Time-reversal $\qquad T\, c\# = (c\#)^{*}\, T$

$T^{-1} = T = T^{+}$ $\qquad$ antiunitary / antilinear

$\vec{p} \to -\vec{p} \qquad \vec{e} \to -\vec{e}$

$a^{s}_{\vec{p}} \to a^{-s}_{-\vec{p}} \qquad b^{s}_{\vec{p}} \to b^{-s}_{-\vec{p}}$

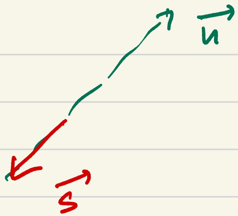
$\vec{n} = (\theta, \varphi)$

$$\xi(+) = \begin{pmatrix} \cos\frac{\theta}{2} \\ e^{i\varphi}\sin\frac{\theta}{2} \end{pmatrix} e^{i\alpha}$$

$\vec{s}\cdot\vec{n} \qquad \vec{s} = \dfrac{\vec{\sigma}}{2}$

$\vec{\sigma}\cdot\vec{n}$

$\vec{n} = (\sin\theta\cos\varphi,\ \sin\theta\sin\varphi,\ \cos\theta)$



$$\xi(+) = \begin{pmatrix} \cos\frac{\theta}{2} \\ e^{i\varphi}\sin\frac{\theta}{2} \end{pmatrix}$$

$$\xi(-) = \begin{pmatrix} -e^{-i\varphi}\sin\frac{\theta}{2} \\ \cos\frac{\theta}{2} \end{pmatrix}$$

$$\xi = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

Coherent state of $s = 1/2, 1, \dots$: $\exists\, \vec{n}$ $\quad \vec{S}\cdot\vec{n}\,|\uparrow\rangle = s\,|\uparrow\rangle$

Thm: spin-$1/2$ is always in a coherent state

For $s = 1, 3/2, 2, \dots$ coherent states — mes $= 0$

$$\xi = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \qquad \xi(\uparrow) = \begin{pmatrix} \cos\frac{\theta}{2} \\ e^{i\varphi}\sin\frac{\theta}{2} \end{pmatrix} e^{i\alpha}$$

$$|c_1|^2 + |c_2|^2 = 1$$

$$c_1 = \cos\frac{\theta}{2}\, e^{i\alpha}$$

$$c_2 = \sin\frac{\theta}{2}\, e^{i(\alpha+\varphi)}$$

Arbitrary S $\begin{pmatrix} \times \\ \vdots \\ \times \end{pmatrix}$ $2S+1$ — complex #s

Coherent state — only 2 real #s — θ & φ

$$2(2S+1) - \underset{\uparrow}{1} - \underset{\uparrow}{1} = 4S \text{ real \#s}$$

normalization — overall phase

Need: $4S = 2 \Rightarrow S = 1/2$

$$\xi^s = \left\{ \overset{\binom{1}{0}}{\xi(+)} \,,\; \overset{\binom{0}{1}}{\xi(-)} \right\} \qquad s = 1, 2 \qquad \vec{n}\cdot\vec{\sigma}\,\xi = \pm\xi$$

Flipped spinor: $\xi^{-s} = -i\,\sigma^2 (\xi^s)^*$ $\qquad \vec{\sigma}\,\sigma^2 = \sigma^2(-\vec{\sigma}^*)$

e.g. $\xi^{-1} = -i \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} \cos\frac{\theta}{2} \\ e^{i\varphi}\sin\frac{\theta}{2} \end{pmatrix}^* =$

$$= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \cos\frac{\theta}{2} \\ e^{-i\varphi}\sin\frac{\theta}{2} \end{pmatrix} = \xi(-)$$

$\xi^{-2} = -\xi(+)$ $\qquad \xi^{-s} = \left\{ \underset{\binom{0}{1}}{\xi(-)} \,,\; \underset{-\binom{1}{0}}{-\xi(+)} \right\}$

$$\xi^{-(-s)} = -\xi^s$$

$a^s_{\vec{p}}$: $\quad v^s(p) \longrightarrow \xi^s$ $\qquad (\partial^\mu p_\mu - m)\, v(p) = 0$

$b^s_{\vec{p}}$ $\qquad v^s(p) = \begin{pmatrix} \sqrt{p\cdot\sigma^*}\;\xi^{-s} \\ -\sqrt{p\cdot\bar{\sigma}^*}\;\xi^{-s} \end{pmatrix}$ $\qquad \begin{pmatrix} \sqrt{p\cdot\bar{\sigma}}\;\xi^{s} \\ \sqrt{p\cdot\sigma}\;\xi^{s} \end{pmatrix}$

$a^s_{\vec{p}} = \{ a^1_{\vec{p}},\ a^2_{\vec{p}} \}$ $\qquad b^s_{\vec{p}} = \dots$

Define $\quad a^{-s}_{\vec{p}} = \{ a^2_{\vec{p}},\ -a^1_{\vec{p}} \}$ $\qquad b^{-s}_{\vec{p}} = \{ b^2_{\vec{p}},\ b^1_{-\vec{p}} \}$

$\tilde{p} = (p^0, -\vec{p})$ $\qquad u^s(p)$ $\qquad v^{-s}(\tilde{p})$

$$v^{-s}(\tilde{p}) = \begin{pmatrix} \sqrt{\tilde{p}\cdot\sigma}\,(-i\,\sigma^2\xi^*) \\ \sqrt{\tilde{p}\cdot\bar{\sigma}}\,(-i\,\sigma^2\xi^*) \end{pmatrix} =$$

Identity

$$\sqrt{\tilde{p}\cdot\sigma}\,\sigma^2 = \sigma^2\sqrt{p\cdot\sigma^*}$$

$$p\cdot\sigma = \tilde{p}\cdot\bar{\sigma}$$

$$= \begin{pmatrix} -i\,\sigma^2\sqrt{\tilde{p}\cdot\sigma^*}\,\xi^* \\ -i\,\sigma^2\sqrt{\tilde{p}\cdot\bar{\sigma}^*}\,\xi^* \end{pmatrix} =$$

$$\Sigma^k = \frac{1}{2}\begin{pmatrix} \sigma^k & 0 \\ 0 & \sigma^k \end{pmatrix}$$

$$= \underbrace{-i\begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}}_{\gamma^1\gamma^3}\left[u^s(p)\right]^* = -\gamma^1\gamma^3\left[u^s(p)\right]^*$$

$$\gamma^3\gamma^1 v^{-s}(\tilde{p}) = +\cancel{\gamma^1}\,\cancel{\gamma^3}\left[u^s(p)\right]^*$$

$$T\, a^s_{\vec p}\, T = a^{-s}_{-\vec p} \qquad T\, b^s_{\vec p}\, T = -b^{-s}_{-\vec p}$$

$$f(x) = \int_{\vec p} \frac{1}{\sqrt{2E_{\vec p}}} \sum_s \left(a^s_{\vec p}\, u^s(p)\, e^{-ip\cdot x} + b^{s\dagger}_{\vec p}\, v^s(p)\, e^{ip\cdot x} \right)$$

$$T f(x) T = \int_{\vec p} \frac{1}{\sqrt{2E_{\vec p}}} \sum_s \left(a^{-s}_{-\vec p} [u^s(p)]^* e^{+ip\cdot x} - \left(b^{-s}_{-p}\right)^\dagger [v^s(p)]^* e^{-ip\cdot x} \right)$$

$$\tilde p = (p^0, -\vec p) \qquad p\cdot x = p^0 t - \vec p\cdot\vec x = -\tilde p \cdot \underbrace{(-t, \vec x)}_{\tilde x}$$

$$p\cdot x = -\tilde p\cdot\tilde x$$

$$T\psi(x)T = \int_{\vec{p}} \frac{1}{\sqrt{2E_{\vec{p}}}} \sum_s \left(a^{-s}_{-\vec{p}} [u^s(p)]^* e^{+ip\cdot x} - \left(b^{-s}_{-p}\right)^{+} [v^s(p)]^* e^{-ip\cdot x} \right)$$

$$-\gamma^1\gamma^3 \int_{\tilde{\vec{p}}} \frac{1}{\sqrt{2E_{\tilde{\vec{p}}}}} \sum_s \left(a^{-s}_{\tilde{\vec{p}}}\, u^{-s}(\tilde{p}) e^{-i\tilde{p}\cdot\tilde{x}} + v^{-s}(\tilde{p})\, b^{-s\dagger}_{\tilde{\vec{p}}}\, e^{i\tilde{p}\cdot\tilde{x}} \right)$$

$$\psi(x) = \int_{\vec{p}} \frac{1}{\sqrt{2E_{\vec{p}}}} \sum_s \left(a^s_{\vec{p}}\, u^s(p)\, e^{-ip\cdot x} + b^{s\dagger}_{\vec{p}}\, v^s(p)\, e^{ip\cdot x} \right)$$

$$T\psi(t,\vec{x})T = -\gamma^1\gamma^3\, \psi(-t,\vec{x}) \qquad \tilde{x} = (-t,\vec{x})$$

$$[v^s(p)]^* = -\gamma^1\gamma^3\, v^{-s}(\tilde{p})$$

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $a+b=b+a$

$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$