

Score-based Direct relation btw FM & Diff

recall: $\frac{dx_t}{dt} = u_t(x_t)$

$$x_t = \sigma_t x_0 + \sigma_t \epsilon$$

$$u_t(x_t) = \int dx_0 u_t(x_t|x_0) p_t(x_t|x_0) p_0(x_0)$$

$$u_t(x_t|x_0) = \frac{\partial}{\partial x_t} \log p_t(x_t|x_0) = \frac{\partial}{\partial x_t} \log \frac{1}{(\sigma_t \sigma_0)} e^{-\frac{(x_t - \sigma_t x_0)^2}{2\sigma_t^2}} = \frac{\partial}{\partial x_t} \left(-\log \sigma_t - \log \sigma_0 - \frac{(x_t - \sigma_t x_0)^2}{2\sigma_t^2} \right)$$

$$= \left(-\frac{1}{\sigma_t} \right) + \frac{\partial}{\partial x_t} \left(-\frac{(x_t - \sigma_t x_0)^2}{2\sigma_t^2} \right) = \left(-\frac{1}{\sigma_t} \right) + \frac{\partial}{\partial x_t} \left(-\frac{x_t^2 + \sigma_t^2 x_0^2 - 2x_t \sigma_t x_0}{2\sigma_t^2} \right)$$

$$= \left(-\frac{1}{\sigma_t} \right) + \int dx_0 \left(-\frac{1}{\sigma_t} \right) \nabla_{x_t} e^{-\dots} \frac{p_0(x_0)}{p_t(x_t|x_0)}$$

$$\int dx_0 \left(-\frac{1}{\sigma_t} \right) \frac{p(x_t|x_0) p(x_0)}{p_t(x_t|x_0)} \nabla_{x_t} p_t(x_t|x_0) = 1$$

$$= \left(-\frac{1}{\sigma_t} \right) x_t + \left(-\frac{1}{\sigma_t} \right) x_0 \quad \text{check coeffs are } f(t) \text{ \& } -\frac{1}{2} S(t)^2$$

but CNF more general orab paths

Score based diff & CNF v. base related
→ basically same!

• So score matching & flow matching

$$\|x_0 - u_t(x_t|x_0)\|^2 \text{ vs. } \|x_0 - \frac{\partial}{\partial x_t} \log p_t(x_t|x_0)\|^2$$

just two diff hyperpar choices, ^{equiv} (loss fns)

• Also, can sample diff model deterministically

"Prob Flow DE" (Song 2021) $\boxed{dx_t = \left(f(t) x_t - \frac{1}{2} S(t)^2 x_t \right) dt}$

Aside: flow matching also works b/w samples
of arbitrary dist'n

given $x_0 \sim p_0$ samples at $x_T = t x_0 + (1-t)x_1$
 $x_1 \sim p_1$ interpolating

argmin $\int_{x_0, x_1, t} \|v_\theta(x_t, t) - u_t(x_t | x_0, x_1)\|^2$ has same minimum
as $u_t(x_t)$

"Rectified Flow"

Liu, Gong, Lu 2209.03003



Super trivial training & model setup!!

Demos: torchfm examples

calochallenge ~~📄~~ - Calo Diffusion paper
Clausius summary

PC - Jedi for jet cloud generation

transformer + diffusion

↓
perm eqn ↓
high dim'l discrete