Summary of C, P & T:

	$\bar{\psi}\psi$	$i\bar{\psi}\gamma^5\psi$	$\bar{\psi}\gamma^\mu\psi$	$\bar{\psi}\gamma^\mu\gamma^5\psi$	$\bar{\psi}\sigma^{\mu\nu}\psi$	∂_μ
P	+1	−1	$(-1)^\mu$	$-(-1)^\mu$	$(-1)^\mu(-1)^\nu$	$(-1)^\mu$
T	+1	−1	$(-1)^\mu$	$(-1)^\mu$	$-(-1)^\mu(-1)^\nu$	$-(-1)^\mu$
C	+1	+1	−1	+1	−1	+1
CPT	+1	+1	−1	−1	+1	−1

$(-1)^\mu = 1$ for $\mu = 0$

$(-1)^\mu = -1$ for $\mu = 1, 2, 3$

$\bar{\psi}\sigma^{\mu\nu}\psi \quad \bar{\psi}\partial_\mu\psi \quad \bar{\psi}\partial_\nu\psi$

$P = 1$, $T = -1$, $C = -1$

$CPT = 1$

Dirac Lagrangian: $\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi$

inv under C, P & T separately

1st term $\sim \bar{\psi}\gamma^\mu\psi \cdot i\partial_\mu$

P: $(-1)^\mu(-1)^\mu = 1$ T: $(-1)^\mu[-(-1)^\mu](-1) = 1$

$C: \quad -1 \longrightarrow +1 \quad (\text{since } \partial_\mu^T = -\partial_\mu)$ $\qquad CPT = +1$

Recall: $C \bar{\psi} \psi C = \psi^T \bar{\psi}^T = (\bar{\psi}\psi)^T = \bar{\psi}\psi$

$$C \bar{\psi} C \partial_\mu C \psi C = \psi^T \partial_\mu \bar{\psi}^T = -(\bar{\psi}\partial_\mu \psi)^T = -\bar{\psi}\partial_\mu \psi$$

$\partial_\mu^T = -\partial_\mu$ $\qquad \partial_\mu = -\partial_\mu^T$

$$\langle \alpha | O \beta \rangle = \langle O^\dagger \alpha | \beta \rangle \qquad O^\dagger = (O^T)^* = (O^*)^T$$

$\partial_\mu^\dagger = -\partial_\mu$ $\qquad \partial_\mu^* = \partial_\mu$

$$\int \alpha^* \partial_\mu \beta = -\int \beta \, \partial_\mu \alpha^* = \qquad (i\partial_\mu)^\dagger = i\partial_\mu$$

Can add $\delta\mathcal{L}$ violating P or C or T

e.g. P & C $\qquad \delta\mathcal{L} = \bar{\psi}\partial^\mu\psi \;\; \bar{\psi}\partial_\mu\partial^5\psi$

$P=-1, \quad C=-1, \quad T=+1 \qquad$ CPT $=+1$

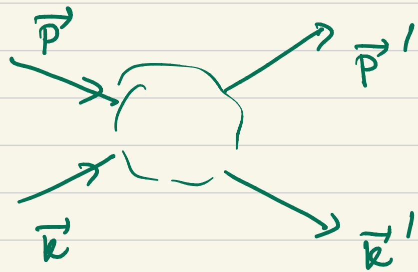
T: $\qquad \bar{\psi}\,\sigma^{\mu\nu}\psi \;\; \bar{\psi}\partial_\mu\psi \;\; \bar{\psi}\partial_\nu\psi$

<u>Thm</u>: $\nexists$ Lorentz-inv QFT with $H^\dagger = H$ that violates CPT

Interacting field theories

KG theory: $\phi(x) = \int_{\vec p} \frac{1}{\sqrt{2E_{\vec p}}}\left(a_{\vec p}\, e^{-ip\cdot x} + a^{+}_{\vec p}\, e^{ip\cdot x}\right)$

2-body int:



$a^{+}_{\vec p\,'}\, a^{+}_{\vec k\,'}\, a_{\vec p}\, a_{\vec k} \in \phi^4$

$\phi^2(x)\,\phi^2(y)$

similarly:

3-body ϕ^6

4-body ϕ^8

$\vdots$

n-body ϕ^{2n}

$\phi^2(x)\,\phi^2(y)$ — non local

Dimensional analysis ~~(poor man's RG)~~

$$S = \int \underbrace{dt\, d^3x}_{M^{-4}} \mathcal{L} \quad \Rightarrow \quad \mathcal{L} \sim M^4$$

S | $\hbar$

$\hbar = c = 1 \qquad L \equiv T \equiv M^{-1} \qquad m \equiv E \equiv \frac{1}{T} \equiv \frac{1}{L}$

$m \equiv E \equiv M$

KG: $\mathcal{L}_0 = \frac{(\partial_\mu \phi)^2}{2} - \frac{m^2 \phi^2}{2}$

Dirac: $\mathcal{L}_0 = \bar{\psi}\,(i\gamma^\mu \partial_\mu - m)\,\psi$

KG: $\mathcal{L}_0 = \frac{(\partial_\mu \phi)^2}{2} - \frac{m^2 \phi^2}{2}$

$$\frac{\partial \phi}{\partial x^\mu} \equiv \frac{\phi}{L}$$

Dirac: $\mathcal{L}_0 = \bar{\psi}\,(i\gamma^\mu \partial_\mu - m)\,\psi$

$$\frac{\phi^2}{L^2} - m^2\phi^2 \sim M^4 \implies \phi \sim M$$

ϕ dimension 1

$$\frac{\psi^2}{L} - m\psi^2 \sim M^4 \implies \psi^2 \sim M^3, \quad \psi \sim M^{3/2}$$

ψ – dim $3/2$

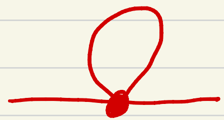
What int can we have?

scalar theory: $\propto \phi^n$

<u>Axiom:</u> Theory must be renormalizable

Consider e.g. $\langle \phi(x)\, \phi(y) \rangle = \int_{\vec{p}} \frac{e^{-i p\cdot(x-y)}}{2 E_{\vec{p}}}$

Let $x = y$



$$\langle \phi^2(x) \rangle \propto \int_0^\infty \frac{p^2\, dp}{\sqrt{p^2+m^2}} \rightarrow \int_0^\Lambda \frac{p^2\, dp}{p} = \frac{\Lambda^2}{2}$$

Need to regularize: cutoff regularization $\infty \rightarrow \overset{\text{cutoff}}{\Lambda}$

In the end, $\Lambda \to \infty$. Physical quantities — should remain finite, i.e, Λ - independent

$$\alpha \phi^n \sim M^4 \Rightarrow \alpha \sim M^{4-n}$$

$\phi \sim M$

$$\text{scat. ampl.} \sim \alpha \Lambda^k = M^{4-n} \Lambda^{n-4} = \left(\frac{M}{\Lambda}\right)^{4-n}$$

$\#$

$$n \le 4 \Rightarrow \mathcal{L}_{int} \sim \mu \phi^3, \ \lambda \phi^4$$

Must have coupling $\sim \mu^k$ with $k \geq 0$

$4-n \geq 0$

Dirac field ψ:

1) ~~self-int~~ $\psi^3 \sim \mu^{9/2}$ $4 - n = 4 - \frac{9}{2} < 0$

Only possible int: $g \bar{\psi} \psi \phi$ ϕ^3 ϕ^4

$\frac{3}{2} + \frac{3}{2} + 1 = 4$ ok

Yukawa theory $\mathcal{L} = \mathcal{L}_{Dirac} + \mathcal{L}_{KG} - g \bar{\psi} \psi \phi$

Scalar QED $\mathcal{L} = |D_\mu \phi|^2 - m^2 |\phi|^2$

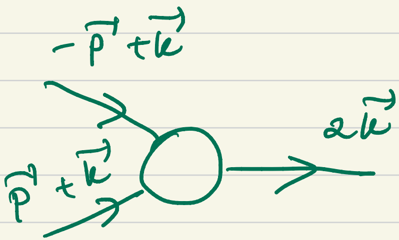
$D_\mu \equiv \partial_\mu + i e A_\mu(x)$ gauge covariant derivative

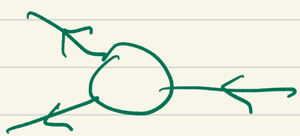
$e^2 |\phi|^2 A^2$ $e A^\mu \phi \partial_\mu \phi$ $e \bar{\psi} \partial^\mu \psi A_\mu$

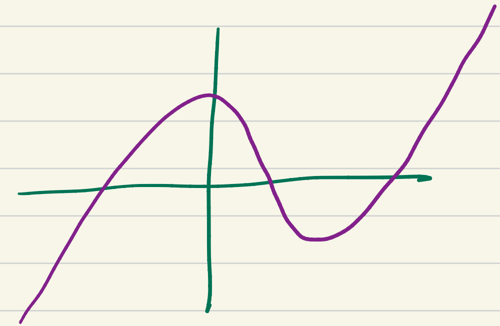
Back to k f:

1) $\mu\phi^3$ $$\mathcal{H} = \frac{\pi^2}{2} + \frac{m^2\phi^2}{2} + \mu\phi^3$$

h. o. $$\frac{p^2}{2} + \frac{\omega^2 x^2}{2} + \mu x^3$$







$E_{gr.st} \to -\infty$

unbounded spectrum

$$\mathcal{H} = \frac{\pi^2}{2} + \frac{m^2 \phi^2}{2} + \frac{\lambda}{4!} \phi^4$$

$$\mathcal{L} = \frac{(\partial_\mu \phi)^2}{2} - \frac{m^2 \phi^2}{2} - \frac{\lambda}{4!} \phi^4$$

phi-fourth theory.