Last time:

$$\mathcal{H} = \frac{\pi^2}{2} + \frac{m^2\phi^2}{2} + \frac{\lambda}{4!}\phi^4 \qquad \mathcal{L} = \frac{(\partial_\mu \phi)^2}{2} - \frac{m^2\phi^2}{2} - \frac{\lambda}{4!}\phi^4$$

phi-fourth theory. Simplest interacting QFT.

$$\mathcal{H} = \frac{\pi^2}{2} + \frac{m^2\phi^2}{2} + \frac{\lambda}{4!}\phi^4 \qquad \mathcal{L} = \frac{(\partial_\mu \phi)^2}{2} - \frac{m^2\phi^2}{2} - \frac{\lambda}{4!}\phi^4$$

phi-fourth theory. Simplest interacting QFT.

<u>Today:</u> Perturbation expansion of correlation functions

2-point correlation fn $\equiv$ 2-pt Green's fn in ϕ^4 theory

$\langle \Omega | T\phi(x)\phi(y) | \Omega \rangle$ - amplitude of propagation between x & y

$|\Omega\rangle$ - gr. st. of int. theory $\qquad |0\rangle$ - gr. st. of free theory

Free theory: $\langle 0 | T\phi(x)\phi(y) | 0 \rangle_{\text{free}} = D_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i e^{-ip\cdot(x-y)}}{p^2 - m^2 + i\varepsilon}$

Now: $H = H_0 + H_{int} = H_{KG} + \int d^3x \, \frac{\lambda}{4!}\phi^4(\vec{x})$

H_{int} enters in 1. $\phi(x) = e^{iHt}\phi(\vec{x})\,e^{-iHt}$

2. $|\Omega\rangle$

Let's begin with $\phi(x)$

$\forall$ fixed time t_0

$$\phi(t_0, \vec{x}) = \int_{\vec{p}} \frac{1}{\sqrt{2E_{\vec{p}}}} \left(a_{\vec{p}}\, e^{i\vec{p}\cdot\vec{x}} + a^{\dagger}_{\vec{p}}\, e^{-i\vec{p}\cdot\vec{x}} \right)$$

@ $t \neq t_0$

$$\phi(t, \vec{x}) = e^{iH(t-t_0)}\phi(t_0, \vec{x})\,e^{-iH(t-t_0)}$$

For $\lambda = 0$ $H \longrightarrow H_0$

$$\phi(t, \vec{x})\Big|_{\lambda=0} = e^{iH_0(t-t_0)}\phi(t_0, \vec{x})\,e^{-iH_0(t-t_0)} \equiv \phi_I(t, \vec{x})$$

Int. picture field

$$\phi_I(t,x) = \int_{\vec{p}} \frac{1}{\sqrt{2E_{\vec{p}}}} \left(a_{\vec{p}}\, e^{-ip\cdot x} + a^{\dagger}_{\vec{p}}\, e^{ip\cdot x} \right) \Big|_{x^0 = t - t^0}$$

$$\phi(t,\vec{x}) = \underbrace{e^{iH(t-t_0)}\, e^{-iH_0(t-t_0)}}_{U^{\dagger}(t,t_0)}\ \phi_I(t,\vec{x})\ \underbrace{e^{iH_0(t-t_0)}\ e^{-iH(t-t_0)}}_{U(t,t_0)} =$$

$$= U^{\dagger}(t,t_0)\ \phi_I(t,x)\ U(t,t_0)$$

$$U(t,t_0) = e^{iH_0(t-t_0)}\ e^{-iH(t-t_0)}$$

– int picture propagator
or time evol. opt

$$U(t,t_0) = e^{iH_0(t-t_0)}\, e^{-iH(t-t_0)} \qquad e^{A} e^{B} \neq e^{A+B}$$

$$i\frac{\partial}{\partial t} U(t,t_0) = e^{iH_0(t-t_0)} \overbrace{(-H_0 + H)}^{H_{int}} e^{-iH(t-t_0)} =$$

$$= \overbrace{e^{iH_0(t-t_0)} H_{int}\, e^{-iH_0(t-t_0)}}^{H_I(t)}\, \underbrace{e^{iH_0(t-t_0)} e^{-iH(t-t_0)}}_{U(t,t_0)}$$

$$e^{iH_0(t-t_0)} \phi(t_0,\vec{x})\, e^{-iH_0(t-t_0)} = \phi_I(x)$$

$$e^{iH_0(t-t_0)} O(t_0,\vec{x})\, e^{-iH_0(t-t_0)} = O(x)$$

$$i\frac{\partial}{\partial t} U(t,t_0) = H_I(t)\, U(t,t_0)$$

$$i \frac{\partial}{\partial t} U(t, t_0) = H_I(t)\, U(t, t_0)$$

$$\frac{\partial}{\partial t} U = (-i) H_I U$$

$$U(t_0, t_0) = 1$$

0. 0-th order $\quad U(t, t_0) = 1$

1. 1st order

$$\frac{\partial}{\partial t} U = (-i) H_I$$

$$U(t, t_0) = 1 + (-i) \int_{t_0}^{t} dt_1\, H_I(t_1)$$

2. 2nd order

$$\frac{\partial}{\partial t} U = (-i) H_I + (-i)^2 H_I(t) \int_{t_0}^{t} dt_1\, H_I(t_1)$$

$$U(t, t_0) = 1 + (-i) \int_{t_0}^{t} H_I(t_1) dt_1 + (-i)^2 \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2\, H_I(t_1) H_I(t_2)$$

$$U(t, t_0) = 1 + (-i)\int_{t_0}^{t} H_I(t_1)\,dt_1 + (-i)^2 \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2\, H_I(t_1)\, H_I(t_2) + \dots$$

$$+ \dots (-i)^n \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \dots \int_{t_0}^{t_{n-1}} dt_n \quad H_I(t_1)\, H_I(t_2) \dots H_I(t_n)$$

$$\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \, H_I(t_1) H_I(t_2) = \frac{1}{2} \int_{t_0}^{t} dt_1 \int_{t_0}^{t} dt_2 \, \underbrace{T H_I(t_1) H_I(t_2)}$$

$t_2 < t_1 \qquad H_I(t_1) H_I(t_2)$

$t_1 < t_2 \qquad H_I(t_2) H_I(t_1)$

Similarly

$$\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \cdots \int_{t_0}^{t_{n-1}} dt_n \; H_I(t_1) \cdots H_I(t_n) = \frac{1}{n!} \int_{t_0}^{t} dt_1 \int_{t_0}^{t} dt_2 \cdots \int_{t_0}^{t} dt_n \, T H_I(t_1) \cdots H_I(t_n)$$

$$U(t, t_0) = 1 + (-i) \int_{t_0}^{t_1} dt_1 \; H(t_1) + \frac{(-i)^2}{2!} \iint_{t_0}^{t} dt_1 \, dt_2 \; T \; H_I(t_1) H_I(t_2)_f$$

$$+ \cdots = T \exp\left[-i \int_{t_0}^{t} dt' \; H_I(t') \right]$$

$$U(t, t') = T \exp\left[-i \int_{t'}^{t} dt'' \; H_I(t'') \right]$$

$$i \frac{\partial}{\partial t} U(t, t') = H_I(t) \; U(t, t') \quad \text{with b.c.} \quad U = 1 \text{ @ } t = t'$$

$$U(t,t') = e^{iH_0(t-t_0)} e^{-iH(t-t')} e^{-iH_0(t'-t_0)}$$

$$U(t,t_0) = e^{iH_0(t-t_0)} e^{-iH(t-t_0)}$$

$$i\frac{\partial}{\partial t} U(t,t') = H_I(t)\, U(t,t')$$

$$U(t',t_0) = e^{iH_0(t'-t_0)} e^{-iH(t'-t_0)}$$

$$U(t,t') = e^{iH_0(t-t_0)} e^{-iH(t-t')} e^{-iH_0(t'-t_0)}$$