Dark Energy: Back to Newton?

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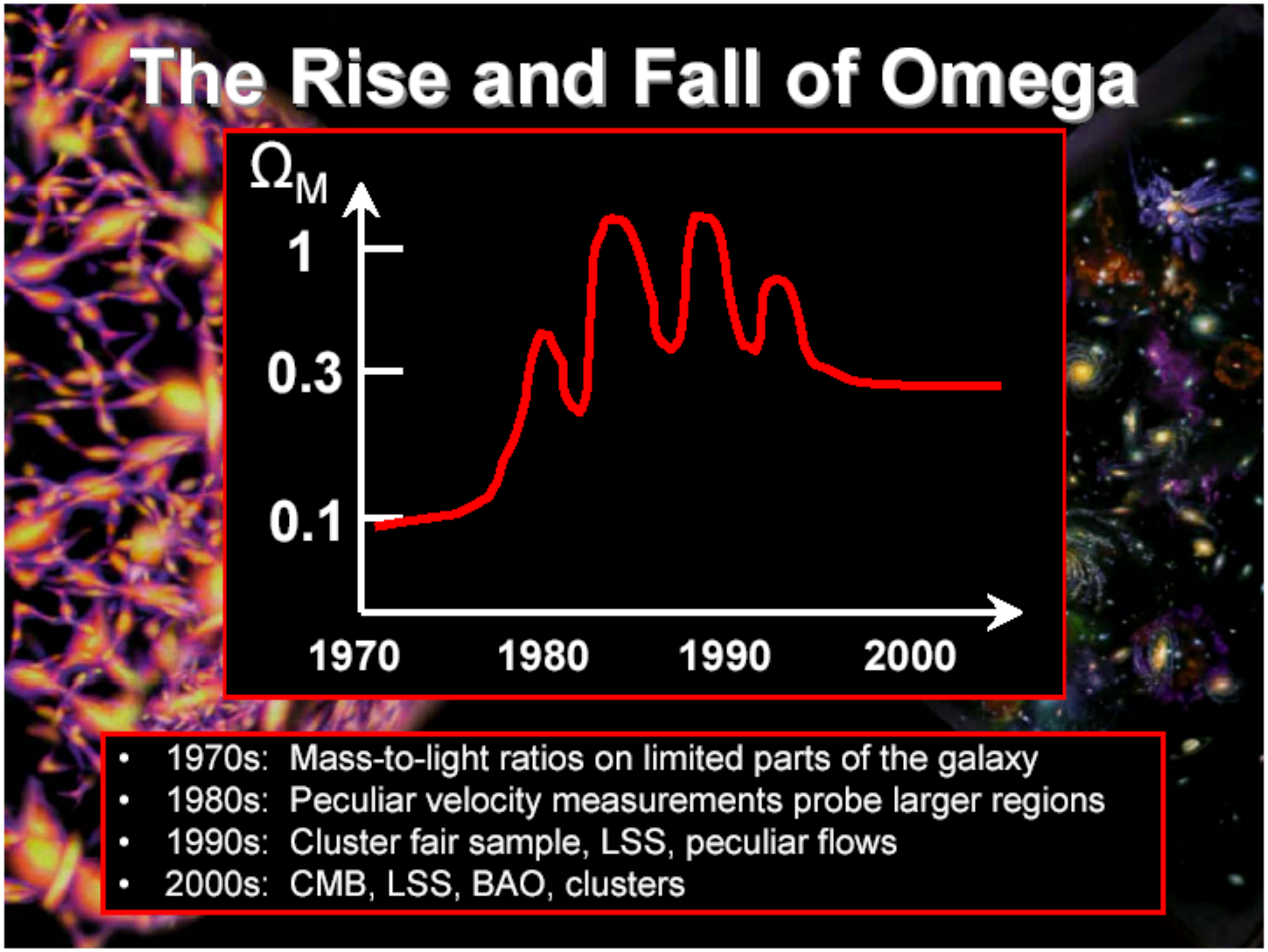
Key Points of the Paper

- The original concept by Newton and Hooke in the 17th century
- Inverse square law plus a force of attraction varies linearly with distance
- A direct link between this term and Einstein's cosmological constant Λ
- A possible relation between Λ and the total mass of the universe
- Mach's Principle

Dark Energy Since 1917

1917	$\Lambda = 4\pi G\rho_\Lambda$	Einstein added Λ to his field equation to allow a static universe
1922	$\Lambda = 0$	Friedmann showed that without Λ the field equation admits nonstatic solution corresponding to an expanding universe
1927	$\Lambda > 0$	The scientific community believed the universe is static
1929		Hubble discovered that galaxies are moving away from us
1931	$\Lambda = 0$	Einstein formally abandoned Λ and stated the biggest blunder he ever made in his life
Mid 1930's to 1950's	$\Lambda = 4\pi G\rho$	1. After Einstein rejected Λ, most scientists retained it. Hubble constant was thought to be 500 kms^{-1} Mpc^{-1}. This made the universe younger than the age of the Earth. Need Λ in the field equation. 2. Hoyle added a C term into the field equation to create a steady state cosmology.
1952	$\Lambda = 0$	The revised Hubble constant of 200 kms^{-1} Mpc^{-1} made Λ unnecessary
1967	$\Lambda > 0$	Λ was revived to explain why quasar appear to have redshifts concentrated near z=2 in the Lemaitre loitering model.
1980	$\Lambda > 0$	Alan Guth inflation theory required a Λ-like scalar field in the early universe
1984	$\Omega_M + \Omega_\Lambda = 1$	Peebles showed that inflation implies vanishing curvature and the universe is flat.
1985	$\Omega_M = 1, \Omega_\Lambda = 0$	Cold dark matter theory can explain the biased distribution of visible galaxies relative to the distribution of all of the mass
1990	$\Omega_M = 0.2, \Omega_\Lambda = 0.8$	
1998	$\Omega_M = 0.3, \Omega_\Lambda = 0.7$	
2003 to 2006	$\Omega_M = 0.27, \Omega_\Lambda = 0.72$	



The Rise and Fall of Omega

- 1970s: Mass-to-light ratios on limited parts of the galaxy
- 1980s: Peculiar velocity measurements probe larger regions
- 1990s: Cluster fair sample, LSS, peculiar flows
- 2000s: CMB, LSS, BAO, clusters

Michael Turner, SSI 2007

Newton and Hooke

Inverse square law

$$F = -\frac{GMm}{r^2}.$$

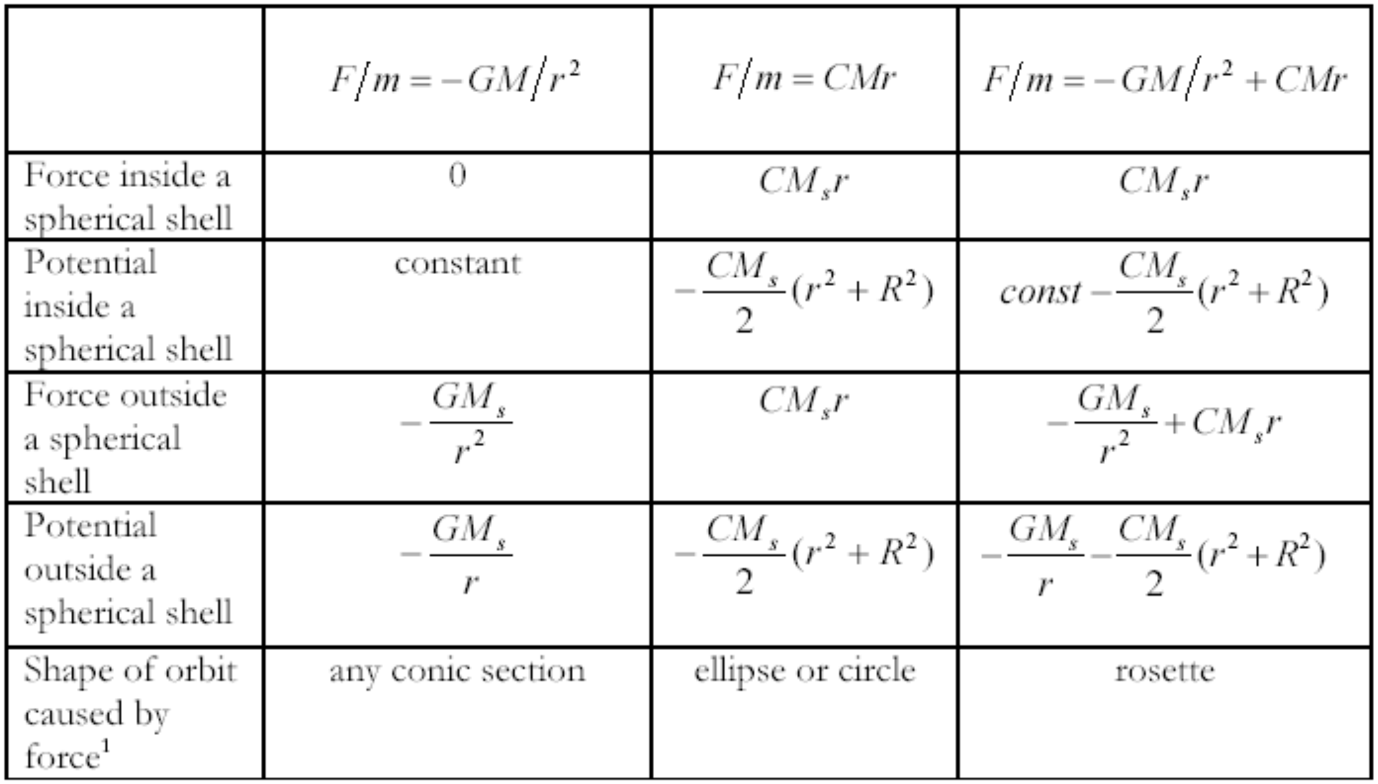
Inverse square law
plus Linear force

$$\frac{F}{m} = \ddot{r} = -\frac{GM}{r^2} + CMr,$$

The linear force term has the same form as the law of elasticity which will cause a particle into simple harmonica oscillation.

Newton later abandoned the linear force term because there was no discernable evidence for it.

Contrasting $1/r^2$ and r forces



	$F/m = -GM/r^2$	$F/m = CMr$	$F/m = -GM/r^2 + CMr$
Force inside a spherical shell	0	$CM_s r$	$CM_s r$
Potential inside a spherical shell	constant	$-\frac{CM_s}{2}(r^2+R^2)$	$const - \frac{CM_s}{2}(r^2+R^2)$
Force outside a spherical shell	$-\frac{GM_s}{r^2}$	$CM_s r$	$-\frac{GM_s}{r^2}+CM_s r$
Potential outside a spherical shell	$-\frac{GM_s}{r}$	$-\frac{CM_s}{2}(r^2+R^2)$	$-\frac{GM_s}{r}-\frac{CM_s}{2}(r^2+R^2)$
Shape of orbit caused by force[1]	any conic section	ellipse or circle	rosette

Orbit velocity $v \sim 1/r^{1/2}$ $v \sim r$

C is an arbitrary (*negative*) constant

A possible relation between Λ and the total mass of the universe

$$\frac{\ddot{a}}{a} = -\frac{4\pi G\rho}{3} + \frac{\Lambda}{3}$$

From Einstein field equations

$$\ddot{r} = -\frac{GM}{r^2} + \frac{\Lambda}{3} r$$

Friedmann Solution

$$\frac{F}{m} = \ddot{r} = -\frac{GM}{r^2} + CMr\,,$$

Newton & Hooke's Law

These two equations have exactly the same form with *CM = Λ / 3*

The linear force, due to any distribution, will act as if the total mass is concentrated at its center of mass.

∴ Λ is proportional to the total mass in the universe.

Mach’s Principle and Einstein’s Λ

Newton’s concept of space is absolute. The inertia forces give a measure of absolute acceleration in space. Newton’s inertia frames are at rest or in uniform motion with respect to absolute space.

Mach’s inertial frames were determined relative to distant celestial bodies.

Einstein believed so strongly in the relativity of inertia that he called it “Mach’s Principle”. The inertia should be fully and exclusively determined by matter. “There can be no inertia relative to space but only an inertia of masses relative to one another”. He defined his new fundamental constant Λ in terms of mass density of the universe.

Mach's Principle in the context of Einstein's relativity

The inertia properties of an object are determined by the energy momentum throughout all space[1]

The spacetime geometry is uniquely determined by the energy-momentum and by the initial conditions on the geometry. [2]

1. Einstein 1918
2. J. A. Wheeler 1964

Problems with Newtonian Cosmology

- Newton's law breaks down when applied to an infinite homogenous universe.

$$-\nabla \cdot F = \nabla^2 \phi = 4\pi G \rho$$

Symmetry considerations yield no direction of F. This leads to an inconsistency of nonzero density ρ on the RHS.

- Einstein argued that the Newton's theory requires that

> the universe should have a kind of center in which the density of the mass is maximum … at great distance, it succeeded by an infinite region of emptiness. The stellar universe ought to be a finite island in the infinite ocean of space.

Solution to Newton’s Problem

- If Newton’s inverse square law, written in the form of Poisson’s equation, is modified by the addition of an extra term Λ, then the solution would correspond an infinite extension of static space filled uniformly with matter.

$$-\nabla \cdot \mathbf{F} + \Lambda = \nabla^2 \phi + \Lambda = 4\pi G \rho$$

- Einstein’s addition of the cosmological constant to his field equation is perfectly analogous to the extension of Poisson’s equation.

Field Equations without Λ, $G_{\mu\nu} = 8\pi G T_{\mu\nu}$

Field Equations with Λ, $G_{\mu\nu} - g_{\mu\nu}\Lambda = 8\pi G T_{\mu\nu}$

$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$

Concluding Remarks of the Paper

320 years ago Newton discovered a term that might be related to Dark Energy.

He could not image the implication of the Λ-like term, as Einstein could not, that the universe is expanding.

It would be ironic if, after all the speculation and complications and theoretical ingenuity, observations show that Newton’s classical equations held the answer.

My Comment on the Paper

Newton's initial intention was to introduce an *attractive* force which is proportional to distance. That is the constant C is negative in the equation

$$\frac{F}{m} = \ddot{r} = -\frac{GM}{r^2} + CMr\,,$$

to cause bodies to revolve in conic section (Newton, Proposition 777, Theorem 37, or page 8 of the paper)

For an acceleration universe, the equation of motion obtained from the Einstein's equations

$$\ddot{r} = -\frac{GM}{r^2} + \frac{\Lambda}{3} r$$

should have a positive Λ.

I do not think Newton thought of a repulsive force as the paper claimed.

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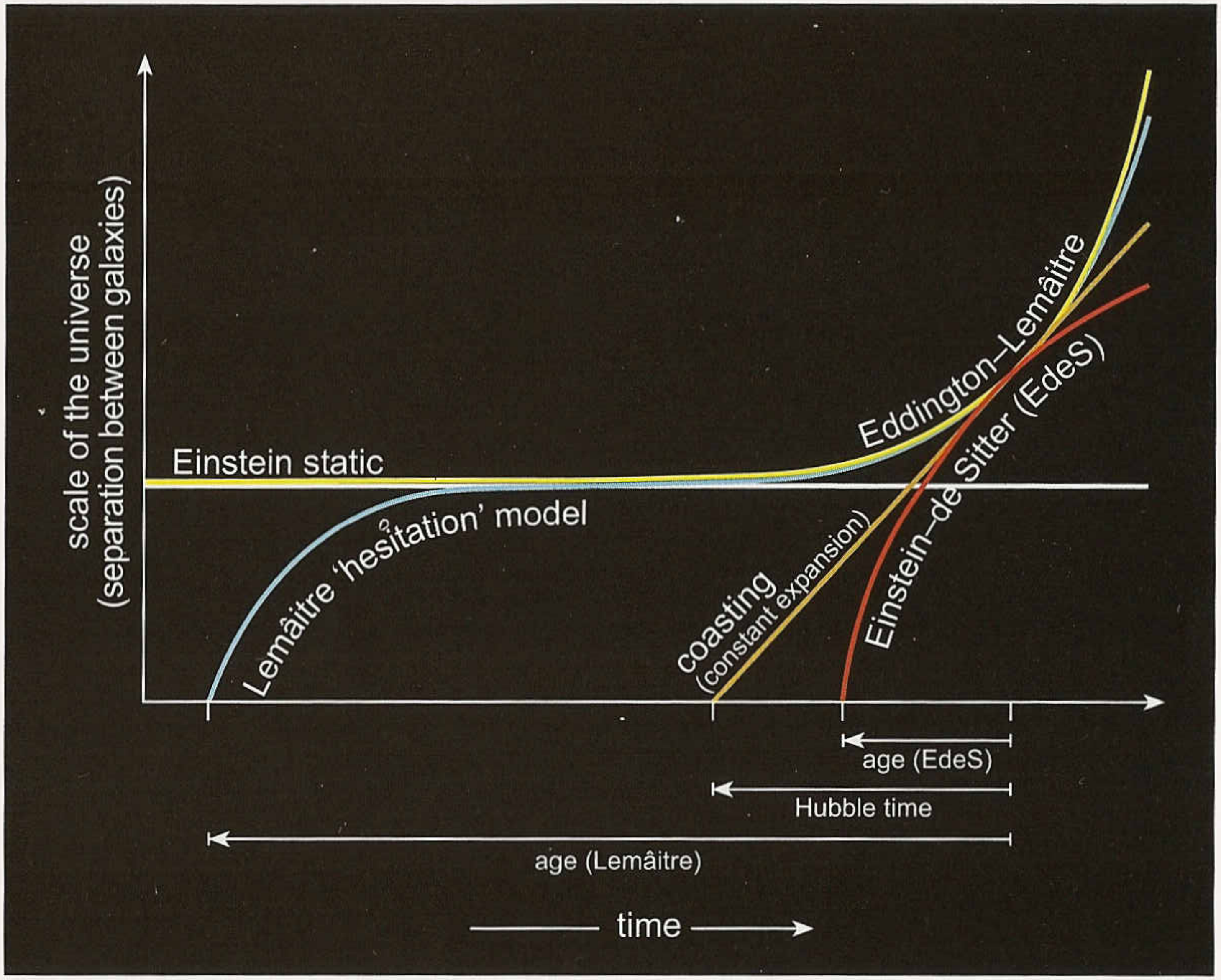

From Iain Nicolson, Dark Side of the Universe