prob/time of scattering into exactly k_f. $P \to dP$

Real prob need to integrate over d^3k_f

proper measure ($k_f \sim \frac{2\pi}{L} n_f$

$\sum_{n_f} \to \frac{V}{(2\pi)^3}\int d^3k$)

$$d\Gamma_{scatt} = \frac{dP_{scatt}}{dt} = \frac{|M|^2 \delta^{(4)}(k_f - k_i)}{E_1 E_2 V} \underbrace{\prod_{i=1}^{n} \frac{d^3k_i}{E_i}}_{\text{L.I. measure!}}$$

Finally need flux Φ for 1 ptcl

in Fixed target frame: $\xrightarrow[v_1]{}\ \bullet \qquad \Phi = \frac{|v_1|}{V}$

in CM frame: $\rightarrow\ \leftarrow \qquad \Phi = \frac{|\vec{v}_1 - \vec{v}_2|}{V}$

$$\boxed{d\sigma_{CM} = \frac{d\Gamma_{scatt}}{\Phi} = \frac{1}{(2E_1)(2E_2)|\vec{v}_1 - \vec{v}_2|} |M|^2 \, d\Pi_n}$$

So — putting in factors

$$\boxed{d\Pi_n = \prod_{j=1}^{n} \frac{d^3p_j}{(2\pi)^3 2E_j} (2\pi)^4 \delta^{(4)}(k_f - k_i)}$$

"Lorentz invariant phase space"

formula for cross section from scattering amplitude

Can also consider <u>decays</u>

$1 \to \{n\}$

$d\Gamma_{decay} = \dfrac{dP}{T}$ = prob to decay / time

same as before $\quad |\langle f|i\rangle|^2 \to |M|^2 \delta^{(4)}(k-k)V\cdot T$

$\langle f|f\rangle\langle i|i\rangle$

$\prod E_i V^n \qquad E_i V$

int over d^3k

Get

$$\boxed{d\Gamma_{decay} = \frac{1}{2E_1}|M|^2 \, d\Pi_n}$$

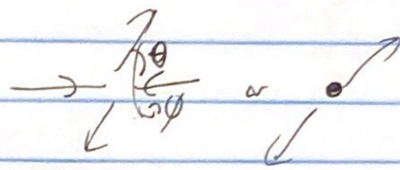
Ex: 2→2 scattering ϕ^3

~~We found~~ $\cancel{M = g^2\left(\frac{1}{s-m^2} + \frac{1}{t-m^2} + \frac{1}{u-m^2}\right)}$

$\cancel{d\sigma_{CM} = \frac{1}{4E_1E_2|\vec{v}_1-\vec{v}_2|}\, g^4(\cdots)^2 \, d\Pi_2}$

Special cases

$d\Pi_2$ in CM frame



$$\vec{p}_3 + \vec{p}_4 = 0 \qquad E_3 = \sqrt{p_3^2+m_3^2} \qquad E_4 = \sqrt{p_4^2+m_4^2}$$

$$d\Pi_2 = \frac{d^3p_3}{(2\pi)^3 2E_3}\frac{d^3p_4}{(2\pi)^3 2E_4}(2\pi)^4 \delta(E_{CM} - E_3 - E_4)\,\delta^{(3)}(\vec{p}_3 - \vec{p}_4)$$

$\downarrow \int d^3p_4 \to E_3 + E_4 = E_{CM}$

$$d\Pi_2 = \frac{d^3p_3}{(2\pi)^2 4E_3E_4}\delta(E_{CM} - E_3 - E_4) = \frac{d\Omega\, dp_3 \cdot p_3^2}{16\pi^2 E_3E_4}\delta(E_{CM} - E_3 - E_4)$$

$$\int dp_3\, \delta(f(p_3)) = \frac{1}{|f'(p_3)|}$$

$$= \frac{1}{\left|\frac{p_3}{\sqrt{p_3^2+m_3^2}} + \frac{p_4}{\sqrt{p_4^2+m_4^2}}\right|} = \frac{E_4E_3}{p_3E_{CM}}$$

$$\boxed{d\Pi_2 = \frac{d\Omega}{16\pi^2}\frac{p_3}{E_{CM}}}$$

$$\frac{d\sigma_{CM}}{d\Omega} = \frac{1}{4E_1E_2\left|\frac{p_1}{E_1}+\frac{p_1}{E_2}\right|}|M|^2\frac{d\Omega\, p_3}{16\pi^2 E_{cm}}$$

$$= \boxed{\frac{1}{64\pi^2 E_{cm}^2}\frac{|\vec{p}_f|}{|\vec{p}_i|}|M|^2} \to \frac{1}{64\pi^2 E_{cm}^2}|M|^2 \text{ (all masses equal)}$$

$1 \to 2$ decay, rest frame

$$d\Gamma = \frac{1}{2m_1}|M|^2 \frac{d\Omega}{16\pi^2}\frac{p_f}{m_1}$$

$$\frac{d\Gamma}{d\Omega} = \frac{1}{32\pi^2}\frac{|\vec{p}_f|}{m_1^2}|M|^2.$$

Ex: $2 \to 2$ in ϕ^3 (

$$iM = g\left(\frac{1}{s-m^2}+\frac{1}{t-m^2}+\frac{1}{u-m^2}\right)$$

$$\boxed{\frac{d\sigma_{cm}}{d\Omega} = \frac{1}{64\pi^2 E_{cm}^2}\, g^2\left(\frac{1}{s-m^2}+\frac{1}{t-m^2}+\frac{1}{u-m^2}\right)^2}$$

$(\sqrt{p_i^2+m^2}, 0, 0, p_i)$

$s = (p_1+p_2)^2$

$= E_{cm}$ $(\sqrt{p_i^2+m^2}, 0, 0, -p_i)$

$= 4(p_i^2+m^2)$

$(E_f = E_i, p_i \sin\theta, 0, p_i \cos\theta)$

$(E_i, 0, 0, p_i)$ $(E_i, 0, 0, -p_i)$

$(E_f, -p_i \sin\theta, 0, -p_i \cos\theta)$

$s = 4(p^2+m^2)$

$t = \left((E_1, p\sin\theta, 0, p\cos\theta) - (E, 0, 0, p)\right)^2$

$= -p^2(\sin^2\theta + (1-\cos\theta)^2)$

$= -2p^2(1-\cos\theta)$

$u = (\ \dots - (E, 0, 0, -p))^2$

$= -2p^2(1+\cos\theta)$

(Note: $s+t+u = \sum m_i^2$ always)