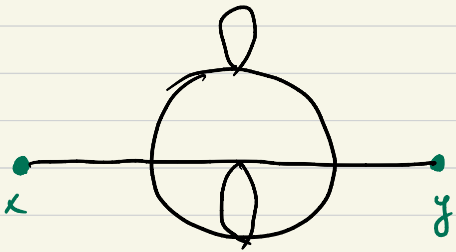
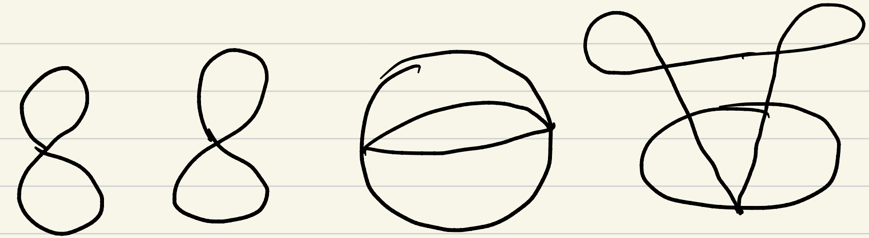
Student Instructional Rating Survey:

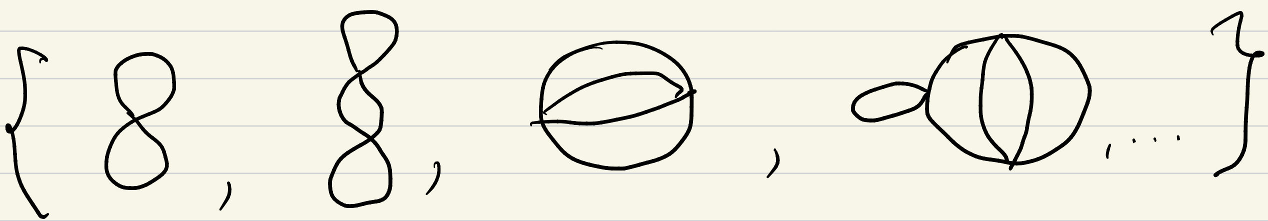
https://sirs.ctaar.rutgers.edu/blue

A typical diagram





Let V_i denote disconnected pieces

$V_i \in$ 

$$\text{Diagram} = \underbrace{(\text{connected piece})}_{ct} + (n_1 \text{ of } V_2) + (n_2 \text{ of } V_2) + \; + \cdots$$

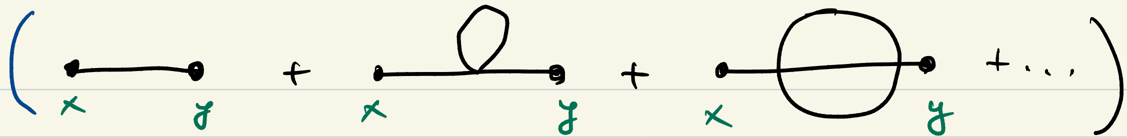
Its value $= c_t \prod_i \frac{1}{k_i!}(v_i)^{k_i}$

numerator $= \sum_{\text{all } c_t} \sum_{\text{all}\{k_i\}} c_t \prod_i \frac{1}{k_i!}(v_i)^{k_i} =$

$$= \sum_{\text{all } c_t} c_t \underbrace{\sum_{\{k_i\}} \prod_i \frac{1}{k_i!}(v_i)^{k_i}}_{\text{Dis}}$$

$$\text{Dis} = \sum_{k_1} \frac{1}{k_1!} v_1^{k_1} \sum_{k_2} \frac{1}{k_2!} v_2^{k_2} \ldots\ldots = e^{v_1} e^{v_2} \ldots =$$

$$= e^{\sum_i v_i}$$

numerator = 

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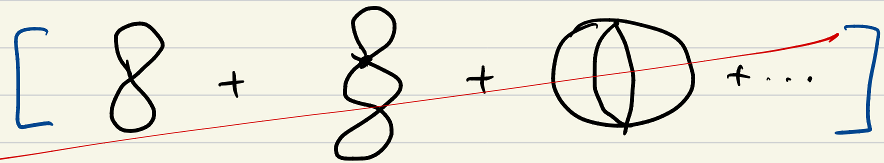
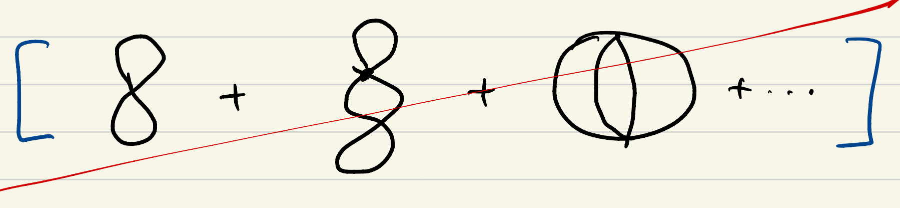
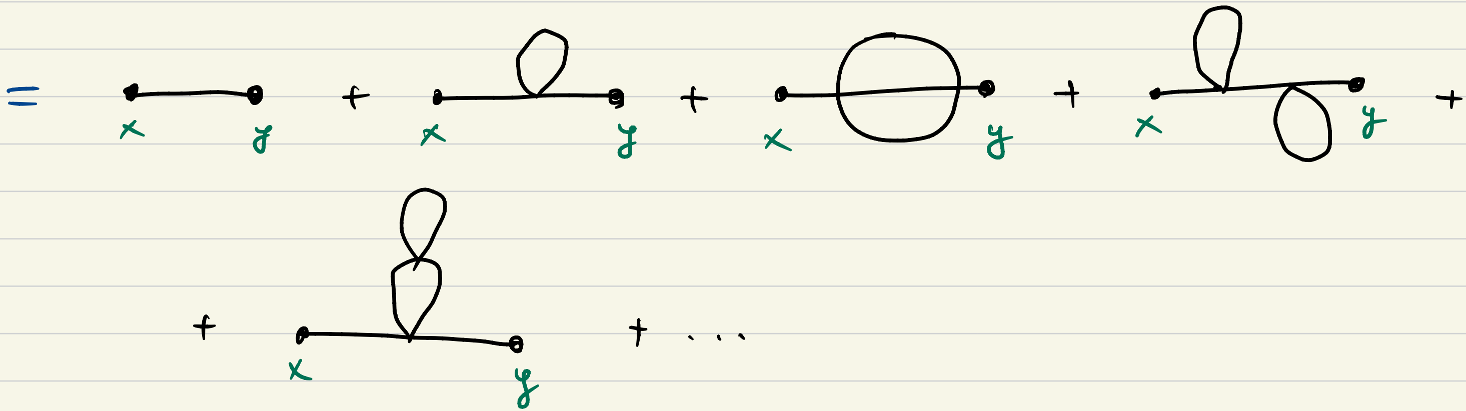
× exp 

denominator = exp 

Thus $\langle \Omega | T \phi(x) \phi(y) | \Omega \rangle = \sum \left(\text{all connected diagrams with 2 external pts} \right)$



Recall:

$$|\Omega\rangle = \lim_{T\to\infty(1-i\epsilon)} \left(e^{-iE_0T}\langle\Omega|0\rangle\right)^{-1} e^{-iHT}|0\rangle$$

$$1 = \langle\Omega|\Omega\rangle = \cancel{\lim_{T\to\dots}} \left(|\langle 0|\Omega\rangle|^2 e^{-2iE_0T}\right)^{-1} \underbrace{\langle 0|U(T,-T)|0\rangle}_{\text{denom}}$$

$$\text{denom} = e^{\sum_i V_i}$$

$$e^{\sum_i V_i} = |\langle 0|\Omega\rangle|^2 e^{-2iE_0T}$$

$$V_i \propto 2T(\mathrm{Vol}) = (2\pi)^4\delta^{(4)}(0)$$

$$\sum_i V_i = -iE_0(2T) + O(1)$$

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$$\frac{E_0}{\text{Volume}} = \frac{i \sum_i V_i}{(2\pi)^4 \delta^{(4)}(0)}$$ - independent of T & Volume

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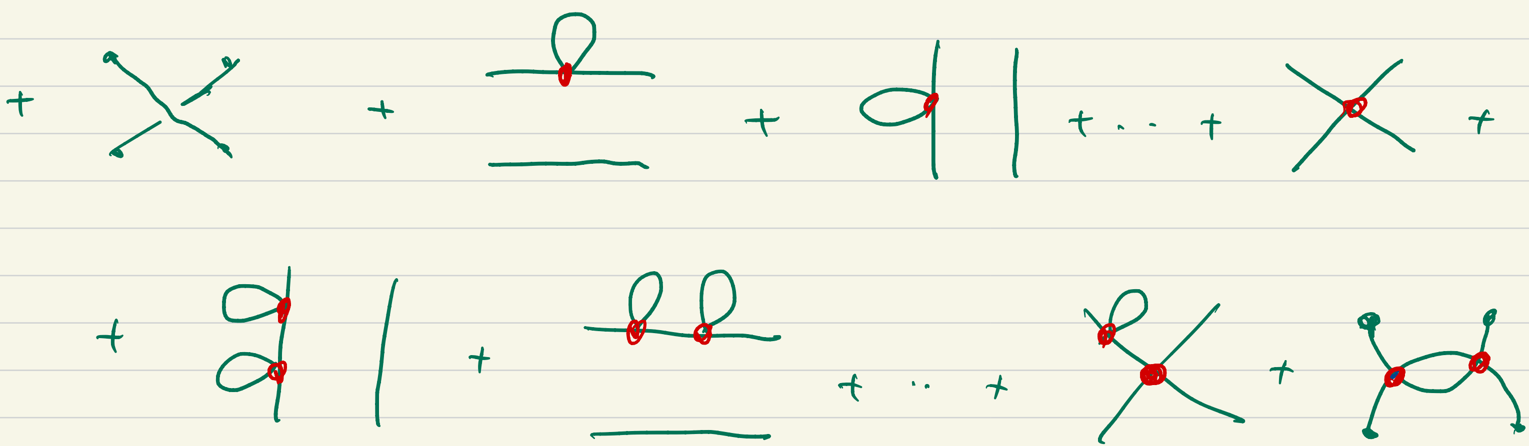
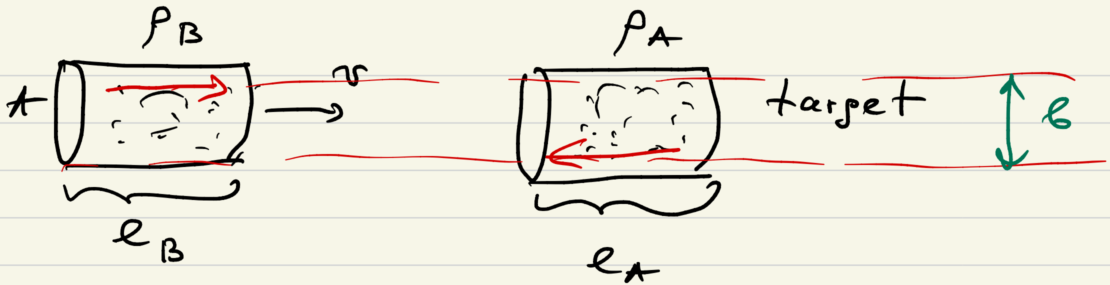
finite energy density

$$\langle \Omega | T \phi(x_1) \ldots \phi(x_n) | \Omega \rangle = \sum \text{all connected diagrams with } n \text{ external pts.}$$

<u>Example:</u> $\langle \Omega | T \phi_1 \phi_2 \phi_3 \phi_4 | \Omega \rangle =$





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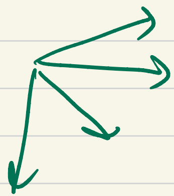
N = # of scattering events

$$N \propto \rho_A \ell_A \rho_B \ell_B A$$

$$\sigma = \frac{N}{\rho_A \rho_B \ell_A \ell_B A} \qquad [\sigma] = \text{area}$$

$$|in\rangle = |p_A \, p_B\rangle$$

$$|out\rangle = |p_1 \dots p_n\rangle$$



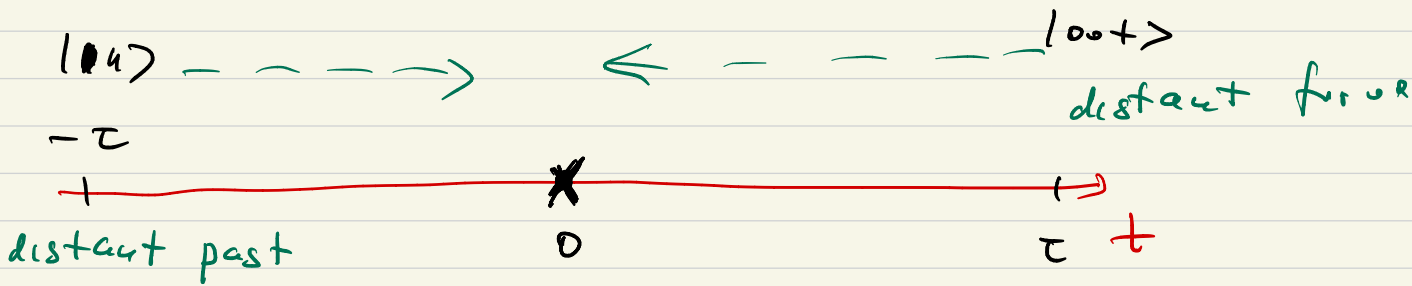
dN — # of scatt. events into $(p_1, p_1 + dp_1), \dots$
$\dots (p_n, p_n + dp_n)$

$$d\sigma = \frac{dN}{\rho_A \, \rho_B \, \ell_A \, \ell_B \, A}$$

$$dN = \sum_{in} \mathcal{P}(in)$$

φ_{in} is related to overlap between

$$e^{-iH\tau}|in\rangle \quad \& \quad e^{iH\tau}|out\rangle$$



$$\lim_{\tau\to\infty} \langle out| e^{-iH(2\tau)} |in\rangle = \langle out| S |in\rangle$$

scattering matrix

$S|in\rangle$

S-matrix

If no interactions $S = \mathbb{1}$

Define T-matrix $S = \mathbb{1} + i\,T$

Momentum conservation $p_A + p_B = \sum p_f$

$$\langle \text{out} | \, i\,T \, | \text{in} \rangle = (2\pi)^4 \, \delta^{(4)}\left(p_A + p_B - \sum p_f\right) \times$$

$$\times \; i\,\mathcal{M}\left(p_A, p_B \to p_f\right)$$

$i\mathcal{M}$ - invariant matrix element

$$\mathcal{P}(AB \to 12\ldots n) = \left(\prod_f \frac{d^3p_f}{(2\pi)^3}\frac{1}{2E_f}\right) \left|\langle \text{out} | iT | \text{in}\rangle\right|^2$$

$$d\sigma = \frac{1}{2E_A\, 2E_B\, |v_A - v_B|}\left(\prod_f \frac{d^3p_f}{(2\pi)^3}\frac{1}{2E_f}\right)\left|\mathcal{M}(p_A, p_B \to \{p_f\})\right|^2 \times (2\pi)^4\,\delta^{(4)}\left(p_A + p_B - \sum p_f\right)$$

$$\int d\Pi_n = \prod_f \int \underbrace{\frac{d^3p_f}{(2\pi)^3}\frac{1}{2E_f}}_{\text{Lorentz inv}} (2\pi)^4\,\delta^{(4)}\underbrace{\left(\underline{P} - \sum p_f\right)}_{\text{Lorentz inv cond}}$$

$\int d\Pi_n$ – invariant phase space volume

$$\frac{1}{E_A E_B |v_A - v_B|} = \frac{1}{|E_B p_A^z - E_A p^z|} =$$

$\vec{p} = \gamma m \vec{v}$

$E = \gamma m$

$v_A = p_A^z / E_A$

$v_B = p_B^z / E_B$

$$= \frac{1}{|\varepsilon_{\mu x y \nu} p_A^\mu p_B^\nu|}$$

– xy comp. of 2nd rank tensor

$T^{\mu\nu} \sim A^\mu A^\nu$

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$A^x A^y$