- Generalizations

  - RQS transform → more expressive than affine!

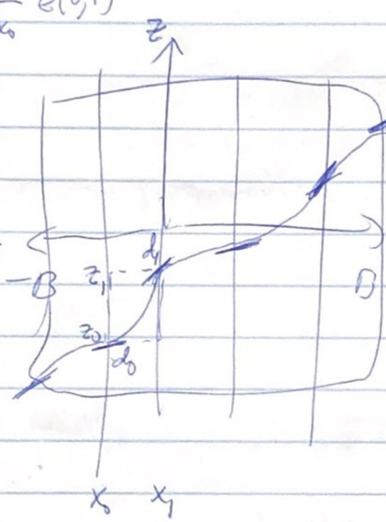
$s = \frac{z_1 - z_0}{x_1 - x_0}$

$\xi = \frac{x - x_0}{x_1 - x_0} \in (0,1)$

$$z = f(x) = z_0 + (z_1 - z_0)\frac{\left(s\xi^2 + d_0\,\xi(1-\xi)\right)}{s + (d_1 + d_0 - 2s)\,\xi(1-\xi)}$$

$\left\{ \begin{array}{l} x_{0,1} \\ z_{0,1} \\ d_{0,1} \end{array} \right.$

left & right bdy & derivatives



Spline
piecewise
continuous & cont. 1st der
monotonic (so invertible)

  - Coupling layers

$$\left\{ \begin{array}{ll} z_1 = x_1 & z_{k+1} = C_{k+1}(x_{k+1}, \mu_{k+1}(x_1, \dots x_k)) \\ \vdots & \vdots \\ z_k = x_k & z_d = C_d(x_d, \mu_d(x_1, \dots x_k)) \end{array} \right.$$

inverse trivial (so sampling & DE equally fast)

can use affine, RQS, ... for C's.

- Sung Hak's 2-moons tutorial
- Calo flow (2106.05285) example

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aka "ABC approx Bayesian computation" & "likelihood-free inference"

Simulation Based Inference

(good ref: Cranmer, Brehmer, Louppe "Frontier of SBI")

Suppose we want to know $p(D|\theta)$ (likelihood) or $p(\theta|D)$ (posterior)

Model θ → Simulation ("forward model") → Data

- Suppose likelihood is unknown (intractable) or slow

  maybe D is too high dimensional
  maybe $p(D|\theta)$ only known for summary statistics (eg χ^2 or Gaussian covariance)

  example Gaussian but v. high dim'l → slow matrix inv.

- Idea of SBI:
  can learn p from samples (θ, D)!

  $\theta \sim p(\theta)$ ← prior
  $D \sim p(D|\theta)$ ← simulation from intractable likelihood

- Different techniques / approaches to SBI
    - Most direct "neural posterior or likelihood estimation"

      train flow on samples, learn $p(D|\theta)$ or $p(\theta|D)$

      can sample from it quickly

      speed up posterior inference by many orders of magnitude (weeks/months $\rightarrow$ seconds!)

    - Neural ratio estimation

      From samples (θ, x)

      produce marginal product $\theta, x \sim p(\theta)p(x)$ (just randomly pair)

      Binary classifier:

      $$\frac{p(\theta,x)}{p(\theta)p(x)} = \frac{p(\theta|x)}{p(\theta)} = \frac{p(x|\theta)}{p(x)}$$

      Given data $x \rightarrow$ get likelihood & posterior up to normalization!

    - pros & cons to both
    - limitations to SBI — only as good as simulation...