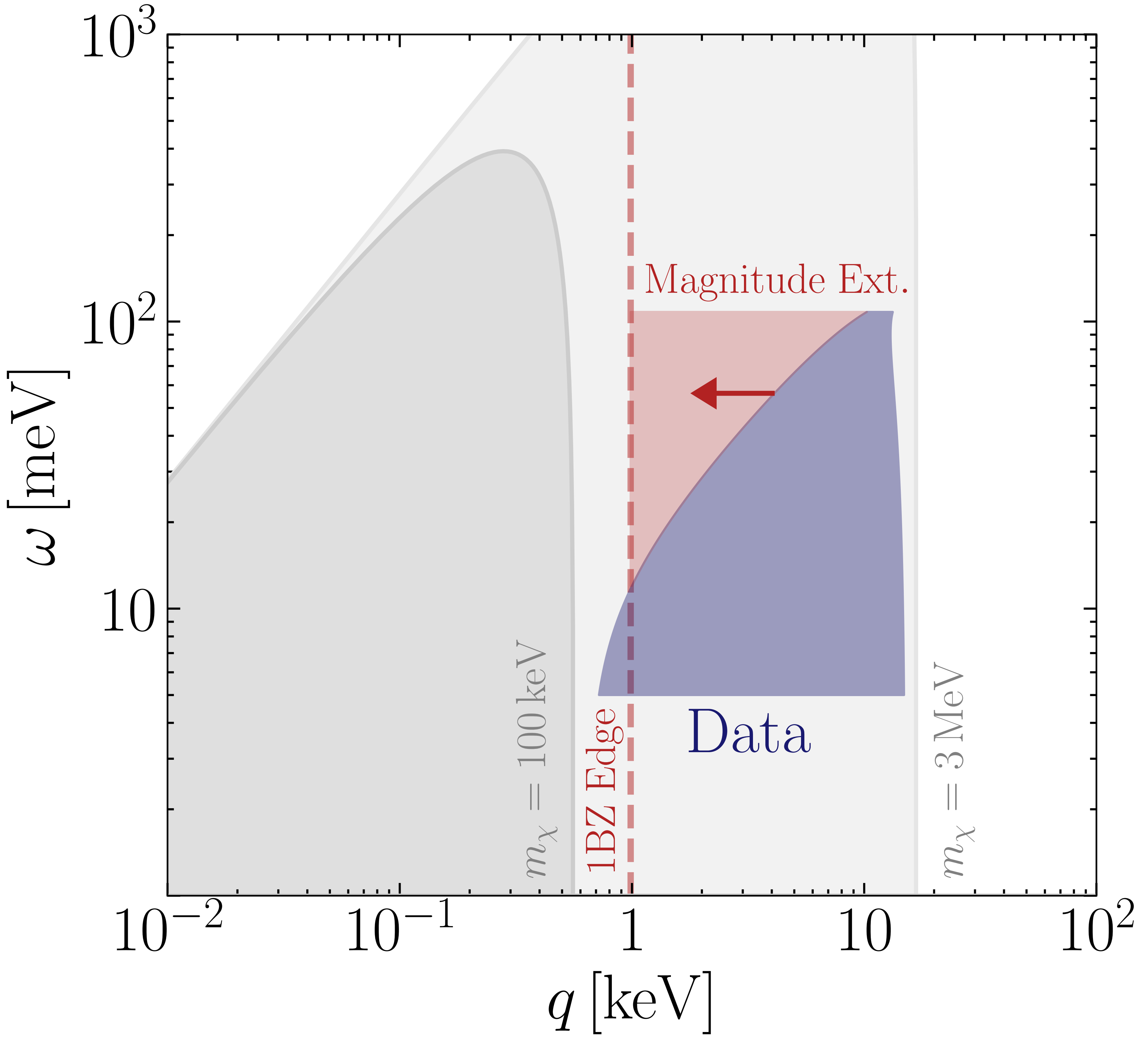
$$S_n(\mathbf{q}, \omega) = \frac{1}{N_{\mathrm{f}}} \frac{\gamma_n^2 e^4}{8\, m_e^2} \frac{\Omega_c}{(2\pi)^3} P_T^{ij} \chi''_{ij}(\mathbf{q}, \omega_{\mathbf{q}})$$

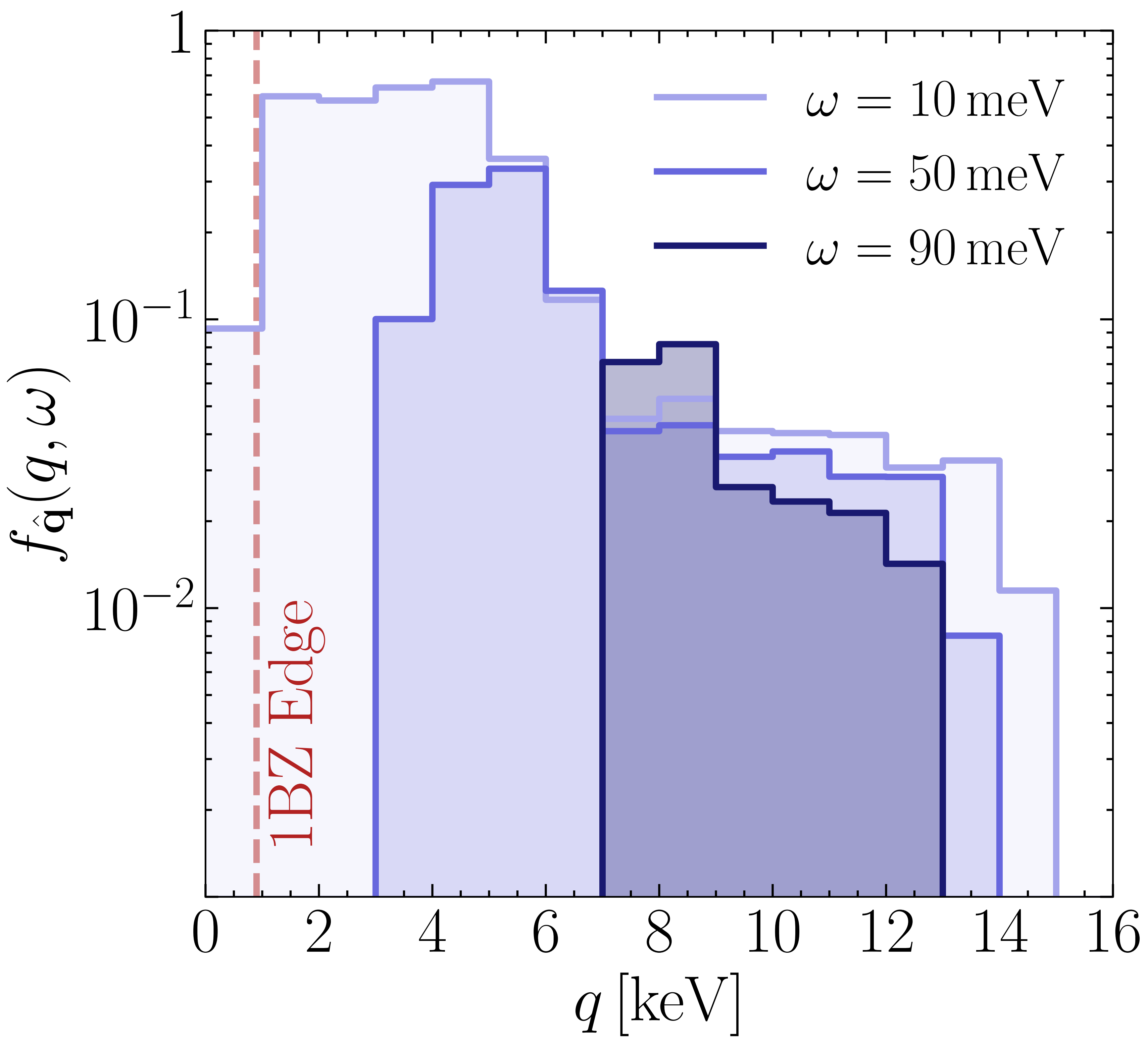
Neutron Scattering





Data Limitations





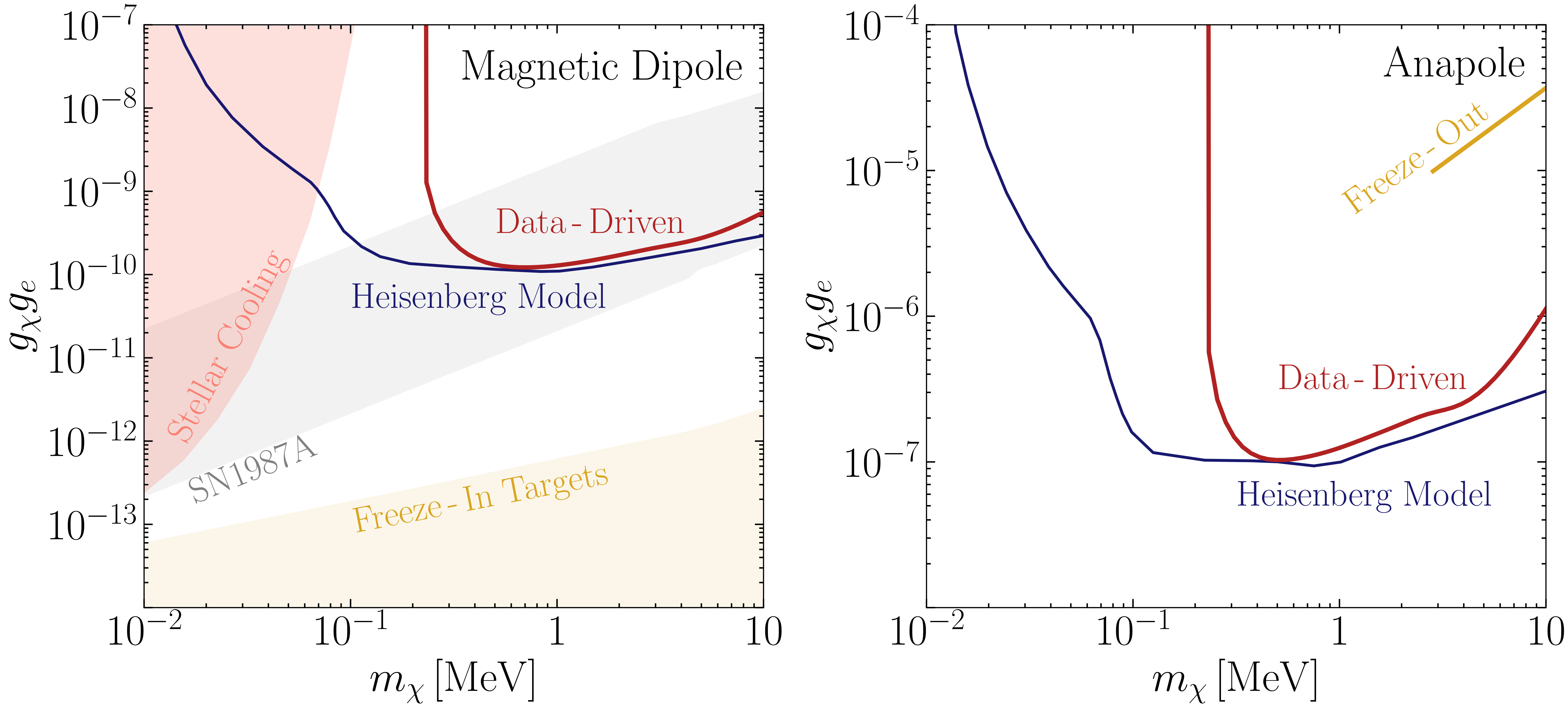
Scattering Rate

- For DM models which only depend on the transverse χ we just get the rate from data (i.e., dipole)

$$R = \frac{\rho_\chi}{2\pi \rho_\mathrm{T}} \frac{g_\chi^2 g_e^2}{m_\chi^3 \gamma_n^2 e^4} \frac{N_\mathrm{f}}{\Omega_c} \int \mathrm{d}\omega \, \mathrm{d}^3\mathbf{q} \, \underbrace{g(\mathbf{q}, \omega)}_{\text{Kinematics}} \boxed{S_n(\mathbf{q}, \omega)}$$

- Longitudinal χ can't be measured without magnetic monopoles, requires additional assumptions

Scattering Rate



Conclusions

Spin dependent couplings still have lots to explore

Absorption and scattering can just be related to ϵ and μ

Magnetized dielectric haloscopes have interesting new phenomenology to explore

Spin dependent dark matter rates can be taken directly from existing experiments

More kinematic coverage needed for more robust rates

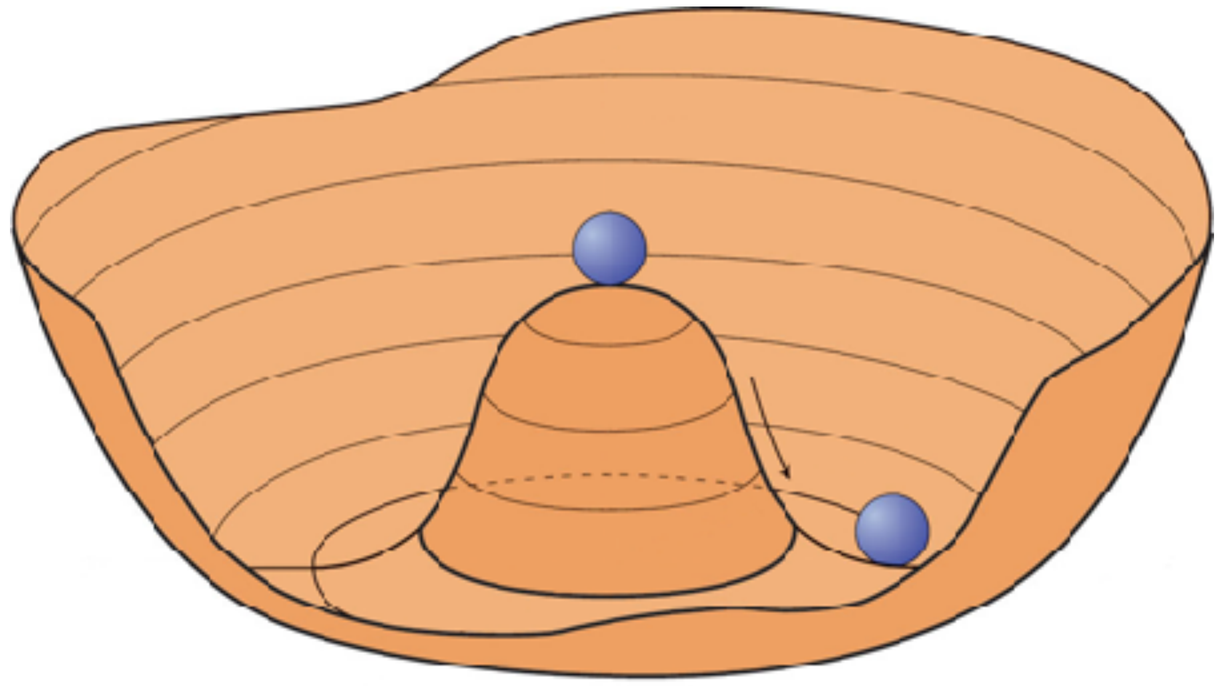
Questions?

Backup Slides

Axions

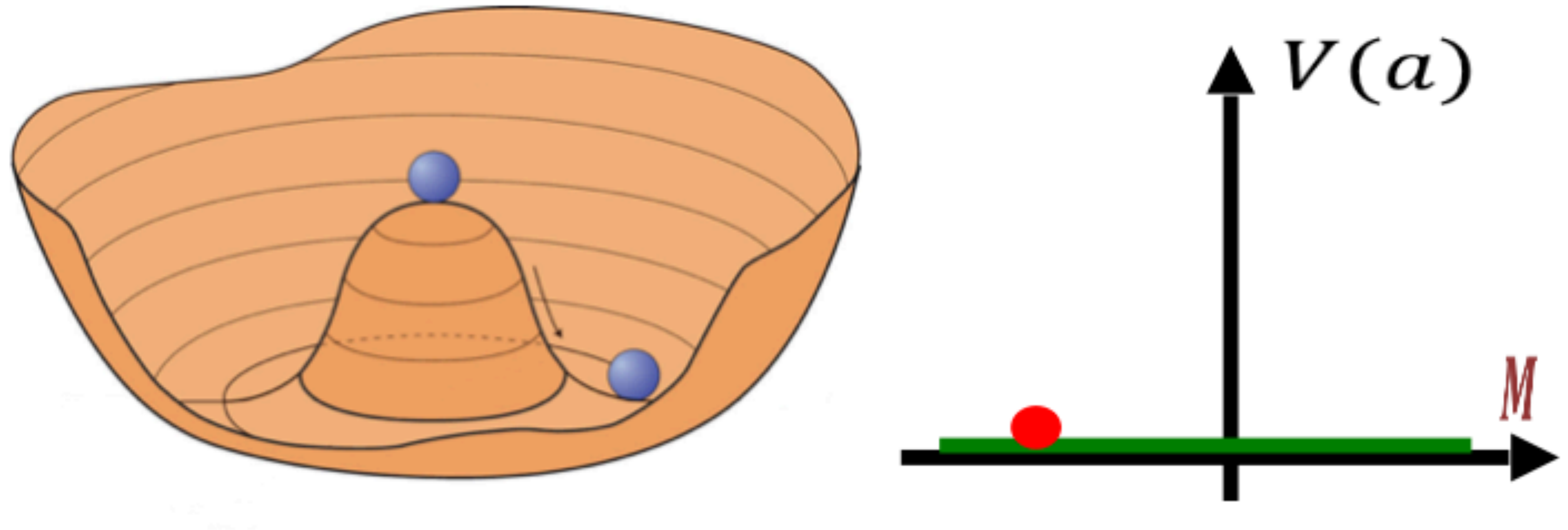
- Solution to the Strong CP problem: make θ a dynamical field so it can minimise the energy and send θ to zero
- Need a new anomalous U(1) chiral symmetry (Peccei-Quinn), which is broken at high temperature ~f_a (around 10^{12} GeV)

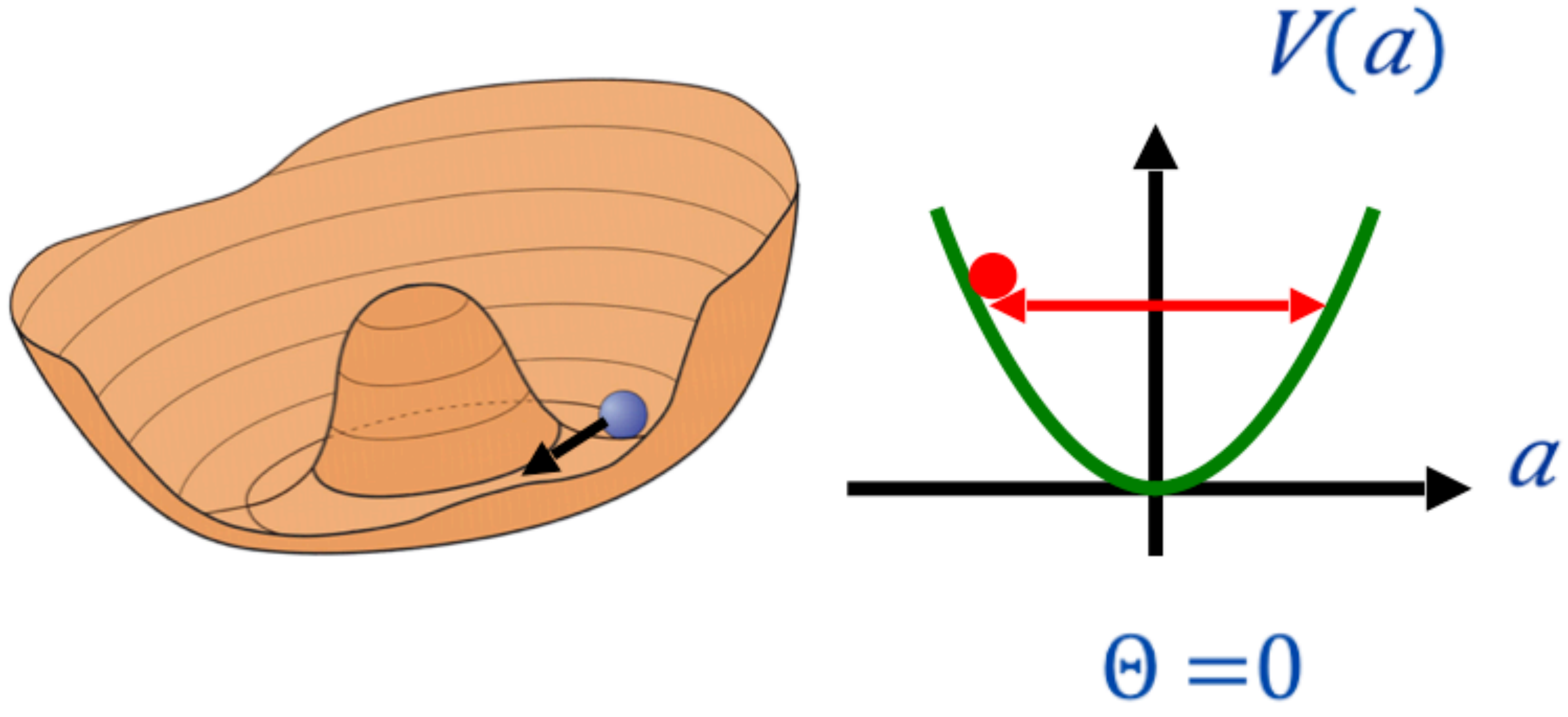
$$\mathcal{L}_{\text{stand mod + axion}} = \ldots + \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{g^2}{32\pi^2}\frac{a(x)}{f_a} G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$$



Axions

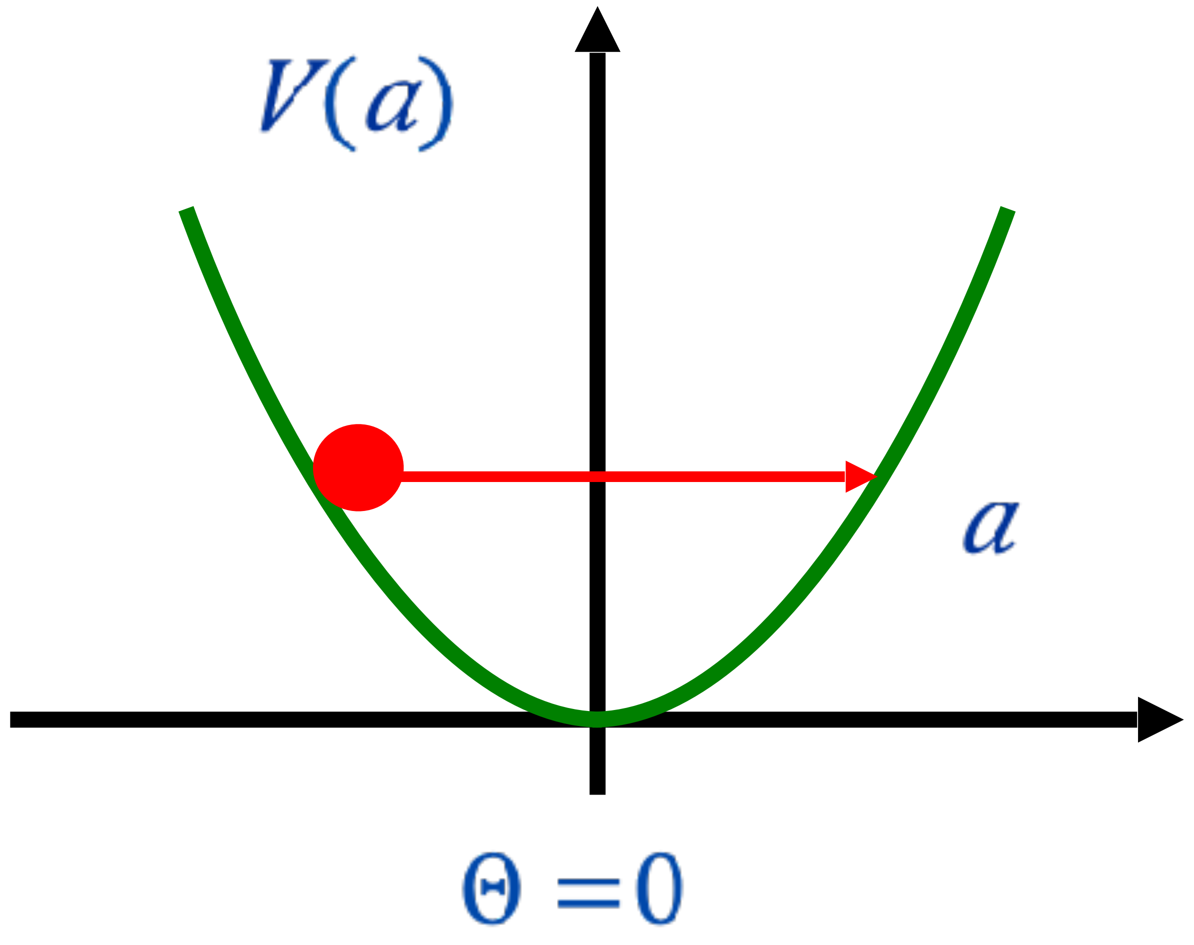
- The "axion" is the angular degree of freedom: goldstone mode!
- At the QCD scale the potential tilts as the axion acquires a mass – axion rolls down to a CP conserving minimum
- Can be produced by misalignment or topological defects



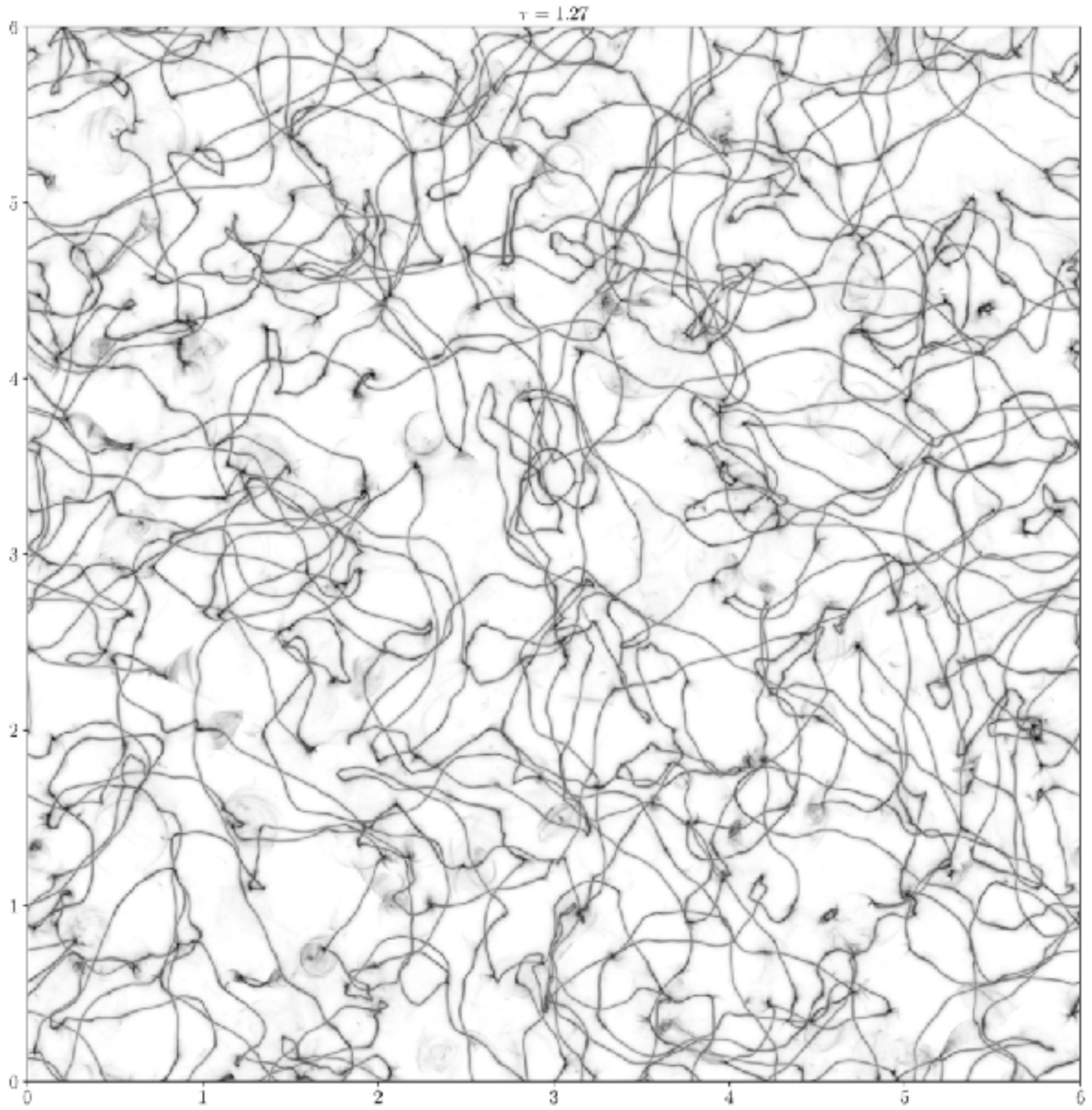


Axion Production Mechanisms

Vacuum Misalignment



Decay of topological defects



arXiv:1809.09241

Axion Production Mechanisms

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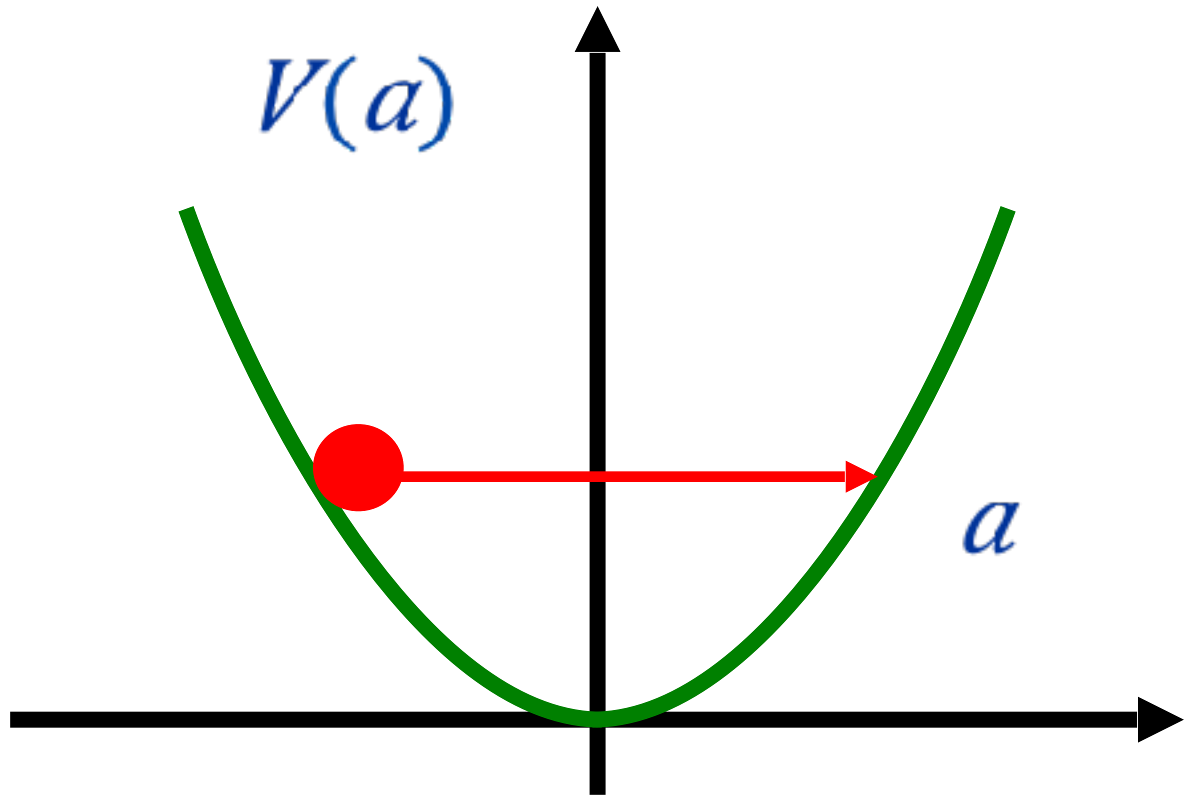
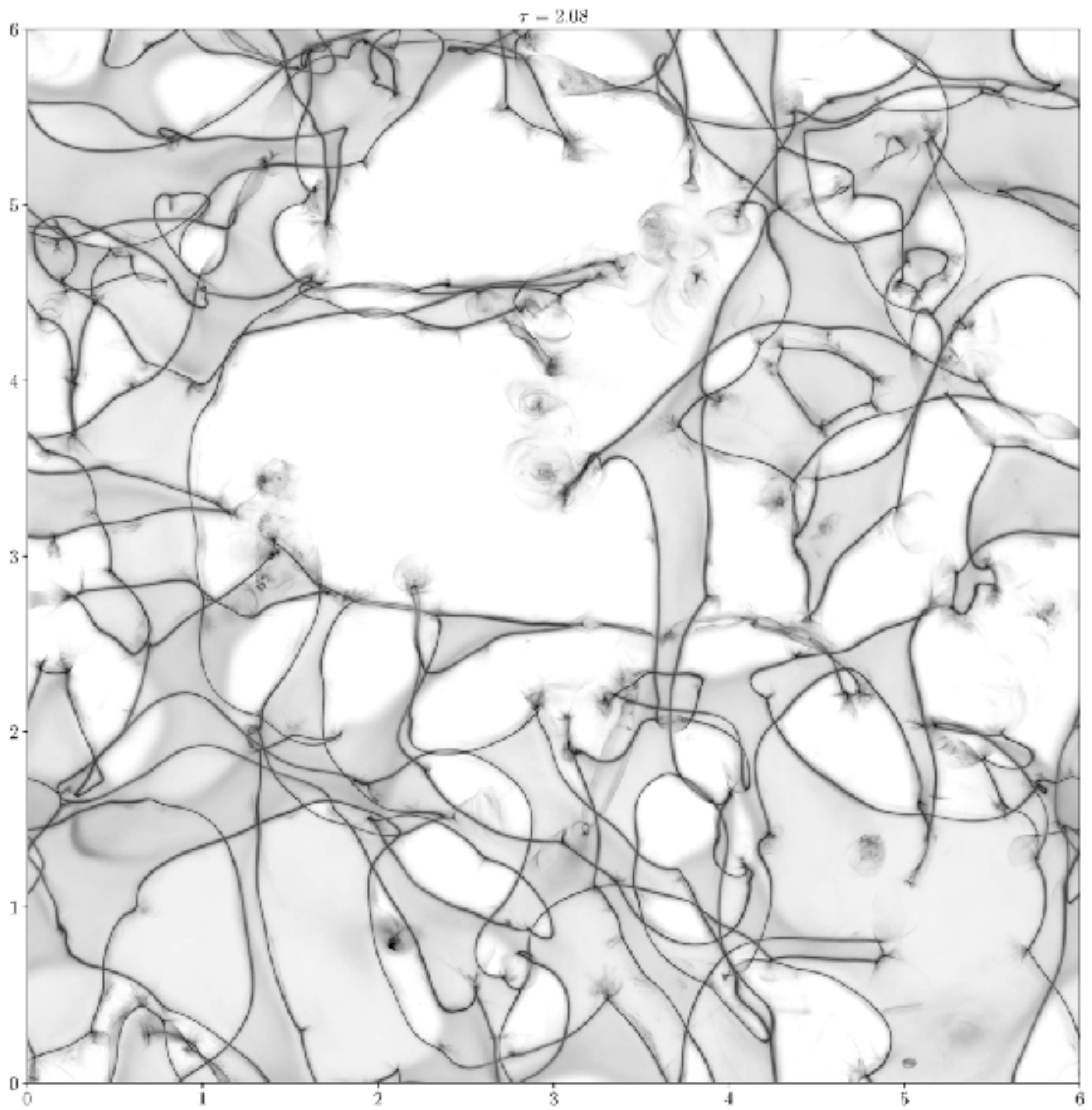
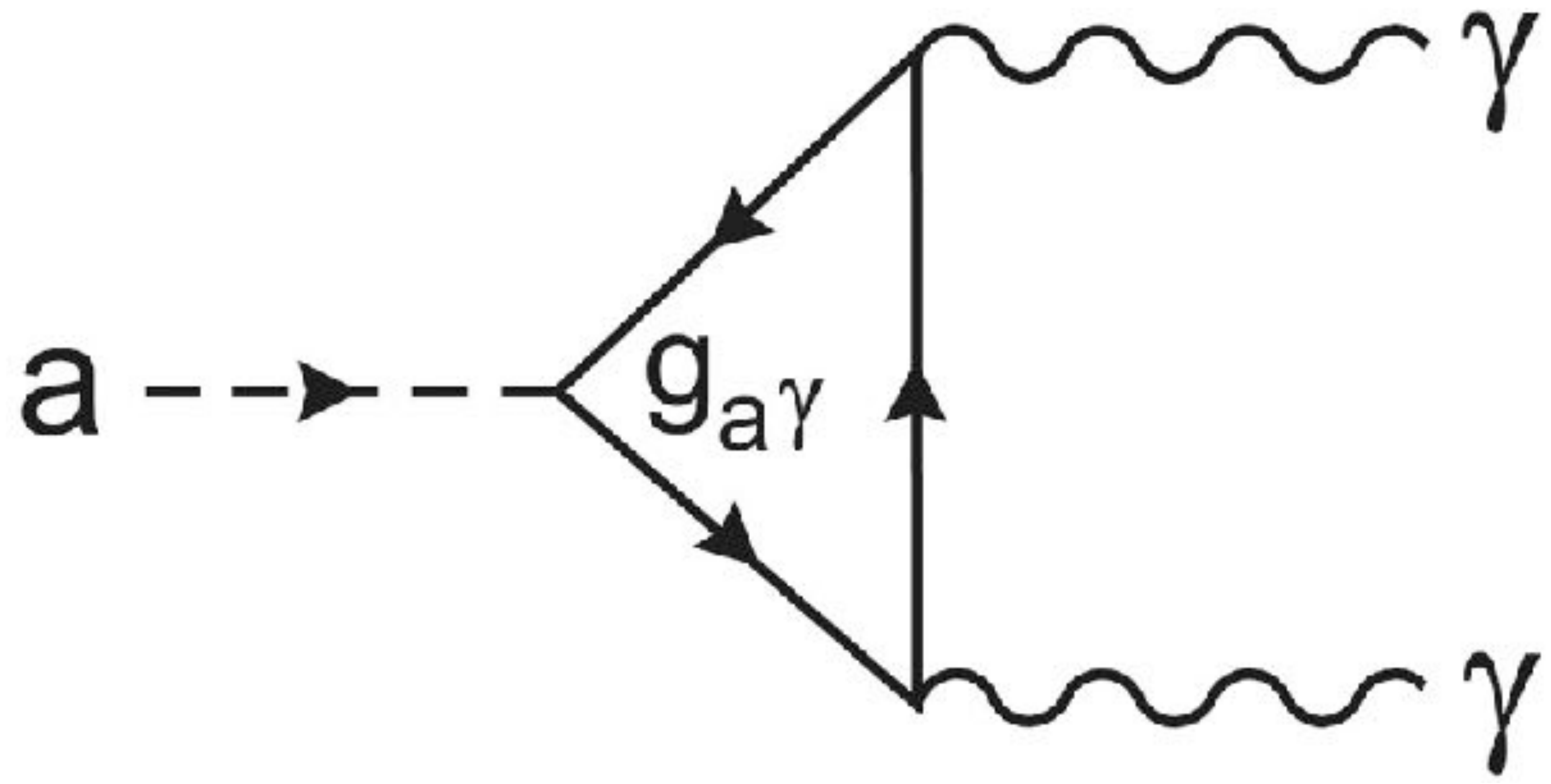


Decay of topological defects



arXiv:1809.09241

Axion-Photon Coupling



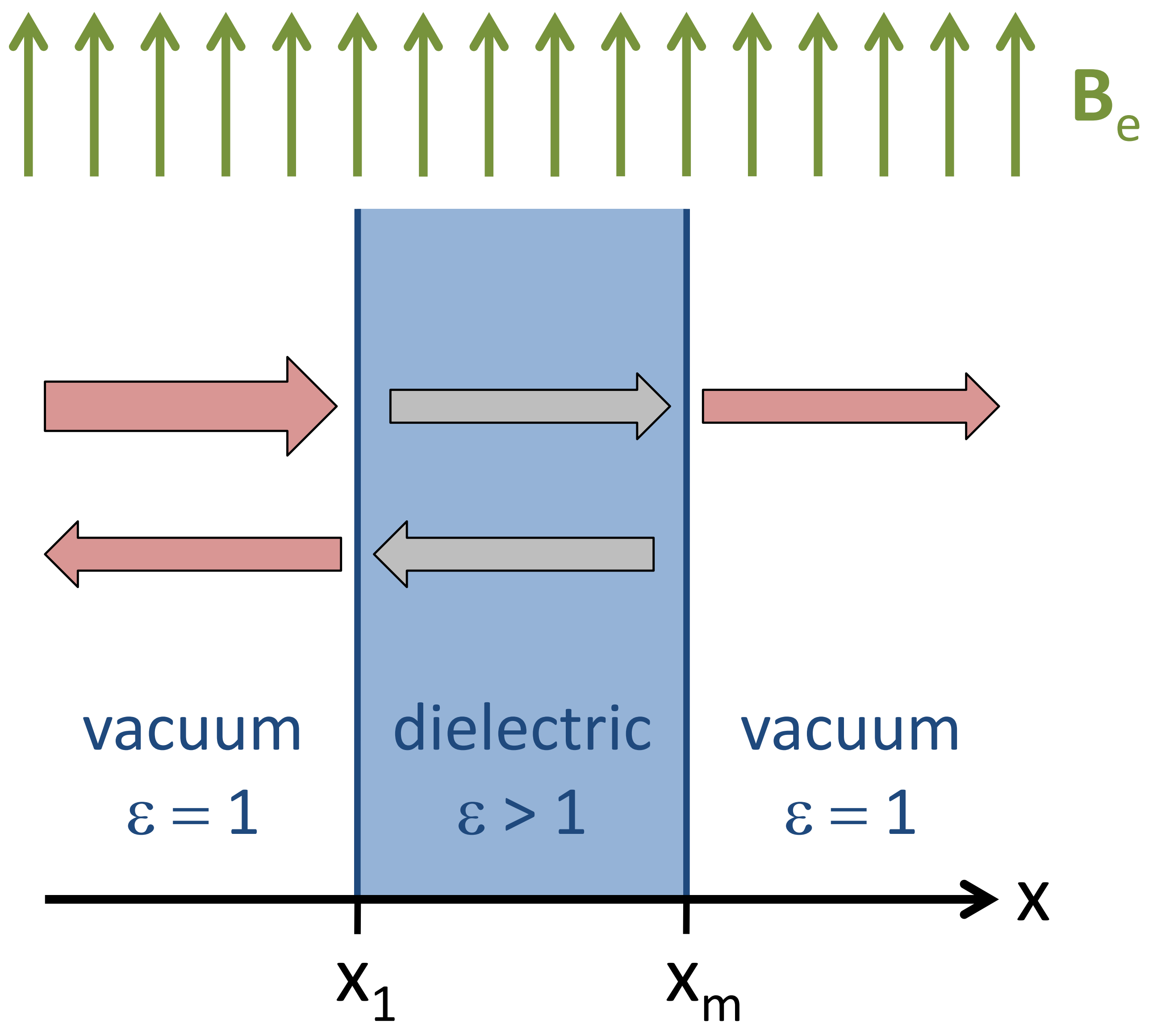
$$
\begin{aligned}
m_a &= 5.70(7)\,\mu\text{eV}\,\frac{10^{12}\text{GeV}}{f_a}\,,\\
g_{a\gamma} &= \frac{\alpha}{2\pi f_a}\,C_{a\gamma} = 2.04(3)\times 10^{-16}\ \text{GeV}^{-1}\,\frac{m_a}{\mu\text{eV}}\,C_{a\gamma}\,,\\
C_{a\gamma} &= \frac{E}{N} - 1.92(4)\,,
\end{aligned}
$$

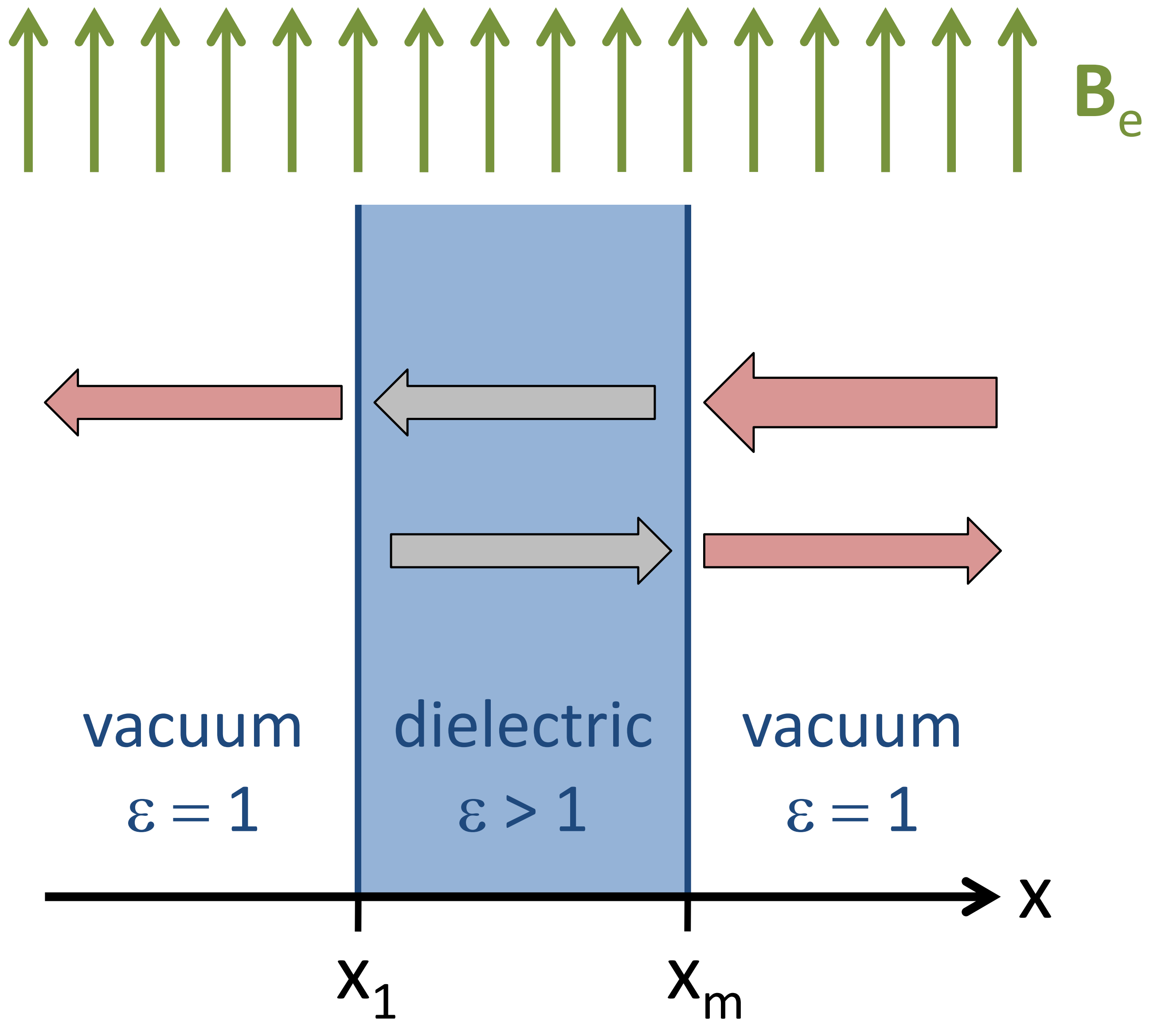
Theoretical formalisms

- **Transfer matrices (classical calculation)**
- All the action is at the interfaces
- Combination of axion and photon field satisfies axion-Maxwell equations: axion-photon wave function
- Solving for the classical E-field everywhere
- **Overlap integral (quantum field calculation)**
- All space is involved
- Axion and photons wave functions treated separately: photon wave function satisfies regular Maxwell equations
- Calculating transition probability

Overlap Integral Formalism

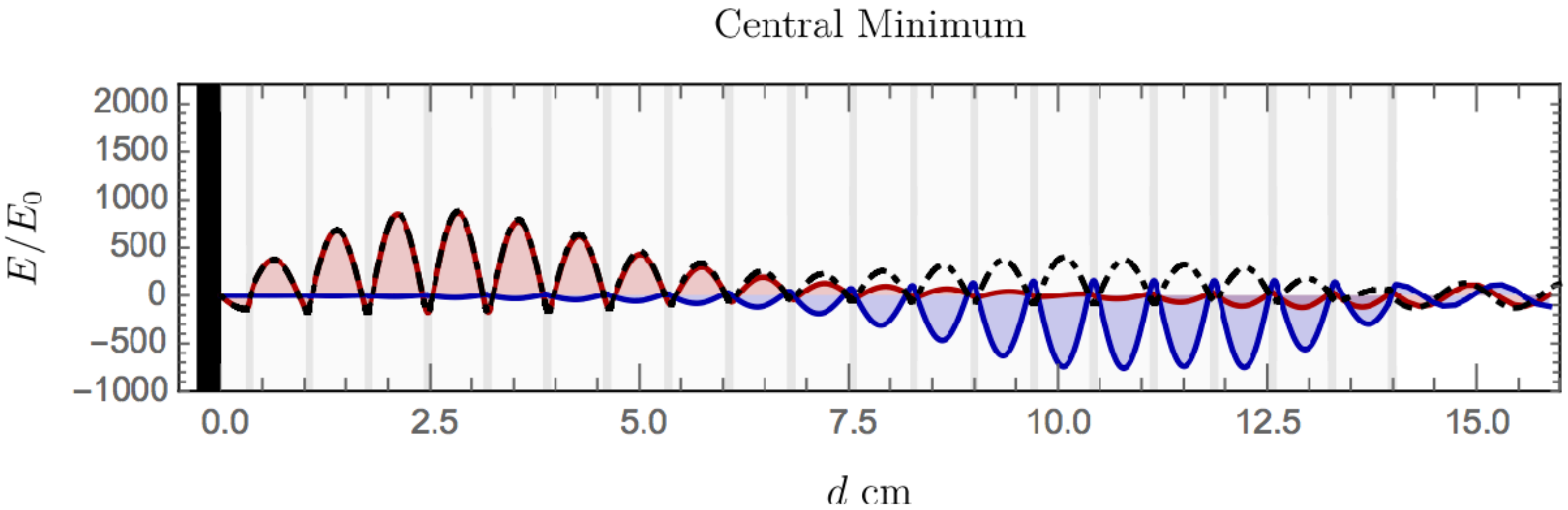
- The main trick is choosing the right free-photon wave functions: Garibian wave functions,





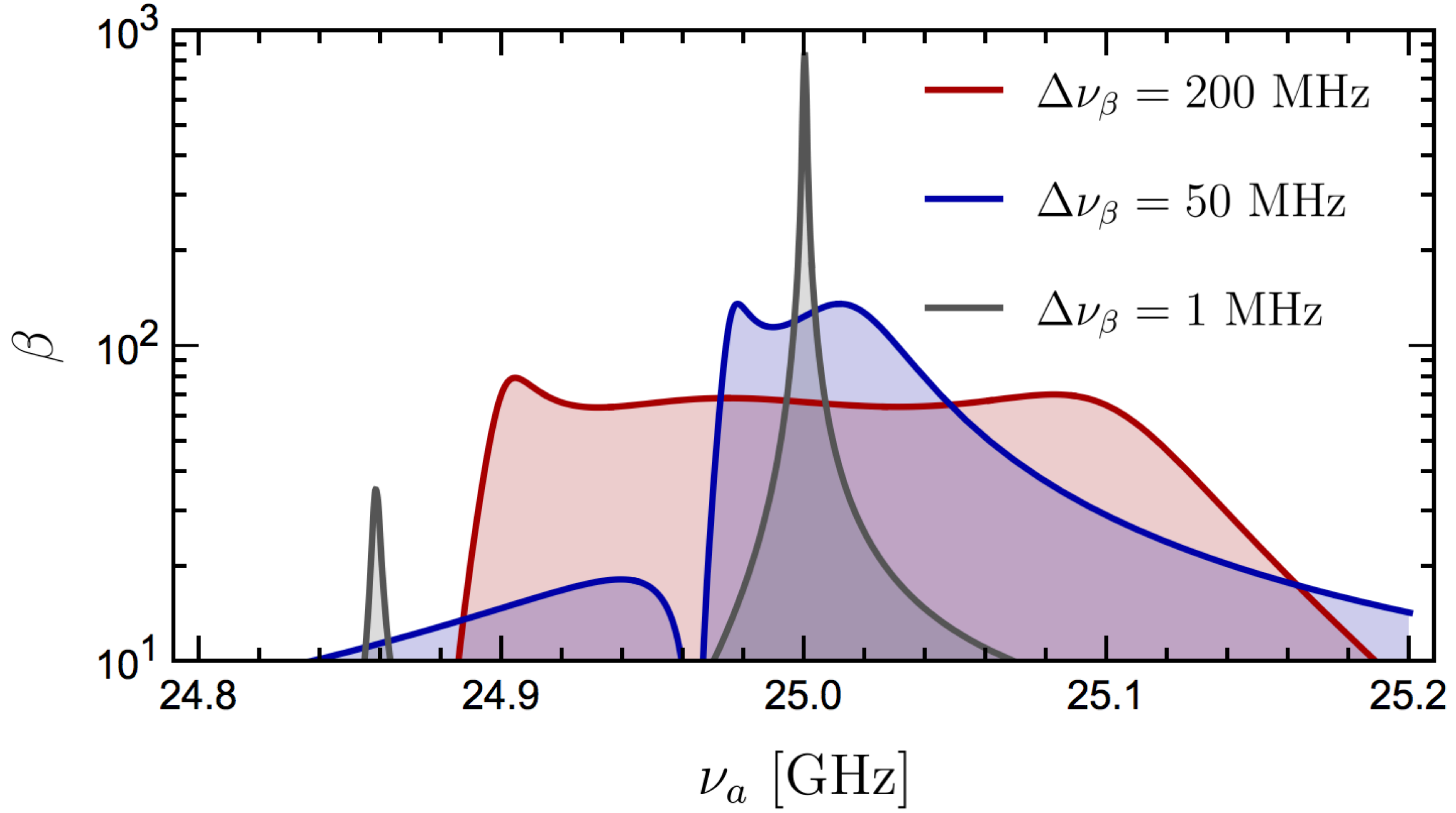
Overlap Integral formalism

- The E-field only encodes boundary conditions: in general it isn't excited

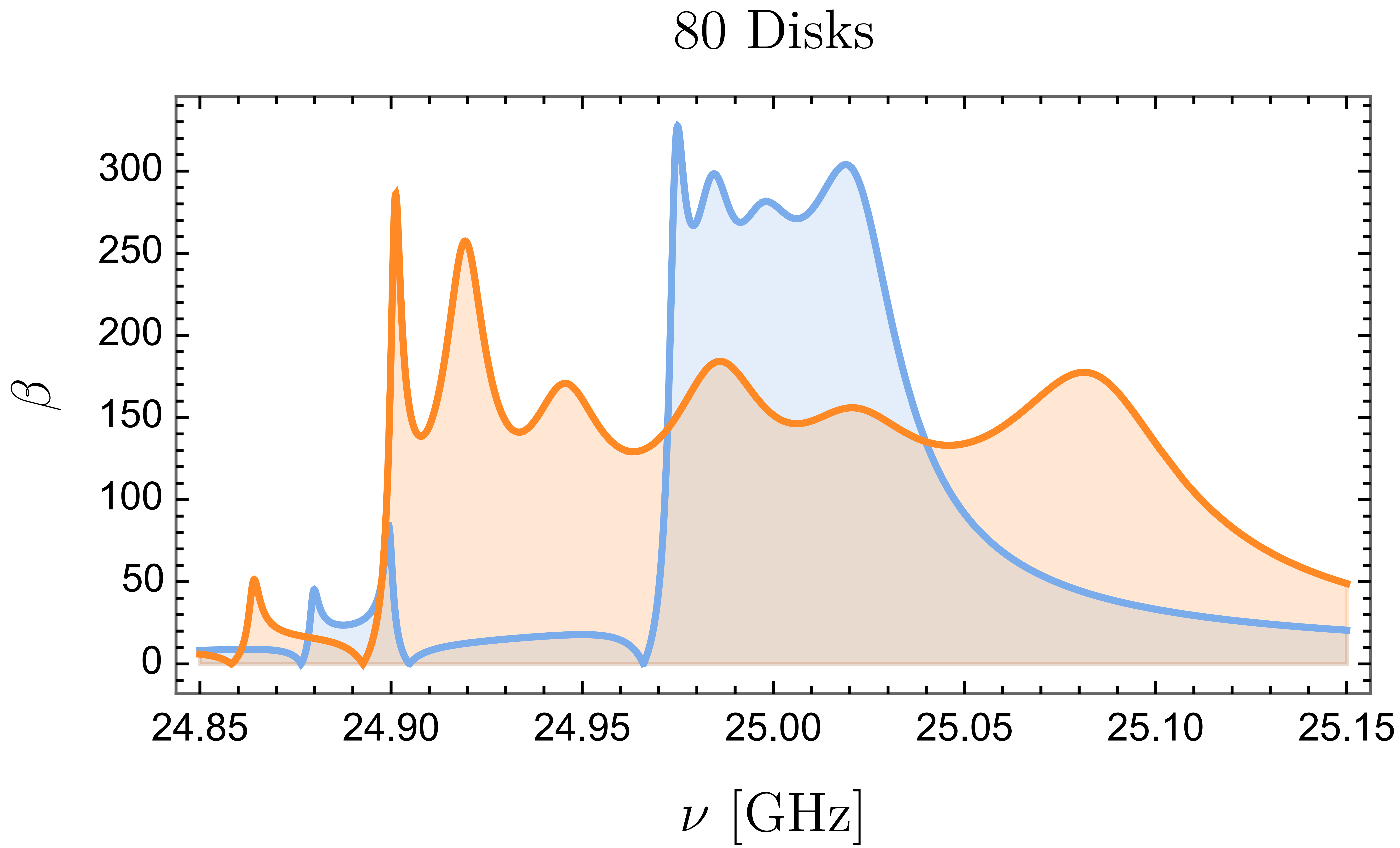


Alex Millar

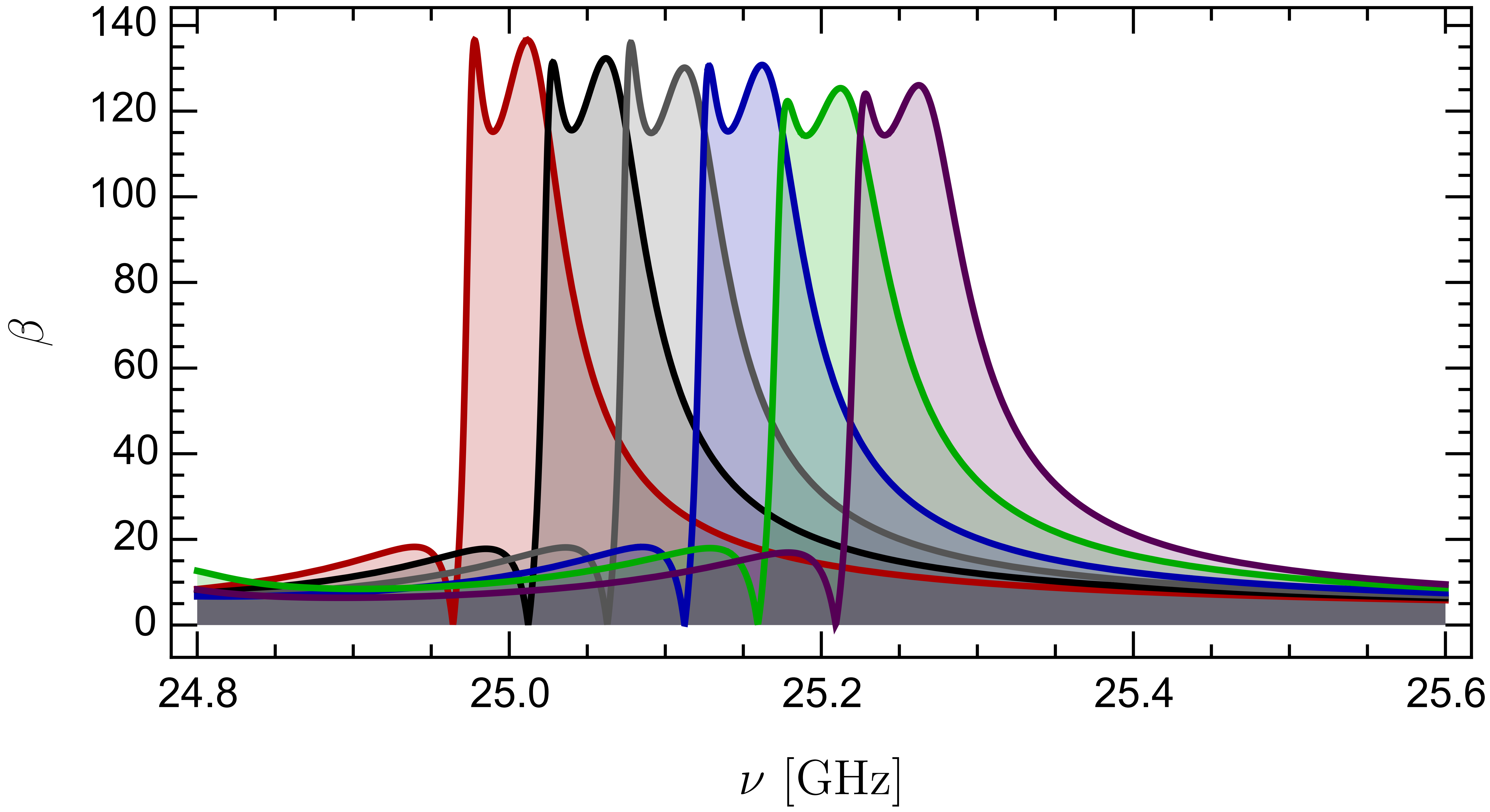
Example Solutions: 20 disks



Optimization



Example Scan



Case Two: Wind

- Full behavior needs a dedicated analysis
- Simple estimate extrapolated from N transparent slabs
- High frequency μ needs an applied B-field (Landau-Liftshitz-Gilbert equation)

$$1 - \mu^{-1} = -\frac{2\mu_B \, M_0 \, \omega_0}{\omega^2 - \omega_M^2 + i\,\omega\,\omega_M / Q_M}$$

$$\omega_0 \equiv 2\mu_B \left(H_0 + \beta M_0\right) \quad , \quad \omega_M \equiv \sqrt{\omega_0 \left(2\mu_B \, M_0 + \omega_0\right)}$$

Projections

- Axio-electric is easy: recast a high frequency haloscope like MuDHI or LAMPOST
- Axion wind is better at lower frequencies
- For the wind term we assume a MADMAX-like setup ignoring O(1) factors and daily modulation

$$\mathrm{SNR} \sim \sqrt{\frac{Q_a}{Q_M}\,\frac{t_e}{m_a}\,\frac{\rho_{\rm DM}\,A}{T_n}}\,N^{3/2}\left(\frac{g_{ae}\,v_{\rm DM}\,\eta}{\mu_B}\right)^2$$

$$\eta = \left|\frac{1-\mu^{-1}}{1+i\,\sqrt{\varepsilon/\mu}\,\cot\left(n\,m_a\,d/2\right)}\right|$$

Spurious EDMs

- Often the axion induced electronic EDM is overestimated (or assumed constant).
- You can do a field redefinition to get

$$\mathscr{L} \supset -2\, m_f\, g_{af}\, a\, \overline{\Psi} i \gamma^5 \Psi .$$

- With non-relativistic Hamiltonian

Looks like EDM

$$H_{\rm alt} \simeq \frac{\boldsymbol{\pi}^2}{2m_f} + q_f\, \phi - \frac{q_f}{2m_f} \mathbf{B} \cdot \boldsymbol{\sigma} - g_{af}\, (\nabla a) \cdot \boldsymbol{\sigma} - \frac{g_{af}}{4m_f} \{\dot{a}, \boldsymbol{\pi} \cdot \boldsymbol{\sigma}\} + \boxed{\frac{q_f\, g_{af}}{2m_f}\, a\, \mathbf{E} \cdot \boldsymbol{\sigma}}$$

Spurius EDMs

- But axion is derivatively coupled: can't have a constant EDM
- Actually the field redefinitions to get the non-relativistic Hamiltonian also redefine the position operator shifting the COM

$$\mathbf{x}_q = \mathbf{x}, \qquad \mathbf{x}'_q = \mathbf{x} + (d/q)\,\boldsymbol{\sigma}$$

- Doesn't reappear at higher order (unlike Schiff's theorem)
- Need to be very careful with non-relativistic derivations
- Actual EDMs are suppressed by $(m_a/m_e)^2$, see arXiv:1312.6667