Dirac propogator

$$S_R(x-y) = (i\not{\partial}_x + m)\, D_R(x-y)$$

Recall: KG retarded Green's fn

$$D_R(x-y) = \theta(x^0-y^0)\langle 0|[\phi(x),\phi(y)]|0\rangle =$$

$$= \int_{\vec{p}} \frac{1}{2E_{\vec{p}}}\left[e^{-ip\cdot(x-y)} - e^{ip\cdot(x-y)}\right]$$

$$(\partial^2 + m^2)\, D_R(x-y) = -i\,\delta^{(4)}(x-y)$$

Define:

$$S_R(x-y) = \theta(x^0-y^0)\langle 0|\{\psi(x),\bar{\psi}(y)\}|0\rangle$$

Recall:

$$\langle 0|\psi(x)\bar{\psi}(y)|0\rangle = (i\not{\partial}_x + m)\int_{\vec{p}} \frac{e^{-ip\cdot(x-y)}}{2E_{\vec{p}}}$$

$$\langle 0|\bar{\psi}(y)\psi(x)|0\rangle = -(i\not{\partial}_x + m)\int_{\vec{p}} \frac{e^{ip\cdot(x-y)}}{2E_{\vec{p}}}$$

$$\gamma^\mu \partial_\mu \gamma_\mu \partial^\mu = \partial_\mu \partial^\mu \mathbb{1}_{4\times4} = \partial^2$$

$$S_R(x-y) = (i\not\partial_x + m)\, D_R(x-y)$$

$$(i\not\partial_x - m)\; S_R(x-y) = i\,\delta^{(4)}(x-y)\,\mathbb{1}_{4\times4}$$

$$\underbrace{(i\not\partial_x - m)(i\not\partial_x + m)}_{=\left(-\not\partial_x^2 - m^2\right)} D_R(x-y) = i\,\delta^{(4)}(x-y)\,\mathbb{1}_{4\times4}$$

$$-(\partial^2 + m^2)\, D_R(x-y) = i\,\delta^{(4)}(x-y)$$

$$(\partial^2 + m^2)\, D_R(x-y) = -i\,\delta^{(4)}(x-y)$$

Find S_R via FT $\quad S_R(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-y)} \tilde{S}_R(p)$

$$(i\not{\partial}_x - m)\, S_R(x-y) = i\,\delta^{(4)}(x-y)\, \mathbb{1}_{4\times4}$$

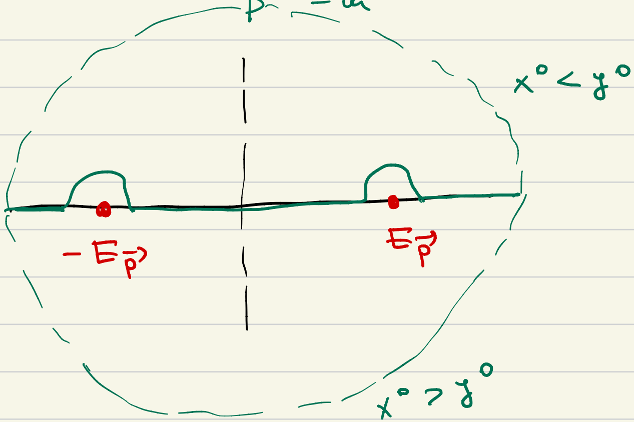
$$\int \frac{d^4p}{(2\pi)^4} (\not{p} - m)\, e^{-ip\cdot(x-y)}\, \tilde{S}_R(p) = i\int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-y)}$$

$$(\not{p} - m)\,\tilde{S}_R(p) = i$$

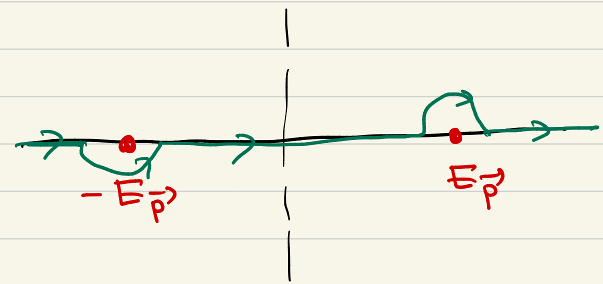
$$\tilde{S}_R(p) = \frac{i}{\not{p} - m} = \frac{i(\not{p} + m)}{p^2 - m^2}$$

$$\tilde{S}_R(p) = \frac{i(\not{p} + m)}{p^2 - m^2}$$

$$S_R(x-y) = \int \frac{d^4p}{(2\pi)^4}\, e^{-ip\cdot(x-y)}\, \tilde{S}_R(p)$$



Feynman b.c.



$$S_F(x-y) = \int \frac{d^4p}{(2\pi)^4}\, \frac{i(\not{p} + m)}{p^2 - m^2 + i\varepsilon}\, e^{-ip\cdot(x-y)} =$$

$$= \begin{cases} \langle \psi(x)\bar{\psi}(y)\rangle, & x^0 > y^0 \\ -\langle \bar{\psi}(y)\psi(x)\rangle, & x^0 < y^0 \end{cases} \equiv \langle T\,\psi(x)\bar{\psi}(y)\rangle$$

Discrete symmetries of Dirac theory

So far: Lorentz transformations $x \to \Lambda x$

$$\psi(x) \to \Lambda_{1/2}\, \psi(\Lambda^{-1} x)$$

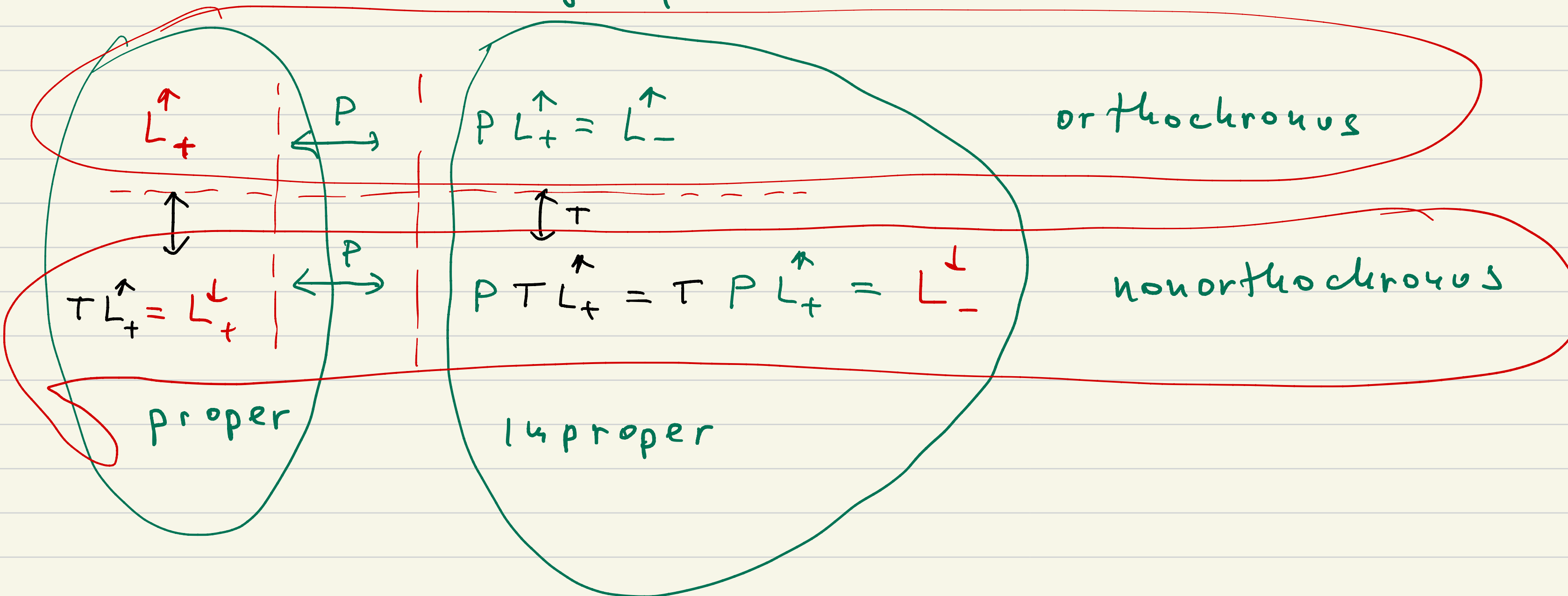
– continous transformation $L_+^\uparrow$

Discrete transformations:

1. parity $P: \quad (t, \vec{x}) \to (t, -\vec{x})$
2. time reversal $(t, \vec{x}) \to (-t, \vec{x})$

Note: $P^2 = 1$, $T^2 = 1 \implies P^{-1} = P \quad T^{-1} = T$

Full Lorentz group



3. Charge conjugation C

particle $\longleftrightarrow$ antiparticle

$C^2 = 1 \qquad C^{-1} = C$

In reality

gravity
EM
strong
} P & T & C

weak int } T, $\cancel{P}$, $\cancel{C}$ and $\cancel{CP}$

CP – symm. between matter & antimatter

Typically symm is a linear operation

e.g. $\vec{x} \to -\vec{x}$ $\quad \vec{y} \to -\vec{y}$ $\quad \alpha\vec{x} + \beta\vec{y} \to -\alpha\vec{x} - \beta\vec{y} =$

$$= \alpha(-\vec{x}) + \beta(-\vec{y})$$

On Hilbert space

- linear
- preserve $\langle \dots | \dots \rangle$

$\Rightarrow$ unitary opt U $\Rightarrow$ $P^\dagger = P^{-1} = P \Rightarrow$ Hermitian

Need : $\quad a^s_{\vec{p}} |0\rangle \longrightarrow a^s_{-\vec{p}} |0\rangle$

$$P\, a^s_{\vec{p}}\, P = \eta_a\, a^s_{-\vec{p}} \qquad P\, b^s_{\vec{p}}\, P = \eta_b\, b^s_{-\vec{p}}$$

$$\eta_{a,b} = e^{i\alpha_{a,b}}$$

$$\mathcal{O} \to \eta_a^2 \,\mathcal{O} \qquad \text{two appl :} \qquad \eta_a^4 \,\mathcal{O} = \mathcal{O}$$

$$\Rightarrow \quad \eta_a^2 = \pm 1 \qquad \eta_b^2 = \pm 1$$

$$M = \gamma^0$$

$$\psi(t, \vec{x}) = M \, \psi(t, -\vec{x})$$

$\hookrightarrow$ const 4×4 matrix

$$M^2 = c \, \mathbb{1} \qquad c^2 = 1$$

$$M \neq \begin{pmatrix} +1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix}$$

$$\psi^\dagger \quad \psi$$

$$\downarrow$$

$$\psi^\dagger (M^\dagger)^2 M^2 \psi$$

$$(M^\dagger)^2 M^2 = \mathbb{1}$$