Student Instructional Rating Survey:

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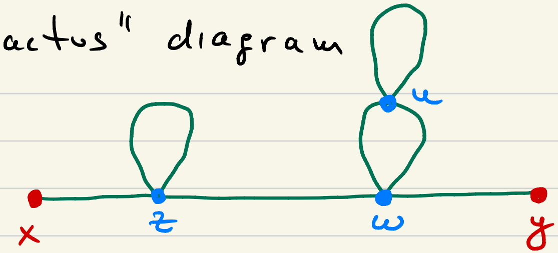
"Cactus" diagram



$\int \underbrace{\phi\phi}\,\underbrace{\phi\phi}$

– represents all $N = 10{,}368$ identical terms

$$\langle 0|\, \phi(x)\, \phi(y)\, \frac{1}{3!}\left(\frac{-i\lambda}{4!}\right)^3 \int d^4z\, \phi\,\phi\,\phi\,\phi \int d^4w\, \phi\,\phi\,\phi\,\phi \int d^4u\, \phi\,\phi\,\phi\,\phi\, |0\rangle$$

N of "different" contractions that give this same expression

$3!$	interchange of z, w, u vertices
$4 \cdot 3$	placement of contractions into z vertex
$4 \cdot 3 \cdot 2$	placement of contractions into w vertex
$4 \cdot 3$	placement of contractions into u vertex
$1/2$	interchange of w & u contractions

$N = 10{,}368$

1. Draw diagram

2. But then $N = ?$

$$\left(\frac{1}{n!} \text{ from } e^{\cdots}\right) \times (n! \text{ permutations of vertices}) = 1$$

$$(\text{Generic vertex} - 4!) \times \left(-\frac{i\lambda}{4!}\right) = -i\lambda$$

$\Rightarrow$ associate $(-i\lambda)$ with each vertex

This scheme: $N_{auto} = n! \, (4!)^n$

For cactus: $N_{auto} = 3! \, (4!)^3$

$$N_{actual} = \frac{3! \, (4!)^3}{2 \cdot 2 \cdot 2} = \frac{N_{auto}}{8} = \frac{N_{auto}}{S}$$

For cactus! $S = 8$

S – symmetry factor of the diagram

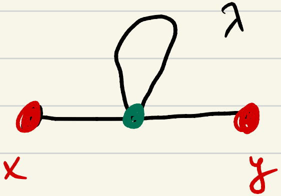
1. 2 – from z loop
   2 – from u loop

2. 2 – from interchanging $D_F(\omega - u)$ propagators

3. equivalence of 2 vertices

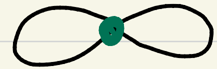
Examples:

λ



$S = 2$

λ



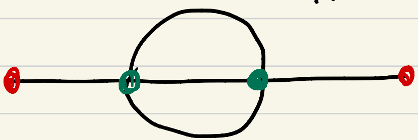
$$\frac{4!}{S} = 3$$

$S = 2 \cdot 2 \cdot 2 = 8$

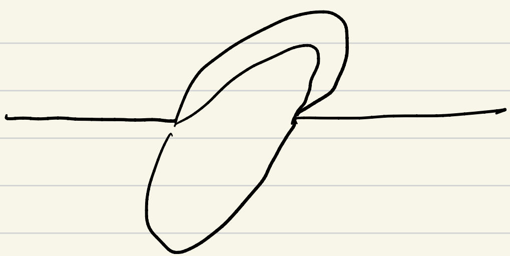
$$\int \phi \phi \phi \phi$$

3

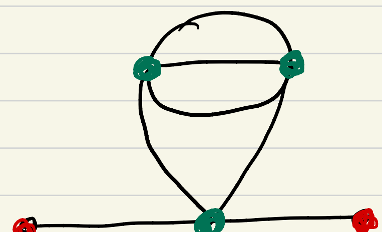
λ^2



$S = 3! = 6$



λ^3



1 2
3 4

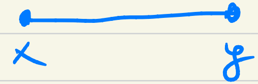
1 3
2 4

1 4
2 3

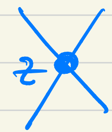
$S = 3! \cdot 2 = 12$

Diagram = propagators / vertices / external pts

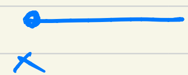
Feynmann rules for ϕ^4 (position space)

1. For each propagator 

$= D_F(x-y)$

2. For each vertex 

$= (-i\lambda)\int d^4 z$

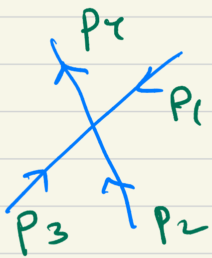
3. For each external pt 

$= 1$

4. Divide by S

Momentum space!

$$D_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2-m^2+i\varepsilon} e^{-ip\cdot(x-y)}$$

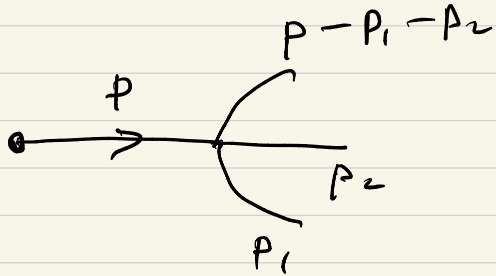


$$\int d^4z \, e^{-ip_1 z} e^{-ip_2 z} e^{-ip_3 z} e^{ip_4 z} =$$

$$= (2\pi)^4 \delta(p_1+p_2+p_3-p_4)$$

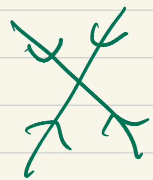
$$\tilde{D}_F(p) = \tilde{D}_F(-p)$$

$\Rightarrow$ momentum is conserved @ each vertex



Momentum space Feynmann rules

1. $\forall$ propagator $\longrightarrow \; = \dfrac{i}{p^2 - m^2 + i\varepsilon}$

2. $\forall$ vertex  $= -i\lambda$

3. $\forall$ external pt $\bullet\!\longleftarrow \; = e^{-ip\cdot x}$

4. Impose momentum conservation $\forall$ vertex

5. Integrate over all undetermined momenta

$$\int \frac{d^4 p}{(2\pi)^4}$$

6. Divide by S

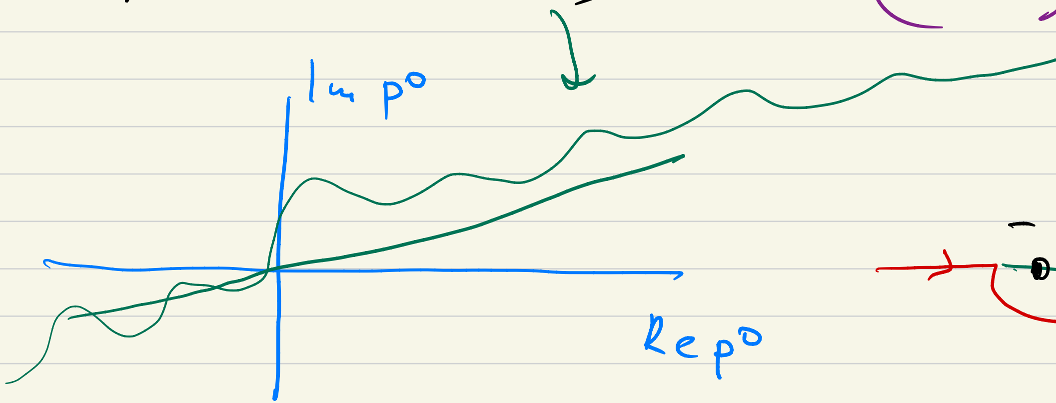
$$\lim_{T\to\infty(1-i\varepsilon)} \int_{-T}^{T} dz^0 \int d^3z \; e^{-i(p_1+p_2+p_3-p_4)\cdot z}$$

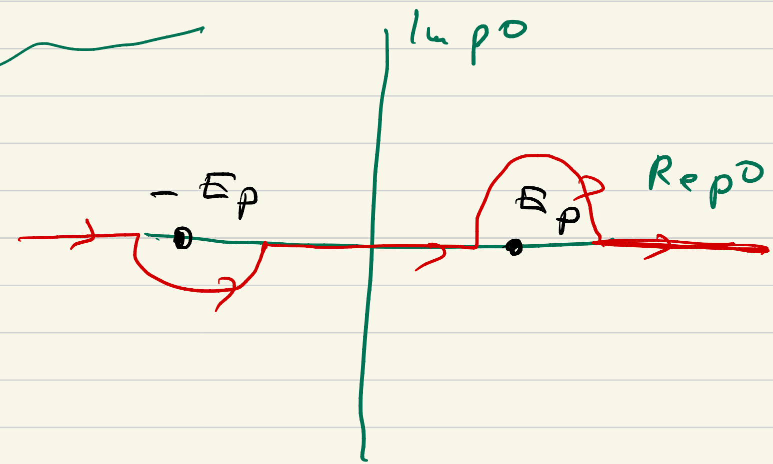
$O(1)$

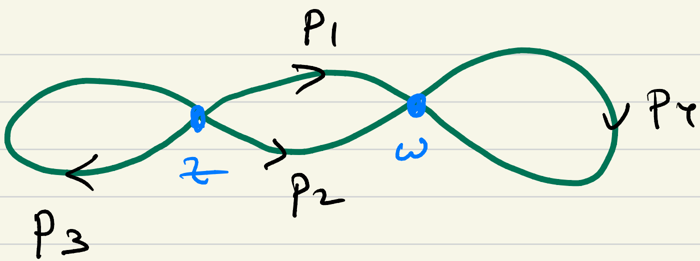
$$(-i)\;\pm\infty(1-i\varepsilon)\,p^0$$

$$p^0 = \#\,(1+i\varepsilon)$$

$(1-i\varepsilon)(1+i\varepsilon)$ – real







z-vertex: $(2\pi)^4\,\delta(p_1+p_2)$

w-vertex: $(2\pi)^4\,\delta(p_1+p_2) =$

$= (2\pi)^4\,\delta(0)$

$$\int dx\ \delta(x)\ \delta(x)\ f(x) = f(0)\,\delta(0)$$

$$\int d^4w\ e^{-i(p_1+p_2)w} = \int d^4w = \int_{-T}^{T} dw^0 \int d^3w =$$

$= 2T$ (volume of space)

$\forall$ disconnected piece $\quad (2\pi)^4 \delta(0) = 2TV$

$E \propto V$