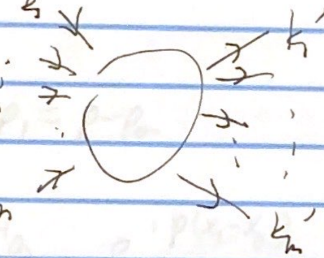
Answer very simple!

→ ~~[illegible]~~ Doing calculation in momentum space much easier!

– for scattering process w/

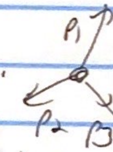
$S = \mathbb{1} + i\delta^{(4)}(\Sigma k_i - \Sigma k_f')\,\mathcal{M}$

"scattering amplitude"



(Momentum space) Feynman Rules (for ϕ^3 theory)

1. write all fully connected diagrams using vertices from $\mathcal{H}_I$ e.g.



value of vtx is g

2. Enforce mom. cons @ each vtx

3. External wavefn factors (1 for scalar)

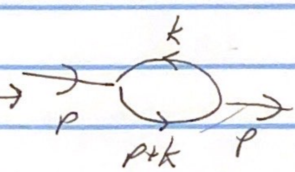
4. ~~Facto~~ Multiply Propagators $\frac{1}{p^2-m^2}$ $\overrightarrow{p}$

That's it! (neglecting $\underline{\text{loops}}$!)

What about loops?

e.g.

$\langle 0|T\phi(x_1)\phi(x_2)|0\rangle \rightarrow$



$\int d^4y_1\, d^4y_2 \;\langle 0|T\,\phi(x_1)\phi(x_2)\,\phi(y_1)^3\phi(y_2)^3|0\rangle$

$D_F(x_1-y_1)\,D_F(x_2-y_2)\,D_F(y_1-y_2)^2$

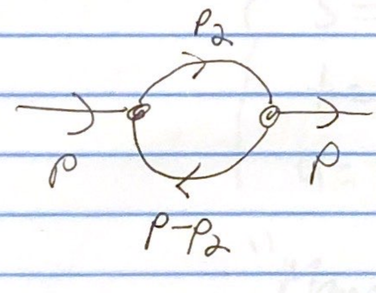
$$\int d^4y_1\, d^4y_2\, d^4p\, d^4q\, d^4p_1\, d^4p_2 \; \frac{\cancel{e^{ip(x_1-y_1)}}\;\cancel{e^{iq(x_2-y_2)}}\;\cancel{e^{i(p_1+p_2)(y_1-y_2)}}}{(p^2-m^2)\,(q^2-m^2)\,\cancel{((p_1+p_2)^2-m^2)}\,(p_1^2-m^2)\,(p_2^2-m^2)}$$

$\delta(p-(p_1+p_2))$

$\delta(q+p_1+p_2)$

so $q=-p \qquad p_1=p-p_2$

$$\int d^4p\, \cancel{d^4}\, d^4p_2 \; \frac{e^{ip(x_1-x_2)}}{(p^2-m^2)^2\,(p_2^2-m^2)\,((p-p_2)^2-m^2)} \xrightarrow{\text{mom space}} \frac{1}{(p^2-m^2)^2}\int \frac{d^4p_2}{(p_2^2-m^2)\,((p-p_2)^2-m^2)}$$



So last Feynman rule: integrate over undetermined "loop" momenta!

Source of infinities in QFT $\longrightarrow$ renormalizati

More on $2 \to 2$ scattering example

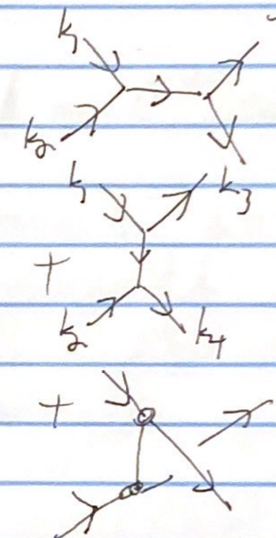
nontrivial part of S-matrix $S = \mathbb{1} + iT$

We found: ~~Turns out $\mathcal{M}$ is~~

$$\langle f|i\rangle = (2\pi)^4 \delta^{(4)}(k_f - k_i)\, \mathcal{M}\big(\phi(k_1)+\phi(k_2) \to \phi(k_3)+\phi(k_4)\big)$$

$$= \frac{1}{(k_1+k_2)^2 - m^2} + \frac{1}{(k_1-k_3)^2 - m^2}$$

$$+ \frac{1}{(k_1-k_4)^2 - m^2}$$



Turns out $\mathcal{M}$ is Lorentz inv't for $2 \to 2$ scattering

only L.I. made out of k_1, k_2, k_3, k_4 are

$$\begin{cases} s = (p_1+p_2)^2 & \to \text{s channel} \\ t = (p_1-p_3)^2 & \text{t channel} \\ u = (p_1-p_4)^2 & \text{u channel} \end{cases}$$

"Mandelstam Invariants"

$$\mathcal{M} = \frac{1}{s-m^2} + \frac{1}{t-m^2} + \frac{1}{u-m^2}$$

Very useful!

From Scattering Amplitude to Cross Section

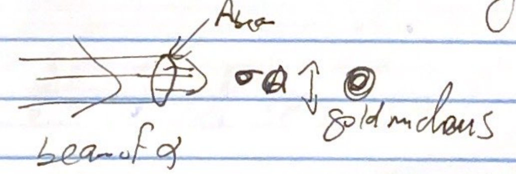
General $2 \to n$ scattering

$$k_1 + k_2 \longrightarrow \{p_{i=1,\dots,n}\}$$

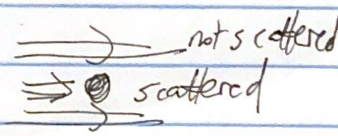
$k_1 = (E_1, \vec{k})$ $\quad k_2 = (m, 0)$ "fixed target"

$k_1 = (E_1, \vec{k})$ $\quad k_2 = (E_2, \vec{k}_2)$ "center of mass"

Consider Rutherford scattering



Simple model: α is scattered if lies in cross sectional area of nucleus otherwise not



$$N_{scatt} = N_{inc} \cdot \frac{\sigma}{A_{beam}}$$

$$\text{Scattering rate} \quad \dot{N}_{scatt} = \frac{N_{inc}}{T\, A_{beam}} \cdot \sigma = \Phi\, \sigma \longrightarrow \text{define cross section more generally!}$$

beam flux Φ: # ptcls/time/area

Also called "instantaneous luminosity" in collider phys $\mathcal{L}$

Γ_{scatt} ← beam of 1 ptcl

Scattering prob rate $\boxed{\dot{\Gamma}_{scatt} = \Phi\, \sigma}$

Also $\frac{d\Gamma_{scatt}}{d\Omega} \to d\sigma/d\Omega$

$L = \int \mathcal{L}\, dt =$ "integrated luminosity" → how much data

$d\Gamma$ more generally $\rightarrow d\sigma$

ptcl physics xsec units: barn $= 10^{-24}\ cm^2$ (joke)

Units

xsecs of various processes @LHC!

$\left\{ \begin{array}{l} \text{femtobarn} = 10^{-15}\ b = 10^{-39}\ cm^2 \\ \text{picobarn} = 10^{-12}\ b \\ \text{nanobarn} = 10^{-9}\ b \\ \text{millibarn} = 10^{-3}\ b \end{array} \right.$

integrated luminosity @LHC $fb^{-1} \rightarrow 139\ fb^{-1}$

amnt of data recorded to date @LHC.

How to get xsec from S-matrix?

Need probability

$$P_{scatt} = \frac{|\langle f|i\rangle|^2}{\langle f|f\rangle\langle i|i\rangle} \rightarrow |M|^2 \left(\delta^{(4)}(k_f-k_i)\right)^2$$

$$= \delta^{(4)}(k_f-k_i)\,\delta^{(4)}(0)$$

regularize: VT

$$\langle k|k\rangle = E_k \underbrace{\delta^{(3)}(0)}_{V}$$

$$\langle i|i\rangle = E_1 E_2 V^2$$

$$\langle f|f\rangle = \prod_{i=1}^{n} E_i V^n$$

$$\boxed{\text{prob/time} = \frac{P_{scatt}}{T} = \frac{|M|^2 \delta^{(4)}(k_f-k_i) \cdot V}{E_1 E_2 \prod E_i\ V^{n+2}}}$$