Quantization of Dirac field $\psi(x)$

$\psi(\vec{x})$. Need $\pi(\vec{x})$

$$\bar{\psi} = \psi^\dagger \gamma^0$$

$$\mathcal{L} = \bar{\psi}\,(i\gamma^\mu\partial_\mu - m)\,\psi = \cancel{i\bar{\psi}\gamma^0\dot{\psi}} + i\bar{\psi}\,\vec{\gamma}\cdot\nabla\psi - m\bar{\psi}\psi$$

$$\gamma^\mu\partial_\mu = \gamma^0\partial_0 + \vec{\gamma}\cdot\nabla \qquad\qquad i\bar{\psi}\gamma^0\dot{\psi} = i\psi^\dagger\dot{\psi}$$

$$h_D = -i\,\vec{\alpha}\cdot\nabla + \beta m$$

$$\pi(\vec{x}) = \frac{\partial \mathcal{L}}{\partial \dot{\psi}(\vec{x})} = i\bar{\psi}\gamma^0 = i\psi^\dagger$$

$$\mathcal{H} = \cancel{\pi\dot{\psi}} - \mathcal{L} = -i\,\psi^\dagger\gamma^0\vec{\gamma}\cdot\nabla\psi + \psi^\dagger\gamma^0 m\,\psi$$

$$H = \int d^3x\; \psi^\dagger \underbrace{\left[-i\gamma^0\vec{\gamma}\cdot\nabla + \gamma^0 m\right]}_{h_D}\psi$$

h_D – Dirac Hamiltonian for one particle

$$\vec{\alpha} = \gamma^0\vec{\gamma} \qquad \beta = \gamma^0$$

$\hat{o}$ - one particle $\qquad \hat{O} = \int d^3x \; \psi^+(x) \; \hat{o} \; \psi(x)$

$$[\psi(\vec{x}), \underset{\substack{\shortparallel \\ i\psi^{+}}}{\pi(\vec{y})}] = i\, \delta^{(3)}(\vec{x}-\vec{y})\, \mathbb{1}_{4\times 4}$$

$$[\psi(\vec{x}), \psi^{+}(\vec{y})] = \delta^{(3)}(\vec{x}-\vec{y})\, \mathbb{1}_{4\times 4}$$

$$[\psi_a(\vec{x}), \psi_b^{+}(\vec{y})] = \delta^{(3)}(\vec{x}-\vec{y})\, \delta_{ab}$$

$$H = \int d^3x \;\; \psi^{+} \underbrace{\left[-i\gamma^0 \vec{\gamma} \cdot \nabla \;\; + \gamma^0 m \right]}_{h_D} \psi$$

$$\left[i\gamma^0 \partial_0 + i\vec{\gamma}\cdot\nabla - m \right] u^s(p)\, e^{-ip\cdot x} = 0$$

$$\underbrace{u^s(\vec{p})\, e^{i\vec{p}\cdot\vec{x}}}_{\Phi}\, e^{-iE_p t}$$

$$p\cdot x = E_p t - \vec{p}\cdot\vec{x}$$

$$\gamma^0 E_p \Phi + (i\vec{\gamma}\cdot\nabla - m)\Phi = 0$$

$$(-i\gamma^0\vec{\gamma}\cdot\nabla + \gamma^0 m)\Phi = E_p \Phi$$

$$h_D \Phi = E_p \Phi$$

$$h_D = \gamma^0 \vec{\gamma}\cdot\vec{p} + \gamma_0 m$$

$u^s(\vec{p})\, e^{i\vec{p}\cdot\vec{x}}$ – eigenstates of h_D

with eigenvalue E_p

$v^s(\vec{p})\, e^{-i\vec{p}\cdot\vec{x}}$ — // — // — eigenvalue $-E_p$

$v^s(-\vec{p})\, e^{i\vec{p}\cdot\vec{x}}$

$$\psi(\vec{x}) = \int_{\vec{p}} \frac{1}{\sqrt{2E_p}} e^{i\vec{p}\cdot\vec{x}} \sum_{s=1,2} \left[a^s_{\vec{p}}\, u^s(\vec{p}) + b^s_{-\vec{p}}\, v^s(-\vec{p}) \right]$$

$$\left[\psi_a(\vec{x}),\ \psi^+_b(\vec{y})\right] = \delta^{(3)}(\vec{x}-\vec{y})\,\delta_{ab}$$

$$\Updownarrow$$

$$\left[a^r_{\vec{p}},\ a^s_{\vec{q}}\right] = \left[b^r_{\vec{p}},\ b^s_{\vec{q}}\right] = (2\pi)^3\,\delta^{(3)}(\vec{p}-\vec{q})\,\delta^{rs}$$

$$\sum_s u^s(p)\,\bar{u}^s(p) = \gamma \cdot p + m$$

$$H = \int d^3x \; \psi^\dagger h_D \psi \qquad \psi(\vec{x}) = \int_{\vec{p}} \frac{1}{\sqrt{2E_p}} e^{i\vec{p}\cdot\vec{x}} \sum_{s=1,2} \left[a^s_{\vec{p}} u^s(\vec{p}) + b^s_{-\vec{p}} v^s(-\vec{p}) \right]$$

$$H = \int d^3x \int_{\vec{p}} \int_{\vec{q}} \frac{1}{\sqrt{2E_p}\sqrt{2E_q}} e^{i(\vec{p}-\vec{q})\vec{x}} \sum_r \left[a^{r\dagger}_q u^{r\dagger}(\vec{q}) + b^{r\dagger}_{-\vec{q}} v^{r\dagger}(-\vec{q}) \right] \times$$

$$\times \sum_s \left[E_p a^s_p u^s(\vec{p}) - E_p b^s_{-p} v^s(-\vec{p}) \right]$$

① $$\int d^3x \; e^{i(\vec{p}-\vec{q})\cdot\vec{x}} = (2\pi)^3 \delta^{(3)}(\vec{p}-\vec{q})$$

$$\frac{1}{(2\pi)^6} \to \frac{1}{(2\pi)^3} \qquad \longrightarrow \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p}$$

$$\sum_r \left[a^{r\dagger}_{\vec{q}}\, u^{r\dagger}(\vec{q}) + b^{r\dagger}_{-\vec{q}}\, v^{r\dagger}(-\vec{q}) \right] \sum_s \left[E_p\, a^s_p\, u^s(\vec{p}) - E_p\, b^s_{-p}\, v^s(-\vec{p}) \right]$$

$$u^{r\dagger}(\vec{p})\, v^s(-\vec{p}) = v^{r\dagger}(-\vec{p})\, u^s(\vec{p})$$

$$u^{r\dagger}(\vec{p})\, u^s(\vec{p}) = v^{r\dagger}(\vec{p})\, v^s(\vec{p}) = 2 E_p\, \delta^{rs}$$

$$H = \int_{\vec{p}} \sum_s \left(E_{\vec{p}}\, a^{s\dagger}_{\vec{p}}\, a^s_{\vec{p}} - E_p\, b^{s\dagger}_{\vec{p}}\, b^s_{\vec{p}} \right)$$

$$E_p = \sqrt{|\vec{p}|^2 + m^2}$$

$$H = \int_{\vec{p}} \sum_s \left(E_{\vec{p}}\, a^{s\dagger}_{\vec{p}} a^s_{\vec{p}} - E_p\, b^{s\dagger}_{\vec{p}} b^s_{\vec{p}} \right)$$

Napoleon

Lenin

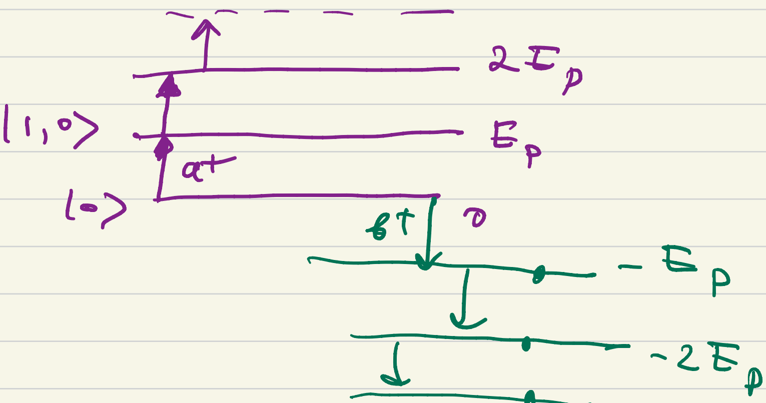
Define $|0\rangle$ $\qquad a^s_{\vec{p}}|0\rangle = b^s_{\vec{p}}|0\rangle = 0$

$b^\dagger \to b$

$[b, b^\dagger] = 1$

b

$[b, b^\dagger] = -1$



$$[\text{Lenin}, \text{Napoleon}] = 1$$