L21: The 3D Kitaev Models + The Yao Lee Spin Liquid

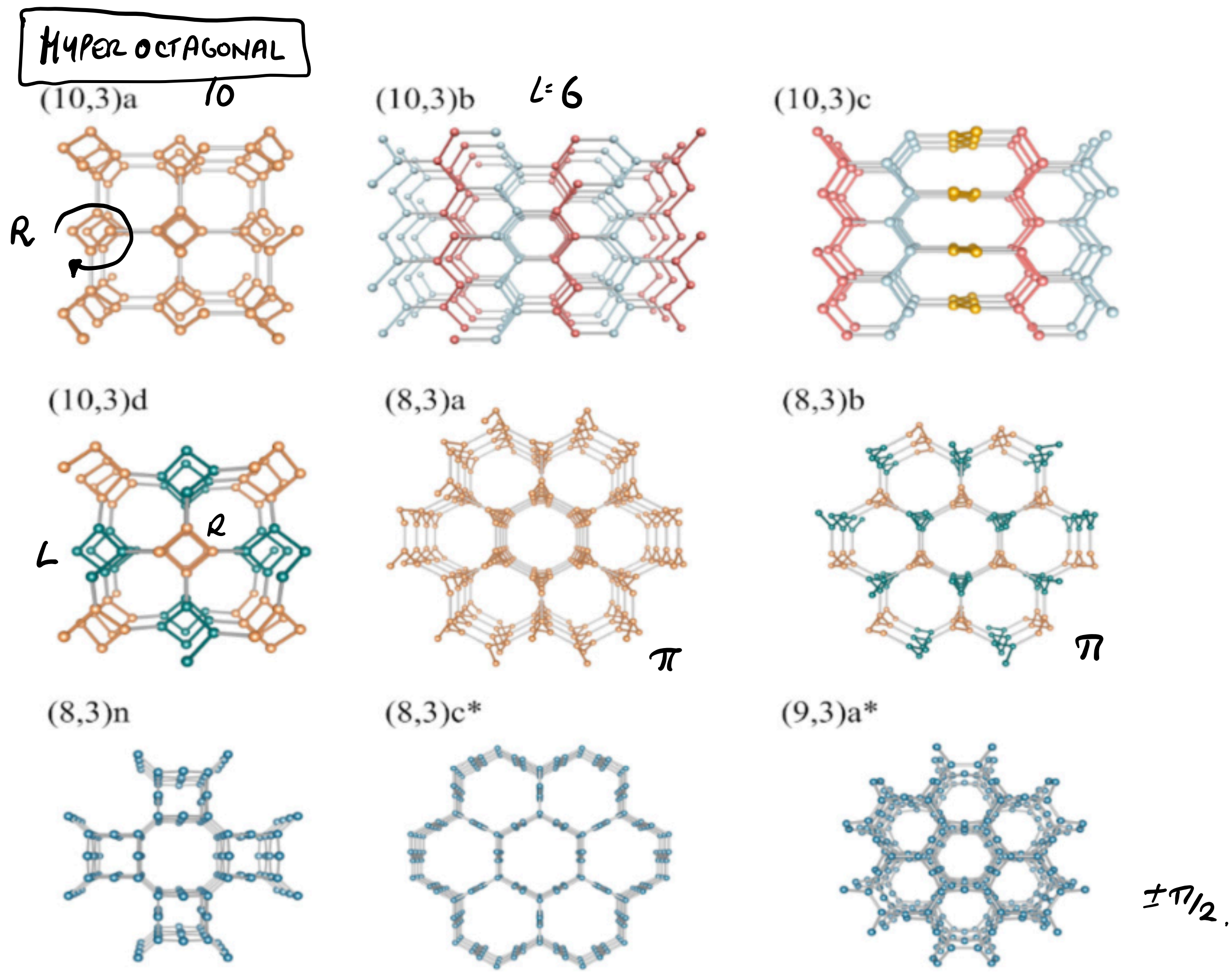


FIG. 1. Elementary tricoordinated 3D lattices considered in this paper [7,21]. The lattices are named and ordered according to the Schläfli notation $(p, c)x$, specifying the elementary plaquette length p and the coordination number c (followed by an index letter x). Spirals colored blue (orange) rotate clockwise (anticlockwise). The colors red/light blue/yellow highlight different directions of 'zigzag' chains. For the lattices (8,3)c and (9,3)a, marked with an asterisk, the gauge sectors behave differently from the other tricoordinated lattices. These systems were regarded separately.

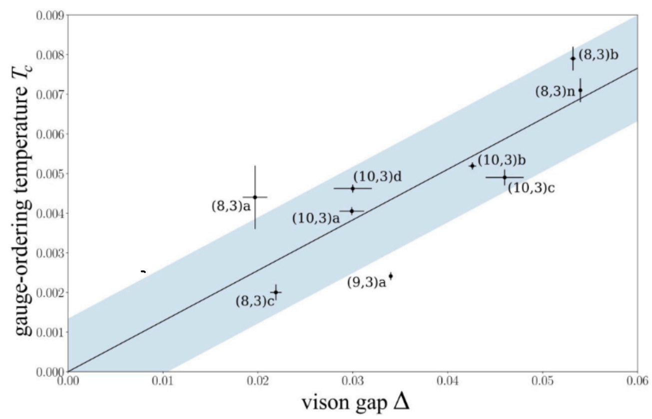
Eschmann et al PRB 102, 075125 (2020)
Trebst Group.

Leib's theorem: If $p \pmod 4 = 2$ the energy is minimized if the plaquettes are flux free.

If $p \pmod 4 = 0$, the gs is a π flux phase

STRICTLY ONLY PROVEN FOR SYSTEM + MIRROR SYMMETRIES. Yet numerically it holds for all 9 3D Kitaev lattices.

3D Models have a gauge ordering temperature

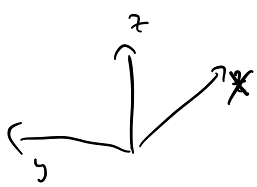


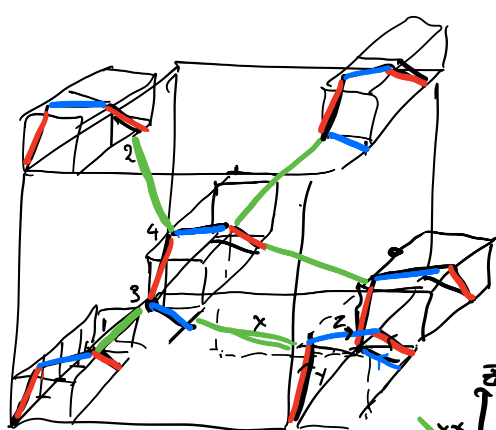
$$W_p = \prod_{\langle i,j \rangle, \gamma \in p} \sigma_i^\gamma \sigma_j^\gamma = \prod_{\langle i,j \rangle_\gamma \in p} -i\,\hat{u}_{ij}$$

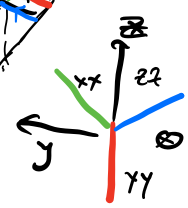
PLANE		α
xz	yy	z
zy	xx	y
yx	zz	x

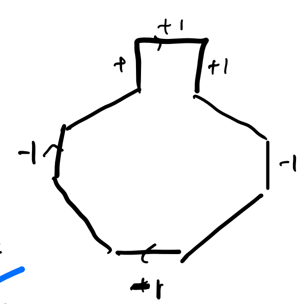
Hyper octagonal

BCC



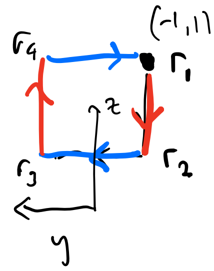


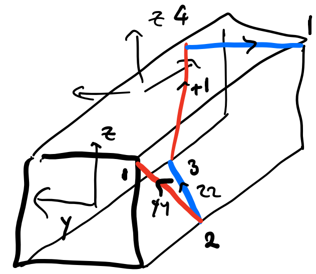




$$-\frac{K}{2}\sum \sigma_i^\alpha \sigma_j^\alpha \longrightarrow iK \sum_{\langle i,j \rangle} \hat{a}_i u_{ij} \hat{a}_j$$

$$\vec{\sigma}_i = -2i a_i \vec{b}_i \qquad u_{ij} = -2i\, b_i^\alpha b_j^\alpha$$

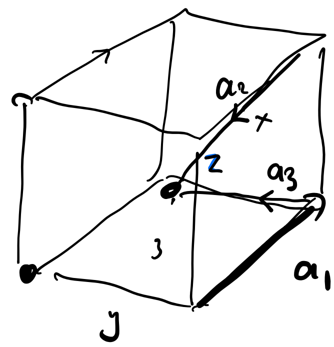




$$\begin{array}{llll} & & & x \quad y \quad z \\ r_1 &=& R + & \frac{1}{8}(-3,-1,1) \\ r_2 &=& R + & \frac{1}{8}(-1,-1,-1) \\ r_3 &=& R + & \frac{1}{8}(1,1,-1) \\ r_4 &=& R + & \frac{1}{8}(3,1,1) \end{array}$$

$\longleftarrow a/4 \longrightarrow$

2→1 i
2→3 i
3→1 i



$$R = l\hat{a}_1 + m\hat{a}_2 + n\hat{a}_3$$

$$\langle k|i\rangle t_{ij} \langle j|k\rangle \sim e^{-ik(R_i - R_j)}$$

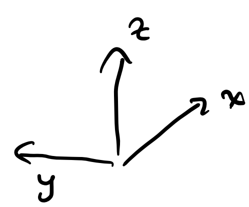
xx — along y direction
yy — " z "
zz — " x direction

$$\mathcal{H} = K\sum a_k^\dagger h(k) a_k$$

$$a_1 = (1,0,0) \quad (x\ y\ z)$$
$$a_2 = \tfrac{1}{2}(-1,1,-1)$$
$$a_3 = \tfrac{1}{2}(1,1,1)$$

$$h(k) = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{cccc} 1 & 2 & 3 & 4 \\ \end{array}\begin{pmatrix} 0 & i & ie^{-ik\cdot a_2} & ie^{-ika_1} \\ -1 & 0 & -i & ie^{-ika_3} \\ -ie^{+ika} & i & 0 & -i \\ -ie^{ika_1} & -ie^{ika_3} & i & 0 \end{pmatrix}$$

x z y
↓ ↓
-x z y



Yao Lee Spin Liquid

$$H_{YL} = \left(\frac{k}{2}\right) \sum_{\langle i,j \rangle} (\vec{\sigma}_i \cdot \vec{\sigma}_j)\, \lambda_i^{\alpha_{ij}} \lambda_j^{\alpha_{ij}}$$

$$\chi_j = \Phi_j^S \sigma_j^\alpha \qquad b_j^\alpha = \Phi_j^T \lambda_j^\alpha$$

$$\sigma^x \sigma^y \sigma^z = \lambda^x \lambda^y \lambda^z = i$$

$$\Phi^S = -2i\, \chi^1 \chi^2 \chi^3 \qquad \Phi^T = -2i\, b\, b\, b$$

$$\vec{\sigma}_j = 2\phi^S \vec{\chi} = -i\, \vec{\chi} \times \vec{\chi}$$

$$\vec{\lambda}_j = 2\Phi^T \vec{b} = -i\, \vec{b} \times \vec{b}$$

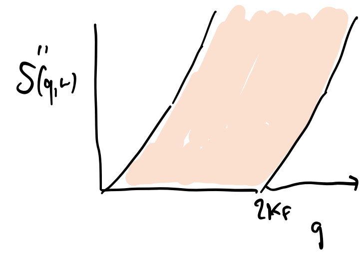
$$D = -2i\, \phi^S \phi^T = 8i\, \chi\chi\chi\, b b b$$

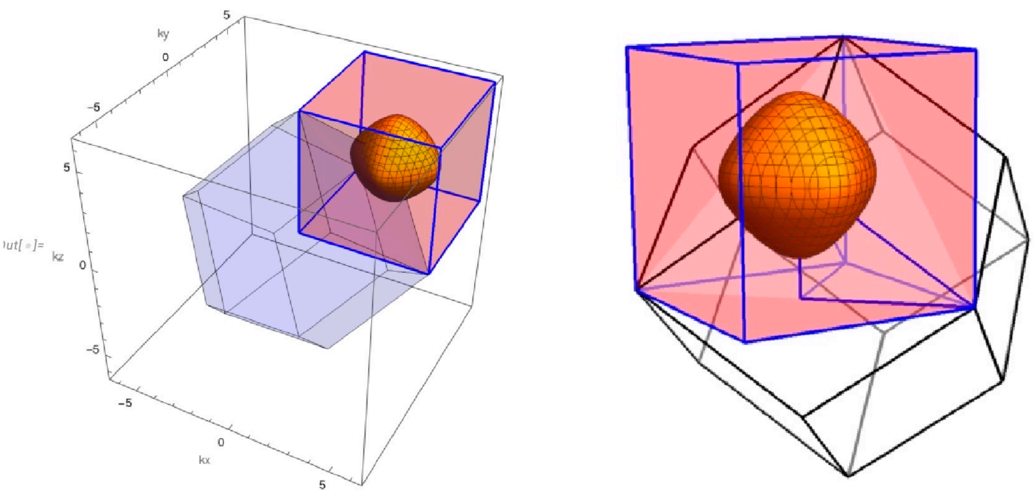
$$|\psi_P\rangle = \prod \frac{1}{2}(1 + D_j)\, |\psi\rangle$$

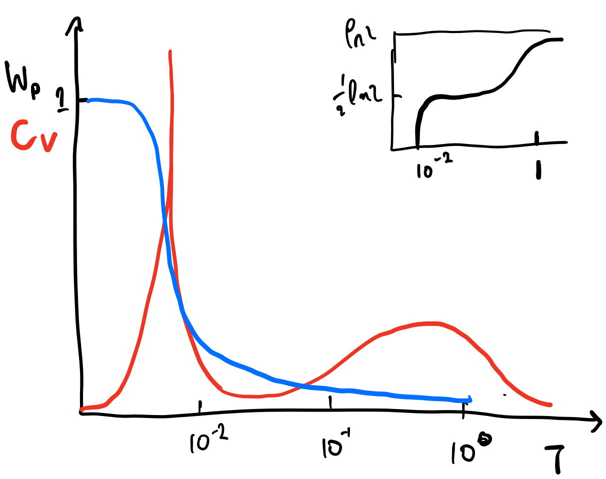
$$\sigma_j^\alpha \lambda_j^\alpha = -2i\chi_j^\alpha b_j^\alpha .$$

$$\boxed{H_{YL} = K\sum u_{ij}\,(i\vec{\chi}_i\cdot\vec{\chi}_j)}$$

Spin liquid with a Fermi surface







$$W_p = \left\langle \prod_{j\in p} \sigma_j^\gamma \right\rangle$$

Why is there a phase transition?

Lattice gauge theory