Accelerated universe from gravity leaking to extra dimensions

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We discuss the idea that the accelerated universe could be the result of gravitational leakage into extra dimensions over Hubble distances rather than the consequence of a nonzero cosmological constant.

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I. INTRODUCTION

A number of recent observations suggest that the universe is accelerating at large scales [1] (see also [2,3]). This may be regarded as evidence for a nonzero but very small cosmological constant. However, before adopting such a conclusion it is desirable to explore alternative possibilities motivated by particle physics considerations. In this respect the models that predict modification of gravity at large distances are particularly interesting. In the present paper we focus on the five-dimensional brane-world model with an infinite-volume extra dimension, which can predict such a modification at cosmological distances [4,5]. In this model the ordinary particles are localized on a three-dimensional surface (three-brane) embedded in infinite-volume extra space to which gravity can spread. Despite the presence of an infinite-volume flat extra space, the observer on the brane measures four-dimensional Newtonian gravity at distances shorter than a certain crossover scale r_c, which can be of astronomical size [4,5]. This phenomenon is due to a four-dimensional Ricci scalar term that is induced on the brane [4,5]. The whole dynamics of gravity is governed by competition between this term and an ordinary five-dimensional Einstein-Hilbert action. At short distances the four-dimensional term dominates and ensures that gravity looks four dimensional. At larger distances, however, the five-dimensional term takes over and gravity spreads into extra dimensions. As a result, the force law becomes five dimensional. Thus, gravity gets weaker at cosmic distances. It is natural that such a dramatic modification should affect the cosmological expansion of the universe. In the present work we will focus on the explicit cosmological solution found in [6]. This solution describes a universe that is accelerated beyond the crossover scale. The acceleration takes place despite the fact that there is no cosmological constant on the brane. Instead, the bulk gravity experiences its own curvature term on the brane as a cosmological constant and accelerates the universe.

In the present paper we shall review this phenomenon in the light of recent astrophysical observations [1,2,3] and confront this model with the conventional cosmological constant scenario. We shall show that the present scenario cannot be mimicked by ordinary 4D gravity with arbitrary high-derivative terms. Therefore, this is intrinsically a high-dimensional phenomenon. Finally, we argue that such scenarios might avoid the difficulties of reconciliation of string theory with the observation of the accelerated universe. This is possible because the bulk metric in the theory is Minkowskian. Moreover, due to the leakage of gravity into extra space there is no infinite future horizon for 4D observers.

Before we proceed we would like to note that other interesting cosmological solutions in this type of model were first studied in Ref. [7]; however, those solutions do not describe an accelerated universe and will not be discussed here.

II. THE FRAMEWORK

The model we will be considering was introduced in Ref. [4]. We start with a $D=(4+1)$-dimensional theory. Let us suppose that there is a three-brane embedded in five-dimensional space-time.[1]

Four coordinates of our world are x_μ, $\mu=0,1,2,3$; the extra coordinate will be denoted by y. Capital letters and subscripts will be used for 5D quantities $(A,B,C=0,1,2,3,5)$; the metric convention is mostly positive.

Following Refs. [4,5] let us consider the action

$$S=\frac{M_{(5)}^3}{2}\int d^5X\sqrt{|\tilde{g}|}\tilde{R}+\frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{|g|}R, \qquad (1)$$

where $M_{(5)}$ denotes the 5D Planck mass, and $M_{\rm Pl}$ is the 4D Planck mass; as they stand in Eq. (1) $M_{(5)}$ and $M_{\rm Pl}$ are independent parameters (in general they could be related). $\tilde{g}_{AB}(X)\equiv\tilde{g}_{AB}(x,y)$ denotes a 5D metric for which the 5D Ricci scalar is $\tilde{R}$. The brane is located at $y=0$. The induced metric on the brane is denoted by

$$g_{\mu\nu}(x)\equiv\tilde{g}_{\mu\nu}(x,y=0). \qquad (2)$$

The 4D Ricci scalar for $g_{\mu\nu}(x)$ is $R=R(x)$. The standard model (SM) fields are confined to the brane. Note that the

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[1]For simplicity we ignore brane fluctuations, in which case the induced metric on the brane takes the simple form given below in Eq. (2).

SM cutoff should not coincide in general with $M_{(5)}$ and, in fact, is assumed to be much higher in our case. For simplicity we suppress the Lagrangian of the SM fields. The brane-world origin of the action (1) and parameters $M_{(5)}, M_{\rm Pl}$ were discussed in details in Refs. [4,5,8].

Let us first study the nonrelativistic potential between two sources confined to the brane. For the time being we drop the tensorial structure in the gravitational equations and discuss the distance dependence of the potential. We comment on the tensorial structure at the end of this section.

The static gravitational potential between the sources in the four-dimensional world volume of the brane is determined as

$$V(r)=\int G_R(t,\vec{x},y=0;0,0,0)\,dt, \tag{3}$$

where $r\equiv\sqrt{x_1^2+x_2^2+x_3^2}$ and $G_R(t,\vec{x},y=0;0,0,0)$ is the retarded Green's function (see below). Let us turn to Fourier-transformed quantities with respect to the world-volume four-coordinates x_μ:

$$G_R(x,y;0,0)\equiv\int\frac{d^4p}{(2\pi)^4}e^{ipx}\tilde{G}_R(p,y). \tag{4}$$

In Euclidean momentum space the equation for the Green's function takes the form

$$[M_{(5)}^3(p^2-\partial_y^2)+M_{\rm Pl}^2p^2\delta(y)]\tilde{G}_R(p,y)=\delta(y). \tag{5}$$

Here p^2 denotes the square of a Euclidean four-momentum $p^2\equiv p_4^2+p_1^2+p_2^2+p_3^2$. The solution with appropriate boundary conditions takes the form

$$\tilde{G}_R(p,y)=\frac{1}{M_{\rm Pl}^2p^2+2M_{(5)}^3p}\exp(-p|y|), \tag{6}$$

where $p\equiv\sqrt{p^2}=\sqrt{p_4^2+p_1^2+p_2^2+p_3^2}$. Using this expression and Eq. (3) one finds the following (properly normalized) formula for the potential:

$$V(r)=-\frac{1}{8\pi^2M_{\rm Pl}^2}\frac{1}{r}\left\{\sin\left(\frac{r}{r_c}\right)\mathrm{Ci}\left(\frac{r}{r_c}\right)+\frac{1}{2}\cos\left(\frac{r}{r_c}\right)\times\left[\pi-2\,\mathrm{Si}\left(\frac{r}{r_c}\right)\right]\right\}, \tag{7}$$

where

$$\mathrm{Ci}(z)\equiv\gamma+\ln(z)+\int_0^z[\cos(t)-1]dt/t,$$

$\mathrm{Si}(z)\equiv\int_0^z\sin(t)dt/t$, $\gamma\simeq0.577$ is the Euler-Mascheroni constant, and the distance scale r_c is defined as follows:

$$r_c\equiv\frac{M_{\rm Pl}^2}{2M_{(5)}^3}. \tag{8}$$

In our model we choose r_c to be of the order of the present Hubble size, which is equivalent to the choice $M_{(5)}\sim10-100$ MeV. We will discuss the phenomenological compatibility of such a low quantum gravity scale in Sec. VI. It is useful to study the short distance and long distance behavior of this expression.

At short distances when $r\ll r_c$ we find

$$V(r)\simeq-\frac{1}{8\pi^2M_{\rm Pl}^2}\frac{1}{r}\left\{\frac{\pi}{2}+\left[-1+\gamma+\ln\left(\frac{r}{r_c}\right)\right]\left(\frac{r}{r_c}\right)+\mathcal{O}(r^2)\right\}. \tag{9}$$

Therefore, at short distances the potential has the correct 4D Newtonian $1/r$ scaling. This is subsequently modified by the logarithmic *repulsion* term in Eq. (9).

Let us turn now to the large distance behavior. Using Eq. (7) we obtain, for $r\gg r_c$,

$$V(r)\simeq-\frac{1}{8\pi^2M_{\rm Pl}^2}\frac{1}{r}\left\{\frac{r_c}{r}+\mathcal{O}\left(\frac{1}{r^2}\right)\right\}. \tag{10}$$

Thus, the long distance potential scales as $1/r^2$ in accordance with the laws of 5D theory.

We would like to emphasize that the behavior (6) is intrinsically higher dimensional and is very hard to reproduce in conventional four-dimensional field theory. Indeed, the would-be four-dimensional inverse propagator should contain the term $\sqrt{p^2}$. In the position space this would correspond in the Lagrangian to the following pseudodifferential operator:

$$\hat{\mathcal{O}}=-\partial_\mu^2+\frac{\sqrt{-\partial_\mu^2}}{r_c}. \tag{11}$$

We are not aware of a consistent four-dimensional quantum field theory with a finite number of physical bosons which would lead to such an effective action.

Finally, we would like to comment on the tensorial structure of the graviton propagator in the present model. In flat space this structure is similar to that of a massive 4D graviton [4]. This points to the van Dam–Veltman–Zakharov (vDVZ) discontinuity [9,10]. However, this problem can in general be resolved by at least two methods. In the present context one has to use the results of [11] where it was argued that the vDVZ discontinuity that emerges in the lowest perturbative approximation is in fact absent in the full nonperturbative theory. The application of similar arguments to our model leads to a result that is continuous in $1/r_c$. This will be discussed in detail elsewhere [12]. Thus, the vDVZ problem is an artifact of using the lowest perturbative approximation.[2]

[2]Note that the continuity in the graviton mass in (A)dS backgrounds was demonstrated recently in Refs. [13,14]. We should emphasize that we are discussing the continuity in the classical 4D gravitational interactions on the brane. There is certainly a discontinuity in the full theory in the sense that there are extra degrees of freedom in the model. These latter can manifest themselves at quantum level in loop diagrams [15].

In general, the simplest possibility to deal with the vDVZ problem, as was suggested in Ref. [8], is to compactify the extra space at scales bigger than the Hubble size with r_c being even bigger, but we do not consider this possibility here.

III. COSMOLOGICAL SOLUTIONS

Below we will mainly be interested in the geometry of the 4D brane world. For completeness of the presentation let us first recall the full 5D metric of the cosmological solution. The 5D line element is taken in the following form:

$$ds^2 = -N^2(t,y)dt^2 + A^2(t,y)\gamma_{ij}dx^i dx^j + B^2(t,y)dy^2, \tag{12}$$

where γ_{ij} is the metric of a three-dimensional maximally symmetric Euclidean space, and the metric coefficients read [6]

$$N(t,y) = 1 + \epsilon|y|\ddot{a}(\dot{a}^2+k)^{-1/2},$$

$$A(t,y) = a + \epsilon|y|(\dot{a}^2+k)^{1/2},$$

$$B(t,y) = 1, \tag{13}$$

where $a(t)$ is a 4D scale factor and $\epsilon = \pm 1$. Knowing the brane-world intrinsic geometry is all that matters as far as 4D observers are concerned. This geometry is given in the above solution. Taking the $y=0$ value of the metric we obtain the usual 4D Friedmann-Lemaître-Robertson-Walker (FLRW) form (enabling us to interpret t as the cosmic time on the brane world)

$$ds^2 = -dt^2 + a^2(t)dx^i dx^j \gamma_{ij}, \tag{14}$$

$$= -dt^2 + a^2(t)[dr^2 + S_k^2(r)d\psi^2] \tag{15}$$

where $d\psi^2$ is an angular line element, $k=-1,0,1$ parametrizes the brane-world spatial curvature, and S_k is given by

$$S_k(r) = \begin{cases} \sin r & (k=1), \\ \sinh r & (k=-1), \\ r & (k=0). \end{cases} \tag{16}$$

In the present case, the dynamics is generically different from the usual FLRW cosmology, as shown in [6]. The standard first Friedmann equation is replaced in our model by

$$H^2 + \frac{k}{a^2} = \left(\sqrt{\rho/3M_{\rm Pl}^2 + 1/4r_c^2} + \epsilon\frac{1}{2r_c}\right)^2, \tag{17}$$

where ρ is the total cosmic fluid energy density. We have in addition the usual equation of conservation for the energy-momentum tensor of the cosmic fluid given by

$$\dot{\rho} + 3H(p+\rho) = 0. \tag{18}$$

Equations (17) and (18) are sufficient to derive the cosmology of our model. In particular, using these relations one can obtain a second Friedmann equation as in standard cosmology.

Equation (17) with $\epsilon=1$ and $\rho=0$ has an interesting self-inflationary solution with a Hubble parameter given by the inverse of the crossover scale r_c. This can be easily understood by looking back at the action (1), where it is apparent that the intrinsic curvature term on the brane appears as a source for the bulk gravity, so that with appropriate initial conditions this term can cause an expansion of the brane world without the need of matter or a cosmological constant on the brane. This self-inflationary solution is the key ingredient for our model to produce late time accelerated expansion.[3] Before discussing this issue in detail let us first compare our cosmology with the standard one.

We first note that the standard cosmological evolution is recovered from Eq. (17) whenever $\rho/M_{\rm Pl}^2$ is large compared to $1/r_c^2$, so that the early time cosmology of our model is analogous to standard cosmology. In this early phase Eq. (17) reduces, at leading order, to the standard 4D Friedmann equation given by

$$H^2 + \frac{k}{a^2} = \frac{\rho}{3M_{\rm Pl}^2}. \tag{19}$$

The late time behavior, however, is generically different, as was shown in [6]: when the energy density decreases and crosses the threshold $M_{\rm Pl}^2/r_c^2$, one has a transition either to a pure 5D regime (see, e.g., [16,17]) where the Hubble parameter is linear in the energy density ρ (this happens for the $\epsilon=-1$ branch of the solutions), or to the self-inflationary solution mentioned above (when $\epsilon=+1$). This latter is the case we would like to investigate in more detail in the rest of this work, and we set $\epsilon=+1$ from now on. In terms of the Hubble radius (and for the flat universe) the crossover between the two regimes happens when the Hubble radius H^{-1} is of the order of the crossover length scale between 4D and 5D gravity, that is, r_c. If we do not want to spoil the successes of the ordinary cosmology, we thus have to assume that r_c is of the order of the present Hubble scale H_0^{-1}. With such a value chosen for r_c, the expansion of the universe is governed at first order by the standard Friedmann equations (18) and (19) whenever $H \gg H_0$, and deviates from standard evolution only recently in cosmic history. In particular, this means that big-bang nucleosynthesis and recombination proceed in the usual way in our scenario.

The conservation equation (18) is the same as the standard one, so that a given component of the cosmic fluid (nonrelativistic matter, radiation, cosmological constant, etc.) will have the same dependence on the scale factor as in standard cosmology. For instance, for a given component, labeled by α, which has the equation of state $p_\alpha = w_\alpha \rho_\alpha$ (with

[3]Note that the nonzero 4D Ricci scalar on the brane makes a seemingly negative contribution to the brane tension [18,6]. In this case, we consider a nonfluctuating brane which is placed at the $\mathbf{R}/\mathbf{Z}_2$ orbifold fixed point.

w_α being a constant), one gets from Eq. (18) $\rho_\alpha = \rho_\alpha^0 a^{-3(1+w_\alpha)}$ (with ρ_α^0 being a constant). The Friedmann equation (17) can be rewritten in terms of the redshift $1+z \equiv a_0/a$ as follows:

$$H^2(z) = H_0^2\left\{ \Omega_k(1+z)^2 + \left(\sqrt{\Omega_{r_c}} + \sqrt{\Omega_{r_c} + \sum_\alpha \Omega_\alpha (1+z)^{3(1+w_\alpha)}} \right)^2 \right\}, \quad (20)$$

where the sum is over all the components of the cosmic fluid. In the above equation Ω_α is defined as follows:

$$\Omega_\alpha \equiv \frac{\rho_\alpha^0}{3M_{\rm Pl}^2 H_0^2 a_0^{3(1+w_\alpha)}}, \quad (21)$$

while Ω_k is given by

$$\Omega_k \equiv \frac{-k}{H_0^2 a_0^2} \quad (22)$$

and Ω_{r_c} denotes

$$\Omega_{r_c} \equiv \frac{1}{4r_c^2 H_0^2}. \quad (23)$$

In the rest of this paper, as far as the cosmology of our model is concerned, we will consider a nonrelativistic matter with density Ω_M, in which case Eq. (20) reads[4]

$$H^2(z) = H_0^2\{\Omega_k(1+z)^2 + [\sqrt{\Omega_{r_c}} + \sqrt{\Omega_{r_c} + \Omega_M(1+z)^3}]^2\}. \quad (24)$$

We can compare this equation with the conventional Friedmann equation

$$H^2(z) = H_0^2\{\Omega_k(1+z)^2 + \Omega_M(1+z)^3 + \Omega_X(1+z)^{3(1+w_X)}\}. \quad (25)$$

Here, in addition to the matter and curvature contributions we have included the density of a dark energy component Ω_X with equation of state parameter w_X. When $w_X = -1$, the dark energy acts in the same way as a cosmological constant, and the corresponding Ω_X will be denoted as Ω_Λ in the following. Comparing Eqs. (24) and (25) we see that Ω_{r_c} acts similarly (but not identically, as we will see below) to a cosmological constant.

The $z=0$ value of Eq. (24) leads to the normalization condition

$$\Omega_k + (\sqrt{\Omega_{r_c}} + \sqrt{\Omega_{r_c} + \Omega_M})^2 = 1, \quad (26)$$

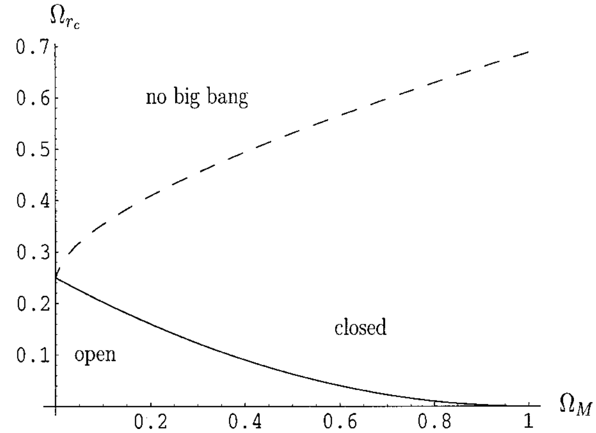

FIG. 1. Different possibilities for expansion as a function of Ω_M and Ω_{r_c}. The solid line denotes a flat universe ($k=0$), with Ω_{r_c} obtained through Eq. (28). The universes above the solid line are closed ($k=1$), the universes below are open ($k=-1$). The universes above the dashed line avoid the big-bang singularity by bouncing in the past.

which differs from the conventional relation

$$\Omega_k + \Omega_M + \Omega_X = 1. \quad (27)$$

For a flat universe ($\Omega_k = 0$) we get from Eq. (26)

$$\Omega_{r_c} = \left(\frac{1-\Omega_M}{2}\right)^2 \quad \text{and} \quad \Omega_{r_c} < 1. \quad (28)$$

This shows in particular that for a flat universe Ω_{r_c} is always smaller than Ω_X; nevertheless, as will be seen below, the effects of Ω_{r_c} and Ω_X can be quite similar. Figure 1 shows the different possibilities for expansion as a function of Ω_M and Ω_{r_c}.

IV. COSMOLOGICAL TESTS

We would like to discuss now, in a qualitative way, a few cosmological tests and measurements. We do not expect that the current experimental precision will enable us to discriminate between the predictions of our model and those of standard cosmology. However, future measurements might enable one to do so.

In order to compare the outcome of our model with various cosmological tests we need first to summarize some results. In the FLRW metric (14), we define, as usual (see, e.g., [19]), the transverse, H_0-independent (dimensionless), comoving distance d_M:

$$d_M = \begin{cases} \dfrac{s_k(\sqrt{|\Omega_k|}d_C)}{\sqrt{|\Omega_k|}} & \text{if } \Omega_k \neq 0, \\ d_C, & \text{if } \Omega_k = 0, \end{cases} \quad (29)$$

where d_C is defined as follows:

$$d_C = \int_0^z H_0 \frac{dx}{H(x)}. \quad (30)$$

[4]Notice that we have set the cosmological constant on the brane to zero, and will do so until the end of this work since we are interested here in producing an accelerated universe without cosmological constant.

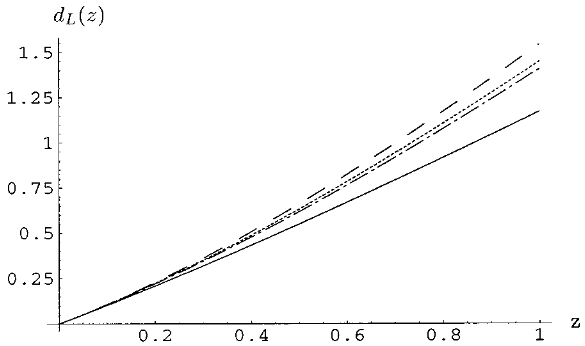

FIG. 2. Luminosity distance as a function of redshift for ordinary cosmology with $\Omega_\Lambda=0.7$, $\Omega_M=0.3$, $k=0$ (dashed line), $\Omega_\Lambda=0$, $\Omega_M=1$, $k=0$ (solid line), with dark energy with $\Omega_X=0.7$, $w_X=-0.6$, $\Omega_M=0.3$, $k=0$ (dot-dashed line), and in our model (dotted line) with $\Omega_M=0.3$ and a flat universe [for which one gets from Eq. (28) $\Omega_{r_c}=0.12$ and $r_c=1.4H_0^{-1}$].

From the expression for d_M one gets the (H_0-independent and dimensionless) luminosity distance d_L and the (H_0-independent) angular diameter distance d_A given by

$$d_L=(1+z)d_M, \tag{31}$$

$$d_A=\frac{d_M}{1+z}. \tag{32}$$

These definitions can be used on the same footing both in standard and in our cosmological scenarios (as they stand above, they rely only on the geometry of the four-dimensional universe experienced by the radiation, which is the same in both cases). The only difference is due to the expression for $H(z)$ which enters the definition of d_C; one should choose either Eq. (25) or Eq. (24) depending on the case considered. Whenever we want to distinguish between the two models, we will put a tilde on the quantities corresponding to our model (e.g., $\tilde{d}_L$).

A. Supernova observations

The evidence for an accelerated universe coming from supernovae observation relies primarily on the measurement of the apparent magnitude of type Ia supernovae as a function of redshift. The apparent magnitude m of a given supernova is a function of its absolute magnitude $\mathcal{M}$, the Hubble constant H_0, and $d_L(z)$ (see, e.g., [20]). Considering the supernovae as standard candles, $\mathcal{M}$ is the same for all supernovae and so is H_0; thus, we need only compare $d_L(z)$ in our model with that in standard cosmology. Figure 2 shows the luminosity distance d_L as a function of redshift in standard cosmology (for zero and nonzero cosmological constant) and in our model. This shows the expected behavior: our model mimics the cosmological constant in producing the late time accelerated expansion. However, as is also apparent from this plot, for the same flat spatial geometry and the same amount of nonrelativistic matter, our model does not produce exactly the same acceleration as a standard cosmological constant, but rather mimics the one obtained from a dark energy component with $w_X>-1$ (or more properly with a z-dependent w_X; see below).

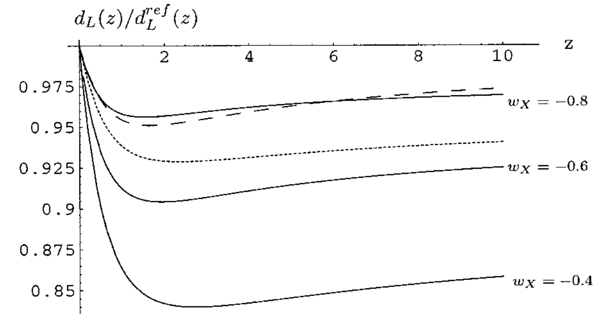

FIG. 3. Plot of $d_L(z)/d_L^{\mathrm{ref}}(z)$ for various models of dark energy with constant equation of state parameters w_X in standard cosmology (solid lines) as compared with the outcome of the model considered in this paper (dashed and dotted lines). All plots correspond to flat universes with $\Omega_M=0.3$ (solid lines and dotted line), and $\Omega_M=0.27$ (dashed line).

B. Comparison with dark energy

We want to compare here the predictions of our model to those of standard cosmology with a dark energy component.

Let us first do so for a dark energy component of constant w_X. For this purpose we choose a reference standard model given by standard cosmology with the parameters $\Omega_\Lambda=0.7$, $\Omega_M=0.3$, and $k=0$ (and denote the associated quantities with the superscript "ref," e.g., d_L^{ref}). Figures 3 and 4 show, respectively, the luminosity distance $d_L(z)$ and $d_C(z)H(z)$ (Alcock-Paczynski test, see, e.g., [21]) for various cases.

One can also mimic the acceleration produced in our model by allowing a z-dependent equation of state parameter $w_X^{\mathrm{eff}}(z)$ in the standard Friedmann equations. Indeed, the standard Friemann equation (19) with a dark energy component of a z-dependent equation of state parameter $w_X^{\mathrm{eff}}(z)$ reads as follows:

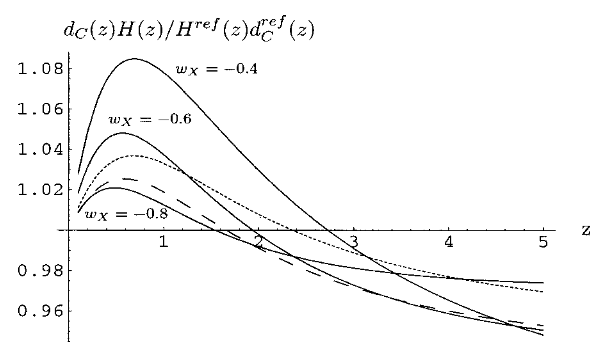

FIG. 4. Plot of $H(z)d_C(z)/H^{\mathrm{ref}}(z)d_C^{\mathrm{ref}}(z)$ (Alcock-Paczynski test) for various models of dark energy with constant equation of state parameters w_X in standard cosmology (solid lines) as compared with the outcome of the model considered in this paper (dashed and dotted lines). All plots correspond to flat universes, with $\Omega_M=0.3$ (solid lines and dotted line), and $\Omega_M=0.27$ (dashed line).

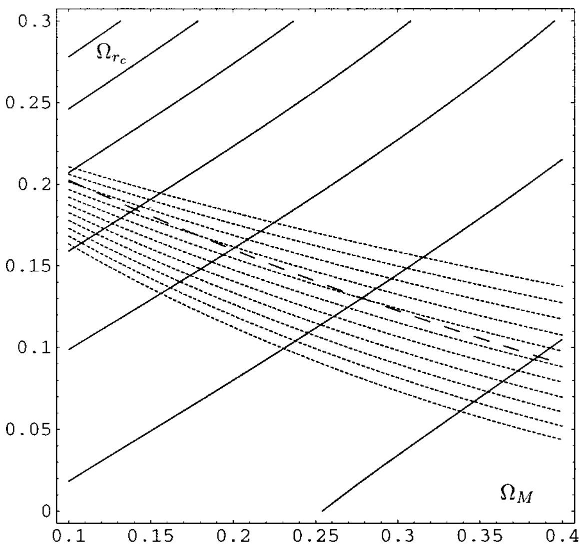

FIG. 5. Solid lines are lines of equal luminosity distance (in our model), $\tilde{d}_L(z=1)/d_L^{\rm ref}(z=1)$, at redshift $z=1$; the contours are drawn at every 5% level. The dashed line corresponds to a flat universe. The dotted lines are the lines of equal $\sqrt{\Omega_M}\tilde{d}_A(z)$ for $z=1100$; the contours are drawn at every 5% level.

$$H^2(z)/H_0^2=\Omega_k(1+z)^2+\Omega_M(1+z)^3 + \Omega_X \exp\left(3\int_0^z [1+w_X^{\rm eff}(y)]\,\frac{dy}{1+y}\right). \quad (33)$$

Requiring that $H(z)$ in this expression equals $H(z)$ in Eq. (24), one finds the following formula for $w_X^{\rm eff}(z)$:

$$w_X^{\rm eff}(z)=\{[\sqrt{4\Omega_{r_c}/\Omega_M(1+z)^3+4}][\sqrt{\Omega_{r_c}/\Omega_M(1+z)^3} + \sqrt{\Omega_{r_c}/\Omega_M(1+z)^3+1}]\}^{-1}-1, \quad (34)$$

with Ω_{r_c}, Ω_M, and Ω_k subject to the constraint (26). At large redshifts $w_X^{\rm eff}$ tends toward $-1/2$, reflecting the fact that the dominant term in Eq. (24) (after matter and curvature terms) redshifts as $(1+z)^{3/2}$ at large z. At low z, however, $w_X^{\rm eff}$ decreases toward an (Ω_k,Ω_M)-dependent asymptotic value. For a flat universe the latter is simply given[5] by $-1/(1+\Omega_M)$.

The preceding discussion shows also that with precision tests one should be able to discriminate between our model and a 4D scenario with a pure cosmological constant.

C. Cosmic microwave background

It is well known that in standard cosmology the location of points of constant luminosity distance at small z is degenerate in the plane $(\Omega_M,\Omega_\Lambda)$. This degeneracy can be lifted through cosmic microwave background (CMB) observations. Figure 5 shows that this is the case in our model as well

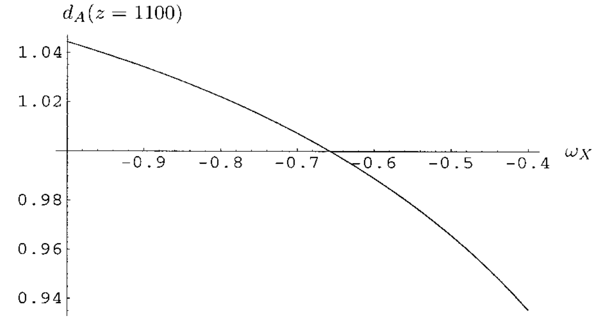

FIG. 6. Angular diameter distance d_A at $z=1100$ of standard cosmology divided by $\tilde{d}_A(z=1100)$ in our model, as a function of w_X for a flat universe and $\Omega_M=0.3$.

(which should not be too much of a surprise, considering the similarities between early cosmology in the two models, as well as between the luminosity distance vs redshift relations). The solid lines of Fig. 5 are lines of constant $\tilde{d}_L$ at redshift $z=1$; the dotted lines are lines of constant $\sqrt{\Omega_M}d_A$ at redshift $z=1100$. This latter quantity roughly sets the position of the first acoustic peak in the CMB power spectrum, since its inverse measures the angular size on the sky of a physical length scale at last scattering proportional to $1/\sqrt{\Omega_M}$ (as is at first approximation the sound horizon at last scattering). Finally, Fig. 6 shows the angular diameter distance d_A, at $z=1100$, of standard cosmology, divided by $\tilde{d}_A$ in our model, as a function of w_X, for a flat universe and $\Omega_M=0.3$. This shows that, for the same content of matter (and a flat universe), the first Doppler peak in our model will be slightly displaced toward the small multipoles in comparison with the one obtained in standard cosmology with a pure cosmological constant.

V. CONFRONTING UNCONVENTIONAL 4D THEORIES OF GRAVITY

One might wonder whether it is possible to obtain a similar cosmological scenario in purely four-dimensional theory by introducing additional generally covariant terms in the Einstein-Hilbert action. The conventional local terms that can be added to the 4D theory contain higher derivatives:

$$M_{\rm Pl}^2\sqrt{g}\left(R+\alpha\,\frac{R^2}{M_{\rm Pl}^2}+\cdots\right). \quad (35)$$

Whatever the origin of these terms might be, their contributions should be suppressed at distances bigger than a millimeter. That is required by existing precision gravitational measurements. This implies that at distances of the present Hubble size their contributions are even more suppressed. For instance, the requirement that the contribution of the R^2 term to the Newtonian interaction be subdominant at distances around a centimeter implies that the relative contribution of the R^2 term at the Hubble scale is suppressed by the factor $({\rm cm}^2 H_0^2)\sim 10^{-56}$. The contributions of other higher terms are suppressed even more strongly.

[5]For $\Omega_M=0.3$ and $k=0$, $w_X^{\rm eff}$ at low z tends towards -0.77 explaining some features of Figs. 3 and 4.

It seems that the only way to accommodate this unusual behavior in a would-be pure 4D theory of gravity is to introduce terms with fractional powers of the Ricci scalar, for instance, the term $\sqrt{gR}$. However, it is hard to make sense of such a theory.

Therefore, we conclude that the scenario discussed in the previous sections is intrinsically a high-dimensional one.

VI. CONSTRAINTS

In our framework such a low five-dimensional Planck scale is compatible with all the observations [8]. In fact, at distances smaller than the present horizon size the brane observer effectively sees a single 4D graviton which is coupled with the strength $1/M_{\rm Pl}$ (instead of a 5D graviton coupled by the strength $1/M_{(5)}^{3/2}$).

As shown in [8] the high-energy processes place essentially no constraint on the scale $M_{(5)}$. This can be understood in two equivalent ways, either directly in a five-dimensional picture, or in terms of an expansion in 4D modes.

As was shown above, in five-dimensional language the brane observer at high energies sees a graviton that is indistinguishable from the four-dimensional one; for short distances the propagator of this graviton is that of a 4D theory:

$$\tilde{G}_R(p,y=0)\propto\frac{1}{p^2}. \tag{36}$$

Moreover, this state couples to matter with the strength $1/M_{\rm Pl}^2$. Therefore in all the processes with typical momentum $p\ll 1/r_c$ the graviton production must proceed just as in 4D theory. For instance, the rate of graviton production in a process with energy E scales as

$$\Gamma\sim\frac{E^3}{M_{\rm Pl}^2}. \tag{37}$$

The alternative language is that of mode expansion. From the point of view of the four-dimensional brane observer a single five-dimensional massless graviton is in fact a continuum of four-dimensional states, with masses labeled by a parameter m:

$$G_{\mu\nu}(x,y)=\int dm\ \phi_m(y)h_{\mu\nu}^{(m)}(x). \tag{38}$$

The crucial point is that the wave functions of the massive modes are suppressed on the brane as follows:

$$|\phi_m(y=0)|^2\propto\frac{1}{4+m^2r_c^2}. \tag{39}$$

This is due to the intrinsic curvature term on the brane which "repels" heavy modes off the brane [8,22]. As a result their production in high-energy processes on the brane is very difficult. Let us once again consider bulk graviton production in a process with energy E (e.g., star cooling via graviton emission at temperature T of order E). This rate is given by [8]

$$\Gamma\sim\frac{E^3}{M_{(5)}^3}\int_0^{m_{\rm max}}dm|\phi_m(0)|^2. \tag{40}$$

Here the integration goes over the continuum of bulk states up to the maximum possible mass that can be produced in a given process, $m_{\rm max}\sim E$. However, since heavier wave functions are suppressed on the brane by a factor $1/m^2r_c^2$, the integral is effectively cut off at $m\sim 1/r_c$, which gives for the rate

$$\Gamma\sim\frac{E^3}{M_{(5)}^3r_c}\sim\frac{E^3}{M_{\rm Pl}^2}. \tag{41}$$

This is in agreement with Eq. (37) and in fact coincides with the rate of production of a single four-dimensional graviton, which is totally negligible. Thus high-energy processes place no constraint on the scale $M_{(5)}$ [8].

For the same reason cosmology places no bound on the scale $M_{(5)}$. Indeed, the potential danger would come from the fact that the early universe may cool via graviton emission in the bulk, which could affect the expansion rate and cause deviation from an ordinary FLRW cosmology. However, due to extraordinarily suppressed graviton emission at high temperatures, the cooling rate due to this process is totally negligible. Indeed, in the radiation-dominated era, the cooling rate due to graviton emission is

$$\Gamma\sim\frac{T^3}{M_{\rm Pl}^2}. \tag{42}$$

At any temperature below $M_{\rm Pl}$ this is much smaller than the expansion rate of the universe $H\sim T^2/M_{\rm Pl}$. Thus essentially until $H\sim M_{(5)}^3/M_{\rm Pl}^2$ (which takes place only in the present epoch) the universe evolves as "normal."

The only constraint in such a case comes from the measurement of the Newtonian force, which implies $M_{(5)} > 10^{-3}$ eV (this will be discussed in more detail elsewhere).

VII. DETERIORATION DUE TO DISSIPATION

In the previous sections we established that classically the asymptotic form of the 4D metric on the brane is that of de Sitter space. Here we would like to ask the question whether this asymptotic form can be modified due to quantum effects. This could happen if there is dissipation of the energy stored in the expectation value of the 4D Ricci scalar into other forms which can either radiate into the bulk or be redshifted away on the brane. Below we shall identify such a mechanism of potential dissipation.

An observer in de Sitter space is submerged in a thermal bath with nonzero temperature due to Hawking radiation from the de Sitter horizon. The temperature of this radiation is $T\sim H$. The crucial point is that the energy stored in this radiation can dissipate into the bulk in the form of very long wavelength graviton emission from the brane. To estimate the rate of this dissipation we can use Eq. (42) with $T\sim H$. The corresponding change of the brane energy density in the absence of other forms of matter and radiation is given by

$$\frac{d\rho_{\rm eff}}{dt} = -\frac{H^3}{M_{\rm Pl}^2}\rho_{\rm eff}, \tag{43}$$

where $\rho_{\rm eff} \equiv M_{\rm Pl}^2 \langle R \rangle$ and the Hubble parameter can be written as $H^2 \propto \langle R \rangle$. The corresponding decay time is huge, $\tau \sim 10^{137}$ sec. Therefore, the 4D metric eventually asymptotes to flat Minkowski space. Note the crucial difference from the conventional 4D de Sitter space, where the vacuum energy cannot dissipate anywhere due to the Hawking radiation. In our case the existence of the infinite-volume bulk is vital.

VIII. INFINITE VOLUME AND STRING THEORY

If the recent observations on the cosmological constant are confirmed it may be extremely nontrivial to describe the accelerated universe within string theory [23,24]. To briefly summarize the concerns let us consider a generic theory with extra dimensions. Usually one is looking for a ground state of the theory with compactified or warped extra dimensions. In both of these cases there is a length scale that defines the volume of the extra space. This scale cannot be bigger than a millimeter [25]. Therefore, at larger distances a conventional four-dimensional space is recovered. Astrophysical observations indicate that this latter asymptotes to a state of four-dimensional accelerated expansion similar to 4D de Sitter space. In this case the following two problems may emerge [23,24].

An observer in dS space sees a finite portion of the space bounded by an event horizon. In fact, the four-dimensional dS interval can be transformed into the form

$$ds_{\rm dS}^2 = -(1-H^2u^2)d\tau^2 + \frac{du^2}{(1-H^2u^2)} + u^2 d\Omega_2. \tag{44}$$

An observer is always inside a finite size horizon. As was argued in [23] the physics for any such observer is described by a finite number of degrees of freedom.[6] On the other hand, there are an infinite number of degrees of freedom in string theory and it is not obvious how string theory can be reconciled with this observation.

Another related difficulty is encountered when one tries to define the string theory *S* matrix on dS space. As we mentioned above, we could think of dS space as a cavity with a shell surrounding it. This shell has nonzero temperature. Thus, particles in the cavity are immersed in a thermal bath and, moreover, there are no asymptotic states of free particles required for the definition of the *S* matrix. It was shown recently that these problems generically persist [26,27] in quintessence models of the accelerating universe.

Both of these difficulties are related to the fact that in dS space the comoving volume of the region that can be probed in the future by an observer is finite (the same discussion applies to any accelerating universe with $-1 < w < -2/3$, where the equation of state is $p = w\rho$).

Theories with infinite-volume extra dimensions might evade these difficulties. The reason is that the accelerating universe in this case can be accommodated in a space that is not simply four-dimensional dS. In fact, as we argued in previous sections, although the space on the brane looks like de Sitter space for long times it will asymptote to space with no dS horizon in the infinite future (to Minkowski space).

Let us briefly discuss these issues. We start by counting the number of degrees of freedom that are in contact with a brane-world observer. It is certainly true that an observer on the brane is bounded in the world-volume dimensions by a dS horizon. However, there is no horizon in the direction transverse to the brane. Thus, any observer on a brane is in gravitational contact with infinite space in the bulk. In this case, the infinite number of bulk modes of higher-dimensional gravitons participate in 4D interactions on the brane [4,8]. Therefore, the number of degrees of freedom needed to describe physics on the brane is infinite.

The problem of definition of the *S* matrix might be more subtle. Below we present the simplest possibility. The key observation is that the metric (13) in the bulk is nothing but the metric of flat Minkowski space. Indeed, performing the following coordinate transformation [28]:

$$Y^0 = A\left(\frac{r^2}{4} + 1 - \frac{1}{4\dot a^2}\right) - \frac{1}{2}\int dt\, \frac{a^2}{\dot a^3}\,\partial_t\left(\frac{\dot a}{a}\right),$$

$$Y^i = A x^i,$$

$$Y^5 = A\left(\frac{r^2}{4} - 1 - \frac{1}{4\dot a^2}\right) - \frac{1}{2}\int dt\, \frac{a^2}{\dot a^3}\,\partial_t\left(\frac{\dot a}{a}\right), \tag{45}$$

where $r^2 = \eta_{ij}x^i x^j$ and $\eta_{ij} = \mathrm{diag}(1,1,1)$, the metric takes the form

$$ds^2 = -(dY^0)^2 + (dY^1)^2 + (dY^2)^2 + (dY^3)^2 + (dY^5)^2. \tag{46}$$

The brane itself in this coordinate system transforms into the following boundary conditions:

$$-(Y^0)^2 + (Y^1)^2 + (Y^2)^2 + (Y^3)^2 + (Y^5)^2 = \frac{1}{H_0^2},$$

$$Y^0(t, y=0) > Y^5(t, y=0). \tag{47}$$

Therefore, the space to the right of the brane is transformed to Minkowski space with the boundary conditions (47).

On this space the *S* matrix could be defined, as there are asymptotic *in* and *out* states of free particles. The same procedure can be applied to the metric on the left of the brane. However, the brane space-time being de Sitter, one encoun-

[6]Indeed, the number of degrees of freedom inside the region bounded by the horizon is finite. Moreover, the physics of the exterior of the horizon can in principle be encoded into the information on the horizon. This latter, according to the Bekenstein-Hawking formula, has finite entropy and, therefore, supports a finite number of degrees of freedom.

ters the same problems in defining in and out states for scattering products localized on the brane. This is true as long as one neglects the dissipation discussed in Sec. VII, because of which the whole space-time will asymptote to Minkowski space-time, for which the mentioned problems do not persist.

Summarizing, the models with infinite-volume extra dimensions might be a useful ground for describing an accelerating universe with no cosmological constant within string theory. In addition, we point out that these models allow us to preserve bulk supersymmetry even if supersymmetry is broken on the brane [29,30].

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