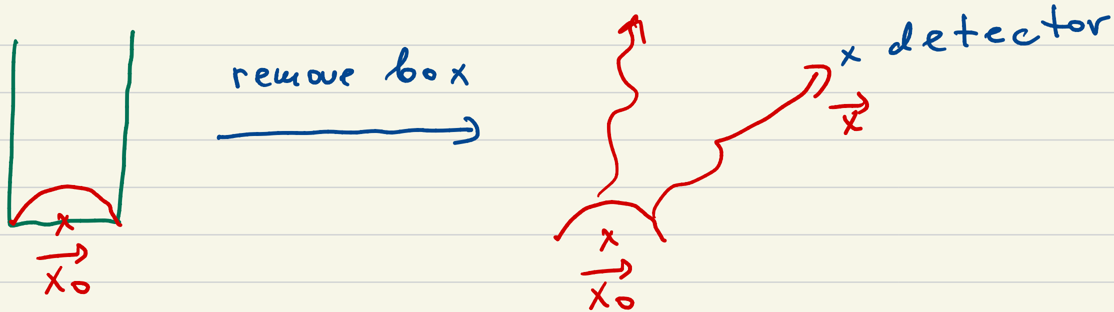
Causality

Recall: QM, free particle

Amplitude to propagate from point $\vec{x}_0$ to point $\vec{x}$

$$\langle \vec{x} | e^{-iHt} | \vec{x}_0 \rangle \neq 0$$

even when $c^2(t-t_0)^2 - |\vec{x}-\vec{x}_0|^2 < 0$ (spacelike) $\dfrac{|x-x_0|}{|t-t_0|} > c$



$$\Delta x \, \Delta p \geq \frac{\hbar}{2}$$

Now QFT: KG field

$$\phi(\vec{x}) = \int_{\vec{p}} \frac{1}{\sqrt{2E_p}} \left(a_{\vec{p}}\, e^{-ip\cdot x} + a^{\dagger}_{\vec{p}}\, e^{ip\cdot x} \right)$$

$$\langle 0 | \phi(x)\, \phi(y) | 0 \rangle = D(x-y)$$

$$D(x-y) \sim e^{-m|\vec{x}-\vec{y}|}$$

PSS $\quad D(x-y) \neq 0 \quad \forall \; x-y$

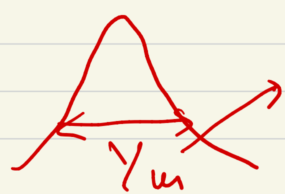
Need $\quad [\phi(x), \phi(y)] = 0 \quad$ if $(x-y)$ - spacelike

$$\phi(x) = e^{iHt}\, \phi(\vec{x})\, e^{-iHt} \qquad \phi(y) = \phi(\vec{y})$$

$$D(x-y) = \langle 0 | \phi(\vec{x})\, e^{-iHt} \phi(\vec{y}) | 0 \rangle = \langle \vec{x} | e^{-iHt} | \vec{y} \rangle$$

$$\phi(\vec{x})|0\rangle \sim |\vec{x}\rangle \qquad \phi(\vec{y})|0\rangle \sim |\vec{y}\rangle$$

$$\int_{\vec{p}} \frac{e^{i\vec{p}\cdot\vec{x}}}{\sqrt{2(p^2+m^2)}}$$



Compton length

$\underset{x}{\bullet}$ $\quad$ $\underset{y}{\bullet}$ $\qquad$ $[O_1(x), O_2(y)] = 0$ if $(x-y)$ - spacelike

Recall QM: $\quad [A, B] = 0 \Rightarrow A$ & B measurable together

$$AB|\psi\rangle = BA|\psi\rangle$$

$[\phi(x), \phi(y)] = 0$ if $(x-y)$ - spacelike

$$\left[F_1(\phi(x)), F_2(\phi(y))\right] = 0$$

$$\pi = \frac{\partial \phi}{\partial t} \qquad [\pi(x), \pi(y)] = 0$$

$$\phi(\vec{x}) = \int_{\vec{p}} \frac{1}{\sqrt{2E_p}} \left(a_{\vec{p}}\, e^{-ip\cdot x} + a^{+}_{\vec{p}}\, e^{ip\cdot x} \right)$$

$$[\phi(x), \phi(y)] = c\text{-}\# = \langle 0 | [\phi(x), \phi(y)] | 0 \rangle =$$

$$= D(x-y) - D(y-x)$$

Opts in $\phi(x)\,\phi(y)$

$$\cancel{a_p a_q} + a_p a_q^{+} + \cancel{a_p^{+} a_q^{+}} + \cancel{a_p^{+} a_q} \sim$$

$$\langle 0 | a_p^{+} a_q^{+} | 0 \rangle$$

$$\sim \delta^{(3)}(\vec{p} - \vec{q})$$

$$D(x-y) = \int_{\vec{p}} \frac{e^{-ip\cdot(x-y)}}{2E_{\vec{p}}}$$

Lorentz inv

$D(s) = D(-s)$ if s - spacelike

$\downarrow$

can take $s \to -s$

with Lorentz transform

$$s^2 = (\Delta t)^2 - (\Delta \vec{x})^2 < 0$$

1. Go to frame where $\Delta t' = 0$

$$s' = (0, \Delta \vec{x}')$$

2. Rotation

$$s'' = (0, -\Delta \vec{x}') \qquad s' = -s''$$

3. Inverse Lorentz [to (1)]

$$\underline{s'''} = (-\Delta t, -\Delta \vec{x}) = -\vec{s}$$

$$\begin{pmatrix} -\Delta t \\ -\Delta x \end{pmatrix} = \begin{pmatrix} \nu & \nu \\ \nu & \nu \end{pmatrix} \begin{pmatrix} 0 \\ -\Delta x' \end{pmatrix}$$

$$\langle 0| [\phi(x), \phi(y)] |0\rangle =$$

$$D(x-y) = \int_{\vec{p}} \frac{e^{-ip\cdot(x-y)}}{2E_{\vec{p}}}$$

$$= D(x-y) - D(y-x) =$$

$$= \int_{\vec{p}} \frac{e^{-ip\cdot(x-y)} - e^{ip(x-y)}}{2E_p} =$$

$$= \int_{\vec{p}} \frac{e^{-ip\cdot(x-y)}}{2E_p}\Bigg|_{p^0=E_p} + \int_{\vec{p}} \frac{e^{-ip(x-y)}}{-2E_p}\Bigg|_{p^0=-E_p} =$$

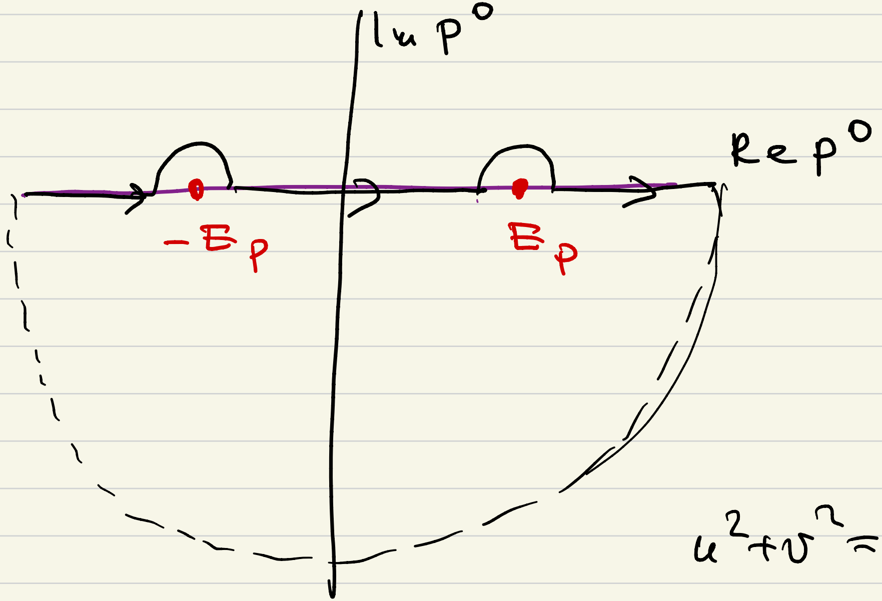
$$\underset{x^0>y^0}{=} \int \frac{d^3p}{(2\pi)^3} \int \frac{dp^0}{2\pi i} \frac{-1}{p^2-m^2} e^{-ip\cdot(x-y)}$$

$$\int_{\vec{p}} \frac{e^{-ip\cdot(x-y)}}{2E_p}\Bigg|_{p^0=E_p} + \int_{\vec{p}} \frac{e^{-ip(x-y)}}{-2E_p}\Bigg|_{p^0=-E_p} \underset{x^0>y^0}{=} \int \frac{d^3p}{(2\pi)^3} \int \frac{dp^0}{2\pi i} \frac{-1}{p^2-m^2} e^{-ip\cdot(x-y)}$$

$$p^0 = \pm E_p$$

$$-ip^0(x^0-y^0)$$

$$p^0 = u + iv$$



$$\sim \frac{e^{v(x^0-y^0)}}{|p^0|^2}$$

$$(p^0)^2 - |\vec{p}|^2 - m^2$$

$$u^2+v^2=R^2$$

$$\int \frac{dp^0}{2\pi i} \frac{-1}{p^2 - m^2} e^{-ip\cdot(x-y)}$$

$$\oint f(z)\,dz = 2\pi i \sum_k r_k$$

$$f(z) \xrightarrow[z \to c_i]{} \frac{r_i}{z - c_i}$$