Time reversal T: $\quad T\,\psi(x)\,T = -\gamma^1\gamma^3\,\psi(\tilde{x})$

$$T\,\bar{\psi}(x)\,T = \bar{\psi}(\tilde{x})\,\gamma^1\gamma^3$$

$x = (t, \vec{x})$

$\tilde{x} = (-t, \vec{x})$

1. scalar

$$T\,\bar{\psi}\underset{\substack{\uparrow \\ T^2=1}}{}\psi(x)\,T = \bar{\psi}\,\gamma^1\gamma^3(-\gamma^1\gamma^3)\,\psi = \bar{\psi}\,\psi(\tilde{x})$$

2. pseudoscalar

$$T\,i\bar{\psi}\,\gamma^5\psi\,T = -i\,\bar{\psi}\,\gamma^1\gamma^3\,\gamma^5(-\gamma^1\gamma^3)\,\psi = -i\,\bar{\psi}\,\gamma^5\psi(\tilde{x})$$

3. vector

$$(\gamma^i)^T = \left(\sigma^{2T} \quad -\sigma^{2T}\right. \qquad \sigma^{2T} = -\sigma^2$$

$$T\,\bar{\psi}\gamma^\mu\psi\,T = \bar{\psi}\,\gamma^1\gamma^3(\gamma^\mu)^*(-\gamma^1\gamma^3)\,\psi = \begin{cases} \bar{\psi}\gamma^\mu\psi(\tilde{x}) & \mu=0 \\ -\bar{\psi}\gamma^\mu\psi(\tilde{x}) & \mu=1,2,3 \end{cases}$$

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \qquad (\gamma^2)^* = -\gamma^2$$

$$(\gamma^{1,3})^* = \gamma^{1,3}$$

$$(E, \vec{p}) \to (E, -\vec{p})$$

$$(t, \vec{x}) \to (-t, \vec{x}) \qquad J = (\rho, \underbrace{\vec{J}}_{\rho\vec{v}})$$

4. pseudovector – same as vector

Charge conjugation C

fermion $\longleftrightarrow$ antifermion

Define $\quad C\, a^s_{\vec{p}}\, C = b^s_{\vec{p}} \qquad C\, b^s_{\vec{p}}\, C = a^s_{\vec{p}}$

C - linear, $\quad C^\dagger = C = C^{-1}$

'' ξ^*

$$\xi^{-s} = -i\,\sigma^2 \left(\xi^s\right)^*$$

$$v^s(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\ \xi^{-s} \\ -\sqrt{p\cdot\bar{\sigma}}\ \xi^{-s} \end{pmatrix}$$

$$u^s(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\ \xi \\ \sqrt{p\cdot\bar{\sigma}}\ \xi \end{pmatrix}$$

$$\left[v^s(p)\right]^* = \begin{pmatrix} \sqrt{p\cdot\sigma}\ \left(-i\,\sigma^2\xi^*\right) \\ -\sqrt{p\cdot\bar{\sigma}}\ \left(-i\,\sigma^2\xi^*\right) \end{pmatrix}^*$$

$$\sqrt{p\cdot\sigma}\ \sigma^2 = \sigma^2\sqrt{p\cdot\bar\sigma}^*$$

$$\sqrt{p\cdot\bar\sigma}\ \sigma^2 = \sigma^2\sqrt{p\cdot\sigma}^*$$

cf. $\sqrt{\tilde p\cdot\sigma}\ \sigma^2 = \sigma^2\sqrt{p\cdot\sigma}^*$

$$A\,B = B\,\tilde{A}$$

Choose $\hat z \parallel \vec p$ $\quad p = (E,\ 0,\ 0,\ p^3)$

$$\sigma = (1,\ \vec\sigma) \qquad \bar\sigma = (1,\ -\vec\sigma)$$

$$\sqrt{p\cdot\sigma} = \begin{pmatrix} \sqrt{E+p^3} & 0 \\ 0 & \sqrt{E-p^3} \end{pmatrix} = \sqrt{E+p^3}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + \sqrt{E-p^3}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\sqrt{p\cdot\sigma} = \sqrt{E+p^3}\ \frac{1+\sigma_3}{2} + \sqrt{E-p^3}\ \frac{1-\sigma_3}{2}$$

$$\overset{\shortparallel}{\frac{1+\sigma^3}{2}} \qquad \frac{1-\sigma^3}{2}$$

$$\sigma_3 \sigma_2 = -\sigma_2 \sigma_3$$

$$\sqrt{p\cdot\sigma}\,\sigma_2 = \sqrt{E+p^3}\,\frac{1+\sigma_3}{2}\,\sigma_2 + \sqrt{E-p^3}\,\frac{1-\sigma_3}{2}\,\sigma_2$$

$$\sqrt{p\cdot\bar{\sigma}}^{*} = \sqrt{E+p^3}\,\frac{1-\sigma_3}{2} + \sqrt{E-p^3}\,\frac{1+\sigma_3}{2}$$

$$\sqrt{p\cdot\sigma}\;\sigma^2 = \sigma^2\sqrt{p\cdot\bar{\sigma}}^{*}$$

$$f(p\cdot\sigma)\;\sigma^2 = \sigma^2 f(p\cdot\bar{\sigma}^{*})$$

$$\left[v^s(p)\right]^* = \begin{pmatrix} \sqrt{p\cdot\sigma}\,\left(-i\sigma^2\xi^*\right) \\ -\sqrt{p\cdot\bar\sigma}\,\left(-i\sigma^2\xi^*\right) \end{pmatrix}^* = \begin{bmatrix} -i\sigma^2\sqrt{p\cdot\bar\sigma}^*\,\xi^* \\ i\sigma^2\sqrt{p\cdot\sigma}^*\,\xi^* \end{bmatrix}^* =$$

$$= \begin{pmatrix} -i\sigma^2\sqrt{p\cdot\bar\sigma}\,\xi \\ i\sigma^2\sqrt{p\cdot\sigma}\,\xi \end{pmatrix} = \begin{pmatrix} 0 & -i\sigma^2 \\ i\sigma^2 & 0 \end{pmatrix}\begin{pmatrix} \sqrt{p\cdot\sigma}\,\xi \\ \sqrt{p\cdot\bar\sigma}\,\xi \end{pmatrix}$$

$$u^s(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\,\xi \\ \sqrt{p\cdot\bar\sigma}\,\xi \end{pmatrix} \qquad \left[v^s(p)\right]^* = -i\gamma^2\,u^s(p)$$

$$v^s(p) = -i\gamma^2 \left(u^s(p)\right)^* \qquad u^s(p) = -i\gamma^2 \left[v^s(p)\right]^* \qquad \begin{pmatrix} v & v \end{pmatrix}$$

$$\psi(x) = \int_{\vec{p}} \frac{1}{\sqrt{2E_{\vec{p}}}} \sum_s \left(a^{s\dagger}_{\vec{p}} \left(u^s(p)\right)^* e^{+ip\cdot x} + b^s_{\vec{p}} \left(v^s(p)\right)^* e^{-ip\cdot x} \right) = \begin{pmatrix} v \\ v \end{pmatrix}$$

$$C\psi(x)C = (-i\gamma^2) \int_{\vec{p}} \frac{1}{\sqrt{2E_{\vec{p}}}} \sum_s \left(b^s_{\vec{p}} \left[v^s(p)\right]^* e^{-ip\cdot x} + \right.$$

$$\left. a^{s\dagger}_{\vec{p}} \left(u^s(p)\right)^* e^{ip\cdot x} \right) = -i\gamma^2 \psi^*(x)$$

$$\psi^\dagger = \left(\psi^*\right)^T \qquad \psi^* = \left(\psi^\dagger\right)^T$$

$$C\bar{\psi}C = C\psi^{+}C\gamma^0 = (C\psi C)^{+}\gamma^0 = (-i\gamma^2\psi)^T\gamma^0 =$$

$$\overset{4}{-i\gamma^2\psi^*} \qquad = (-i\gamma^0\gamma^2\psi)^T$$

$$C\psi^{+}C = (-i\gamma^2\psi)^T$$

$$C\psi C = -i\gamma^2(\psi^{+})^T = -i(\bar{\psi}^{+}\gamma^2)^T \qquad \gamma^{2T} = -\gamma^2$$

$$C\bar{\psi}CC\psi C = (-i\gamma^0\gamma^2\psi)^T(i\bar{\psi}\gamma^0\gamma^2)^T =$$

$$= -\psi^T\gamma^2\gamma^0\gamma^2\gamma^0\bar{\psi}^T = +\psi^T\bar{\psi}^T = +(\bar{\psi}\psi)^T =$$

$$= \bar{\psi}\psi$$