Welcome to Physics 615: Overview of QFT

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Text: Peskin & Schroder "An Introduction to QFT"

website: dep. website $\longrightarrow$ grad. courses

QM (nonrelativistic)

$$\hat{H} = \frac{\hat{\vec{p}}^{\,2}}{2m} + V(\vec{x})$$

Special Relativity

$\Big\}$ QFT

<u>CM:</u> free particle $E = H = \dfrac{\vec{p}^{\,2}}{2m}$

<u>QM:</u> observables $\longrightarrow$ opts

state of system $\longrightarrow$ ψ

$$\vec{p} \longrightarrow \hat{\vec{p}} = -i \frac{\partial}{\partial \vec{x}} = -i \vec{\nabla}$$

$$E \to i\,\partial_t \qquad \partial_t \equiv \frac{\partial}{\partial t}$$

$$E = H \quad \longrightarrow \quad \hat{E}\,\psi = \hat{H}\,\psi$$

$$\hbar\, i\,\partial_t \psi = \frac{\hat{\vec{p}}^{\,2}}{2m}\,\psi \qquad \text{free particle}$$

Schrödinger eq.

$$\hat{H} = \frac{\hat{\vec{p}}^{\,2}}{2m} = -\frac{\hbar^2 \nabla^2}{2m} \qquad \hbar \sim p x \sim E t$$

$\hbar = 1$ units L, t, m

$\hbar = c = 1$ $\quad t = L \quad E = 1/L$

Lorentz inv generalization of Schrödinger eq.?

→ leads to inconsistencies?

QFT solves thru particle creation / annihilation

Two ways

1. Klein-Gordon eq

$$E^2 - p^2 = m^2 \qquad E = \pm\sqrt{p^2+m^2}$$

$$\left(-\partial_t^2 + \nabla^2\right)\phi = m^2\phi$$

✓ states with arbitrary $E<0$

✓ prob. dens. can be <0

$$E = \sqrt{p^2+m^2} > 0$$

$$i\partial_t \psi = \sqrt{-\nabla^2+m^2}\,\psi$$

- nonlocal
- time & space assymetric

$\partial_x \leftrightarrow i\partial_t$

2. Dirac eq.

$$\nabla^2 - \partial_t^2 = (A\partial_x + B\partial_y + C\partial_z + iD\partial_t)^2$$

$$a^2 - b^2 = (\alpha a + \beta b)^2$$

$$\nabla^2 - \partial_t^2 = \left(A\partial_x + B\partial_y + C\partial_z + iD\partial_t \right)^2$$

cross-terms $\quad \partial_x\partial_y \quad \partial_x\partial_z \; \ldots$ $\qquad (\nabla^2 - \partial_t^2)\phi = m^2\phi$

$$\left.\begin{aligned} &AB + BA = 0, \; \ldots \\ &A^2 = B^2 = C^2 = D^2 = 1 \end{aligned}\right\} \text{ need } 4\times4 \text{ matrices}$$

$\times\, e^{i\vec{p}\cdot\vec{x} - iEt}$

$$\left(A\partial_x + B\partial_y + C\partial_z + iD\partial_t \right)\psi = m\psi \quad (1)$$

apply $(\ldots)$ again $\qquad (\nabla^2 - \partial_t^2)\psi = m^2\psi \quad (2)$

$(1) \Rightarrow (2)$ but $(2) \not\Rightarrow (1)$ $\qquad -p^2 + E^2 = m^2$

$A = i\gamma^1, \quad B = i\gamma^2, \quad C = i\gamma^3, \quad D = \gamma^0$

$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

$\partial_\mu = \dfrac{\partial}{\partial x^\mu}$ $\qquad x^0 = t \quad x^1 = x, \; x^2 = y$

$x^3 = z$

$\rightarrow$ states with arbitrary $E < 0$

Causality

$\vec{x}_0 \to \vec{x}$

$$U(t) = \langle \vec{x} | e^{-iHt} | \vec{x}_0 \rangle$$

nonrelativistic QM $\quad E = \frac{p^2}{2m}$ free part.

$$U = \langle \vec{x} | e^{-i\frac{\vec{p}^{\,2}}{2m}t} | \vec{x}_0 \rangle =$$

$$\int d^3p \; |p\rangle\langle p| = 1$$

$$= \int d^3p \; \langle \vec{x} | e^{-i\frac{\vec{p}^{\,2}}{2m}t} | \vec{p} \rangle \langle \vec{p} | x_0 \rangle$$

$$e^{-i\frac{p^2}{2m}}$$

$$\langle \vec{x} | \vec{p} \rangle = \frac{e^{i\vec{p}\cdot\vec{x}}}{(2\pi)^{3/2}}$$

$$U = \frac{1}{(2\pi)^3} \int d^3p \; e^{-i\frac{p^2}{2m}t} \, e^{i\vec{p}\cdot(\vec{x}-\vec{x}_0)}$$

$$= \left(\frac{m}{2\pi i t}\right)^{3/2} e^{im(\vec{x}-\vec{x}_0)^2/2t}$$

Nonzero for $\forall$ $\vec{x}$ & t – violates causality in special relativity

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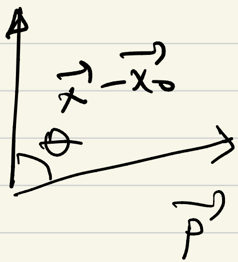
What if $E = \sqrt{p^2+m^2}$?

$$U = \frac{1}{(2\pi)^3} \int d^3p \; e^{-it\sqrt{p^2+m^2}} \; e^{i\vec{p}\cdot(\vec{x}-\vec{x_0})}$$

$$\vec{p}\cdot(\vec{x}-\vec{x_0}) = p\,|\vec{x}-\vec{x_0}|\cos\theta$$

$$d^3p = 2\pi\, p^2\, dp\, d(\cos\theta)$$



$$U = \frac{1}{2\pi^2 |\vec{x} - \vec{x}_0|} \int_0^\infty p\, dp \, \sin\left(p\,|\vec{x}-\vec{x}_0|\right) e^{-it\sqrt{p^2+m^2}}$$

Bessel

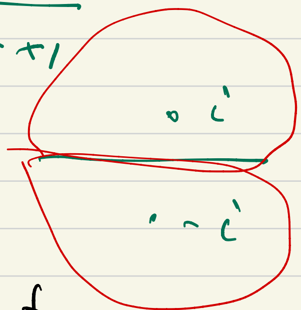
$x >> t$

$$e^{i\left(px - it\sqrt{p^2+m^2}\right)}$$

$\vec{x}_0 = 0$

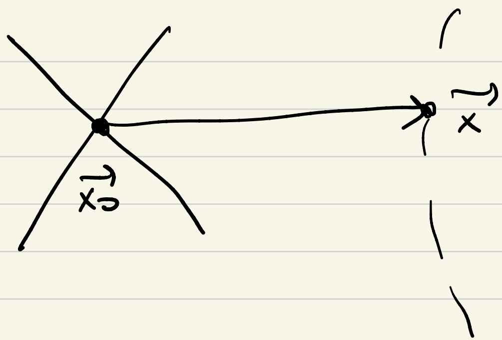
$$\int_0^\infty \frac{dx}{x^2+1}$$

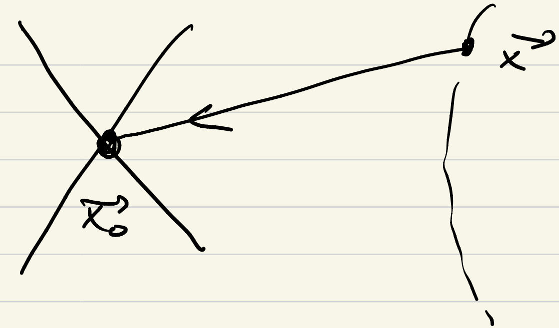
st. phase $\qquad p = \dfrac{imx}{\sqrt{x^2-t^2}}$



$$U \sim e^{-m\sqrt{x^2-t^2}} \neq 0$$

causality violated





particle

$|\text{particle}\rangle + |\text{antipart}\rangle = 0$

causality
is fine