Classical field theory $\xrightarrow{\text{quantize}}$ QFT $\qquad \phi \to \hat{\phi}$

$\pi \to \hat{\pi}$

2nd quantization

$$\left(-\partial_t^2 + \nabla^2\right)\phi = m^2 \phi$$

1st quantization

$$\phi \to \hat{\phi}$$

2nd

Discrete sys.
$$[q_i, p_j] = i\delta_{ij} \qquad [q_i, q_j] = [p_i, p_j] = 0$$

$$p = -i\partial_x$$

Continuous sys.

$$[\phi(\vec{x}), \pi(\vec{y})] = i\,\delta^3(\vec{x}-\vec{y})$$

$$[\phi(\vec{x}), \phi(\vec{y})] = [\pi(\vec{x}), \pi(\vec{y})] = 0$$

$$H = \frac{p^2}{2m} - F x \rightarrow \frac{\hat{p}^2}{2m} - F \hat{x}$$

$$\dot{p} = F \qquad \dot{\hat{p}} = F$$

$$\left(-\partial_t^2 + \nabla^2\right)\hat{\phi} = m^2 \hat{\phi}$$

FT

$$\hat{\phi}(\vec{x}, t) = \int \frac{d^3 p}{(2\pi)^3} e^{i \vec{p} \cdot \vec{x}} \hat{\phi}(\vec{p}, t)$$

$$\phi^{\dagger}(-\vec{p}) = \phi(\vec{p})$$

$$\int \frac{d^3 p}{(2\pi)^3} = \int_{\vec{p}}$$

$$\left(\partial_t^2 + |\vec{p}|^2 + m^2\right)\phi(\vec{p}, t) = 0$$

$$\ddot{\phi}(\vec{p},t) + \omega_p^2 \phi(\vec{p},t) = 0$$

$$\omega_p = \sqrt{|\vec{p}|^2 + m^2}$$

Harmonic osc. $\quad H_{ho} = \frac{p^2}{2} + \frac{\omega^2 \phi^2}{2}$

Ladder opts

$$\phi = \frac{1}{\sqrt{2\omega}} (a + a^+) \qquad p = -i\sqrt{\frac{\omega}{2}} (a - a^+)$$

$$[\phi, p] = i \quad \Rightarrow \quad [a, a^+] = 1$$

Heisenberg algebra

$$H_{ho} = \omega\left(a^+ a + \frac{1}{2}\right)$$

$$[H_{ho}, a^+] = \omega a^+ \qquad [H_{ho}, a] = -\omega a$$

$$[S_z, S_+] = S_+ \qquad [S_z, S_-] = -S_- \qquad su(2)$$

$$a^+ |n\rangle \propto |n+1\rangle \qquad a|n\rangle \propto |n-1\rangle$$

$$a|0\rangle = 0 \qquad E_0 = \omega/2$$

$$|n\rangle = (a^+)^n |0\rangle \qquad E_n = \left(n + \frac{1}{2}\right)\omega$$

$$\phi(\vec{x}) = \int_{\vec{p}} \frac{1}{\sqrt{2\omega_p}} \left(a_p e^{i\vec{p}\cdot\vec{x}} + a_p^{+} e^{-i\vec{p}\cdot\vec{x}} \right)$$

$$\pi(\vec{x}) = \int_{\vec{p}} (-i) \sqrt{\frac{\omega_p}{2}} \left(a_p e^{i\vec{p}\cdot\vec{x}} - a_p^{+} e^{-i\vec{p}\cdot\vec{x}} \right)$$

$$\phi = \frac{1}{\sqrt{2\omega}} (a + a^{+}) \qquad p = -i\sqrt{\frac{\omega}{2}} (a - a^{+})$$

$$\left[a_{\vec{p}}, a^{+}_{\vec{p}'} \right] = (2\pi)^3 \delta^{(3)}(\vec{p} - \vec{p}')$$

$$\hat{\phi}(\vec{x}, t) = \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p}\cdot\vec{x}} \hat{\phi}(\vec{p}, t)$$

$$e^{i\vec{p}\cdot\vec{x}} \phi(\vec{p}, t) \longrightarrow$$

$$\longrightarrow \frac{1}{\sqrt{2\omega_p}} \left(a_p e^{i\vec{p}\cdot\vec{x}} + a_p^{+} e^{-i\vec{p}\cdot\vec{x}} \right)$$

$$\tilde{\phi}^{+}(\vec{p}) = \tilde{\phi}(-\vec{p}) \qquad \pi^{+}(\vec{p}) = \pi(-\vec{p})$$

$$\phi(\vec{x}) = \phi^{+}(\vec{x}) \qquad \pi(\vec{x}) = \pi^{+}(\vec{x})$$

$$\phi(\vec{x}) = \int_{\vec{p}} A_p \, e^{i\vec{p}\cdot\vec{x}}$$

$$\phi(\vec{x}) = \frac{\phi(\vec{x}) + \phi^{+}(\vec{x})}{2} = \int_{\vec{p}} \left(A_p e^{i\vec{p}\cdot\vec{x}} + A_p^{+} e^{-i\vec{p}\cdot\vec{x}} \right)$$

Let $A_p = \frac{1}{\sqrt{2\omega_p}} a_{\vec{p}}$

$$\downarrow$$

$$\frac{1}{\sqrt{2\omega_p}} \left(a_p e^{i\vec{p}\cdot\vec{x}} + a_p^{+} e^{-i\vec{p}\cdot\vec{x}} \right)$$

$$\pi(\vec{x}) = \int (-i) \sqrt{2\omega_p} \left(a_p e^{i\vec{p}\cdot\vec{x}} - a_p^{+} e^{-i\vec{p}\cdot\vec{x}} \right)$$

$$\phi(\vec{x}) = \int_{\vec{p}} \frac{1}{\sqrt{2\omega_p}} \left(a_p + a^{+}_{-p} \right) e^{i\vec{p}\cdot\vec{x}}$$

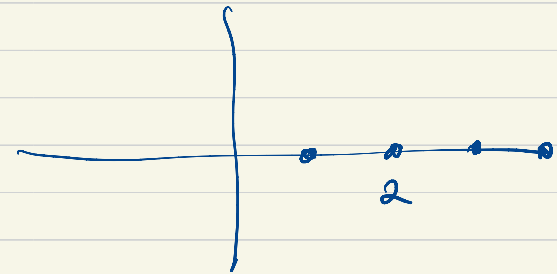
$$\pi(\vec{x}) = \int_{\vec{p}} (-i) \sqrt{\frac{\omega_p}{2}} \left(a_p - a^{+}_{-p} \right) e^{i\vec{p}\cdot\vec{x}}$$

$$H = \int d^3x \left[\frac{\pi^2}{2} + \frac{(\nabla\phi)^2}{2} + \frac{m^2\phi^2}{2} \right] =$$

$$= \int_{\vec{p}} \omega_p \left(a^{+}_p a_p + \frac{1}{2} \underbrace{[a_p, a^{+}_p]}_{=\,\delta(0)} \right)$$

$$\zeta(-1) = 1 + 2 + 3 + \cdots = -\frac{1}{12}$$

$$\zeta(p) = \sum_{n=1}^{\infty} \frac{1}{n^p} \qquad p > 1$$



$$[H, a_p^\dagger] = \omega_p a_p^\dagger \qquad [H, a_p] = -\omega_p a_p$$

$$a_p |0\rangle = 0 \quad \forall \vec{p} \qquad \text{Set } E_{g.s.} = 0$$

excited states $\quad a_p^\dagger a_q^\dagger \cdots |0\rangle$

$$E = \omega_p + \omega_q + \cdots$$

$$P^i = \int T^{0i} \, d^3x = -\int \pi \, \partial_i \phi \, d^3x$$

$$\vec{P} = -\int d^3x \, \pi(\vec{x}) \, \nabla \phi(\vec{x}) = \int_{\vec{p}} \vec{p} \, a_p^\dagger a_{\vec{p}}$$

$$a_{\vec{p}} a_{\vec{p}}^\dagger$$

$$\vec{P} = \vec{p} + \vec{q} + \cdots$$

$$\nabla \phi \, \pi(\vec{x})$$

$$[a_p^\dagger, a_q^\dagger] = 0 \qquad a_p^\dagger a_q^\dagger |0\rangle = a_q^\dagger a_p^\dagger |0\rangle$$

$$(a_p^\dagger)^n |0\rangle \neq 0$$

Bose-Einstein statistics