Parity P: $(t, \vec{x}) \to (t, -\vec{x})$

$1, P$

1) discrete group $P^2 = 1$ [Z_2 cyclic group]

Unitary $\Leftarrow$ { 2) linear
3) inner product preserving } on Hilbert space $a^{s\dagger}_{\vec{p}}|0\rangle$

$P^\dagger = P^{-1} = P$ $\Rightarrow$ Hermitian

$\uparrow$ unitary

$\uparrow$ follows from $P^2 = 1$

$\eta_{a,b} = e^{i\alpha_{a,b}}$

Need: $a^{s\dagger}_{\vec{p}}|0\rangle \to a^{s\dagger}_{-\vec{p}}|0\rangle \Rightarrow P a^s_{\vec{p}} P = \eta_a a^s_{-\vec{p}}$, $P b^s_{\vec{p}} P = \eta_b b^s_{-\vec{p}}$

Observables: $O \xrightarrow[P]{} \eta_a^2 O \xrightarrow[P^2]{} \eta_a^4 O \Rightarrow \eta_a^2 = \pm 1$

Action on Dirac field: $\psi(t,\vec{x}) \xrightarrow[P]{} M\psi(t,-\vec{x}) \xrightarrow[P^2=1]{} M^2\psi(t,\vec{x})$

If M is a rep of P, $\quad M^2=1$

$$P\, a^s_{\vec{p}}\, P = \eta_a\, a^s_{-\vec{p}}$$

$$P\, b^s_{\vec{p}}\, P = \eta_b\, b^s_{-\vec{p}}$$

$$\psi(x) = \int_{\vec{p}} \frac{1}{\sqrt{2E_{\vec{p}}}} \sum_s \left[a^s_{\vec{p}}\, u^s(p)\, e^{-ip\cdot x} + b^{s\dagger}_{\vec{p}}\, v^s(p)\, e^{ip\cdot x} \right]$$

$$\Big\downarrow P$$

$$P\psi(x)P = \int_{\vec{p}} \frac{1}{\sqrt{2E_{\vec{p}}}} \sum_s \left[P a^s_{\vec{p}} P\, u^s(p)\, e^{-ip\cdot x} + P\, b^{s\dagger}_{\vec{p}} P\, v^s(p)\, e^{ip\cdot x} \right]$$

$$\|$$

$$P\psi(x)P = \int_{\vec{p}} \frac{1}{\sqrt{2E_{\vec{p}}}} \sum_s \left[\eta_a a^s_{-\vec{p}}\, u^s(p)\, e^{-ip\cdot x} + \eta_b^{*}\, b^{s\dagger}_{-\vec{p}}\, v^s(p)\, e^{ip\cdot x} \right]$$

$$P\psi(x)P = \int_{\vec{p}} \frac{1}{\sqrt{2E_{\vec{p}}}} \sum_s \left[\eta_a a^s_{-\vec{p}}\, u^s(p)\, e^{-i\tilde{p}\cdot\tilde{x}} + \eta_b^* b^{s\dagger}_{-\vec{p}}\, v^s(p)\, e^{i\tilde{p}\cdot\tilde{x}} \right]$$

$$\tilde{p} = (p^0, -\vec{p}) \qquad\qquad p\cdot x = \tilde{p}\cdot \underbrace{(t, -\vec{x})}_{\tilde{x}}$$

Recall: $\sigma = (1, \vec{\sigma}) \qquad \bar{\sigma} = (1, -\vec{\sigma})$

$$\tilde{p}\cdot\sigma = p\cdot\bar{\sigma} \qquad\qquad \tilde{p}\cdot\bar{\sigma} = p\cdot\sigma$$

$$u(p) = \begin{pmatrix} \sqrt{p\cdot\sigma}\,\xi \\ \sqrt{p\cdot\bar{\sigma}}\,\xi \end{pmatrix} = \begin{pmatrix} \sqrt{\tilde{p}\cdot\bar{\sigma}}\,\xi \\ \sqrt{\tilde{p}\cdot\sigma}\,\xi \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}_{\gamma^0} \begin{pmatrix} \sqrt{\tilde{p}\cdot\sigma}\,\xi \\ \sqrt{\tilde{p}\cdot\bar{\sigma}}\,\xi \end{pmatrix}$$

$$u(p) = \gamma^0\, u(\tilde{p})$$

$$v(p) = -\gamma^0\, v(\tilde{p})$$

$$P\psi(x)P = \int_{\tilde{\vec{p}}} \frac{1}{\sqrt{2E_{\vec{p}}}} \sum_s \left[\eta_a a^s_{-\vec{p}} \gamma^0 u^s(\tilde{p}) e^{-i\tilde{p}\cdot\tilde{x}} \cancel{+} - \eta_b^* b^{s\dagger}_{-\vec{p}} \gamma^0 v^s(\tilde{p}) e^{i\tilde{p}\cdot\tilde{x}} \right]$$

$$\psi(t,\vec{x}) = \int_{\vec{p}} \frac{1}{\sqrt{2E_{\vec{p}}}} \sum_s \left[a^s_{\vec{p}} u^s(p) e^{-ip\cdot x} + b^{s\dagger}_{\vec{p}} v^s(p) e^{ip\cdot x} \right]$$

$$\eta_a = -\eta_b^* \qquad\qquad -\vec{p} \equiv \tilde{\vec{p}}$$

$$P\psi(x)P = \eta_a \gamma^0 \int_{\tilde{\vec{p}}} \frac{1}{\sqrt{2E_{\tilde{\vec{p}}}}} \sum_s \left[a^s_{\tilde{\vec{p}}} u^s(\tilde{p}) e^{-i\tilde{p}\cdot\tilde{x}} + b^{s\dagger}_{\tilde{\vec{p}}} v^s(\tilde{p}) e^{i\tilde{p}\cdot\tilde{x}} \right]$$

$$P\psi(t,\vec{x})P = \eta_a \gamma^0 \psi(t,-\vec{x})$$

$$M = \eta_a \gamma^0$$

Set $\eta_a = -\eta_b = 1$ $\qquad\qquad\qquad$ $\tilde{\psi} = \psi(t, -\vec{x})$

Recall: 5 Dirac billinears

$\bar{\psi}\psi$, $\quad \bar{\psi}\gamma^\mu\psi$, $\quad i\bar{\psi}[\gamma^\mu, \gamma^\nu]\psi$, $\quad \bar{\psi}\gamma^\mu\gamma^5\psi$, $\quad i\bar{\psi}\gamma^5\psi$

$$P\,\bar{\psi}(t,\vec{x})\,P = P\,\psi^\dagger(t,\vec{x})\,P\,\gamma^0 = \left(P\,\psi(t,\vec{x})\,P\right)^\dagger\gamma^0 =$$

$$= \left[\gamma^0\psi(t,-\vec{x})\right]^\dagger\gamma^0 = \psi^\dagger(t,-\vec{x})\,\gamma^0\gamma^0 = \bar{\psi}(t,-\vec{x})\,\gamma^0$$

$$P\,\bar{\psi}\underset{P^2}{\psi}\,P = \bar{\tilde{\psi}}\,\tilde{\psi}$$ true scalar

01 $\quad$ 02 $\quad$ 03 $\qquad\qquad$ 00

12 $\quad$ 13 $\quad$ 23 ...

$$P\,\bar{\psi}\,\gamma^{\mu}\,\psi\,P = \bar{\tilde{\psi}}\,\gamma^0\gamma^{\mu}\gamma^0\,\tilde{\psi} = \begin{cases} +\bar{\tilde{\psi}}\,\gamma^{\mu}\,\tilde{\psi} & \mu=0 \\ -\bar{\tilde{\psi}}\,\gamma^{\mu}\,\tilde{\psi} & \mu\neq 0 \quad \mu=1,2,3 \end{cases}$$

$$x^{\mu} = (t, \vec{x}) \to (t, -\vec{x})$$

true vector $\uparrow L_+$

$$P\,i\,\bar{\psi}\,\gamma^5\,\psi\,P = i\,\bar{\tilde{\psi}}\,\gamma^0\gamma^5\gamma^0\,\tilde{\psi} = -i\,\bar{\tilde{\psi}}\,\gamma^5\,\tilde{\psi}$$

pseudoscalar

$$P\,\bar{\psi}\,\gamma^{\mu}\gamma^5\,\psi\,P = \bar{\tilde{\psi}}\,\gamma^0\gamma^{\mu}\gamma^5\gamma^0\,\tilde{\psi} = \begin{cases} -\bar{\tilde{\psi}}\,\gamma^{\mu}\gamma^5\,\tilde{\psi} & \mu=0 \\ +\bar{\tilde{\psi}}\,\gamma^{\mu}\gamma^5\,\tilde{\psi} & \mu\neq 0 \quad \mu=1,2,3 \end{cases}$$

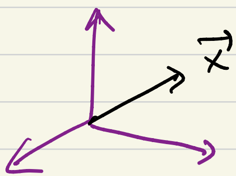
pseudovector

$x^{\mu}x^{\nu}$

$a^{\mu}x^{\nu}$ $\quad a^{\mu}a^{\nu}$

Axial & polar vectors

pseudo   true



$$\vec{x} \to -\vec{x}$$

polar

$$\vec{x} \times \vec{y} \to \vec{x} \times \vec{y}$$

axial

$$\dot{\vec{x}} \to -\dot{\vec{x}} \qquad \vec{p} \to -\vec{p}$$

$$\vec{x} \times \vec{p} \to \vec{x} \times \vec{p}$$

$\vec{x} \cdot \vec{p}$ — true scalar

$\vec{x} \cdot (\vec{y} \times \vec{p})$ — pseudoscalar

$(\vec{x}_1 \times \vec{p}_1) \cdot (\vec{x}_2 \times \vec{p}_2)$ — true scalar

$$a^{s\dagger}_{\vec{p}}\, b^{r\dagger}_{\vec{q}}\, |0\rangle \xrightarrow{P} -\, a^{s\dagger}_{-\vec{p}}\, b^{r\dagger}_{-\vec{q}}\, |0\rangle$$

Useful for bound states

onium s

- e^+ positronium
- e^-

Time reversal

$(t, \vec{x}) \to (-t, \vec{x})$ $\qquad O \xrightarrow{S} S^{-1} O S$

$\| \leftarrow$ if unitary

$S^\dagger$

Want $\quad T \psi(t, \vec{x}) T = M \psi(-t, \vec{x})$

$T = T^{-1}$

$p \cdot x = E_p t - \vec{p} \cdot \vec{x}$

$$\psi(x) = \int_{\vec{p}} \frac{1}{\sqrt{2E_{\vec{p}}}} \sum_s \left[a^s_{\vec{p}} u^s e^{-iE_p t} e^{i\vec{p}\cdot\vec{x}} + b^{s\dagger}_{\vec{p}} v^s e^{iE_p t} e^{-i\vec{p}\cdot\vec{x}} \right]$$

$$T \psi(t, \vec{x}) T |0\rangle = M \psi(-t, \vec{x}) |0\rangle$$

$$e^{iE_p t} \neq e^{-iE_p t}$$

Solution: $\quad T\, c\# = (c\#)^* \, T$

$$a_{\vec{p}} \longrightarrow a_{-\vec{p}} \qquad a_{\vec{q}} \longrightarrow a_{-\vec{q}}$$

$$\alpha\, a_{\vec{p}} + \beta\, a_{\vec{q}} \longrightarrow \alpha^* a_{-\vec{p}} + \beta^* a_{-\vec{q}} \neq$$

$$\neq \alpha\, a_{-\vec{p}} + \beta\, a_{-\vec{q}}$$

Antilinear | antiunitary

QM

$$i\frac{\partial \psi}{\partial t} = H\psi$$

$$\psi \to \psi^*$$

Let $H = H^*$

$$-i\frac{\partial \psi^*}{\partial t} = H\psi^*$$

$$\left(-i\hbar\nabla + q\vec{A}\right)^2$$

$$i\frac{\partial \psi^*}{\partial(-t)} = H\psi^*$$

$$t \to -t$$

If @ $t=0$ $\quad \psi^*(0) = \psi(0) \implies \psi^*(-t) = \psi(t)$

Homework # 2 : 3.2, 3.4, 3.7 Due: Nov 19