Friday, January 28, 2022 10:27 AM

$$\sum_{i,j} \nabla_j A_i \, \nabla_j A_i$$

$$\boxed{\mathcal{H} = \underbrace{\frac{1}{2}\Pi_i^2 + \frac{1}{2}(\nabla_j A_i)^2}_{} - J_i A_i + \mathcal{H}_{Coul}}$$

→ calculate many effects in atomic physics from this

$\frac{1}{2}\Pi_i^2 + \frac{1}{2}(\nabla_j A_i)^2$: SHO "free theory"

$J_i A_i$: treat as pert'n.

$\mathcal{H}_{Coul}$: $\frac{1}{2}\int \frac{\rho(x)\rho(y)}{|\vec{x}-\vec{y}|} d^3y$

- mode expansion of free theory
- treat $J_i A_i$ as pert'n.
- use perturbation theory.

1. – $\vec{J}(x)$ just a fn – classical source
2. – $\vec{J}(x)$ as made of QM ptcls – quantum source but non relativistic. → atomic physics
3. – $\vec{J}(x)$ made of Qtm fields

   A_μ & matter on equal footing both Qtm & relativistic.

describe emission/absorption of γ rays in atomic transitions

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Want $J(x)$ as qtm field, cannot describe matter creation/annihilation

e.g. $e^+e^- \to 2\gamma$ electron/positron annihilation

$\gamma + \gamma \to e^+e^-$ " creation | $n \to p + e^- + \bar{\nu}_e$ β decay

Making $J(x)$ into QFT $\longrightarrow$ "QED".

So we want a relativistic QFT for $J(x)$ $\xrightarrow{\text{be concrete}}$ electron.

- What is Lagrangian ? $\xrightarrow{\text{proper}}$ spin ½ representations of Lorentz group
  Hamiltonian ? construct simplest $\mathcal{L}$ consistent w/ symmetries
  eqns of motion ? $\longrightarrow \mathcal{L}_{Dirac}$
  ⋮
  heuristic
  intuitive
  historical development, Dirac originally discovered

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Relativistic QM: what is a relativistic Schrödinger eqn?

ordinary QM: $i\frac{\partial}{\partial t}\psi = H\psi \left(\stackrel{\text{e.g.}}{=} \frac{p^2}{2m}\psi\right)$.

NR

t & x not treated same

$\downarrow$

Try: $i\frac{\partial}{\partial t}\psi = H\psi = \sqrt{p^2+m^2}\,\psi$ still not Lorentz covariant

Try: $-\frac{\partial^2}{\partial t^2}\psi = (p^2+m^2)\psi = (-\nabla^2+m^2)\psi$.

$$\left(-\frac{\partial^2}{\partial t^2}+\nabla^2\right)\psi = m^2\psi$$

$\partial_\mu \partial^\mu \psi = m^2\psi$.

K-G eqn. ↗ cf. Maxwell.

relativistic scalar wave eqn.

Lorentz inv't but not QM

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Problem : 2nd order in $\frac{\partial}{\partial t}$ $\longrightarrow$ $|\psi|^2$ is not a probability (prob. not conserved)

ψ does not specify state of system

need ψ and $\dot{\psi}$

$\downarrow$

KG not eqn for wavefn.

$\downarrow$

turns out it describes qtm fields field eqn. not wavefns.

Try: Dirac make KG linear by enlarg $\psi \longrightarrow \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \end{pmatrix}$

$p^2 + m^2 = \left(\sqrt{p^2+m^2}\right)^2$

$\frac{d^2}{dx^2} f = \cdots$

$\frac{d}{dx}\begin{pmatrix} f \\ \frac{df}{dx} \end{pmatrix} = \cdots$

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$N \times N$

$$(*) \quad \mathbb{1}_N (p^2 + m^2) = (\alpha_x p_x + \alpha_y p_y + \alpha_z p_z + \beta m)^2$$

are there any matrices $\alpha_x, \alpha_y, \alpha_z, \beta$ s.t. $(*)$ is true?

$$N \left\{ \underbrace{\begin{pmatrix} p^2+m^2 & & 0 \\ 0 & p^2+m^2 & \\ & & \ddots \end{pmatrix}}_{N} \right.$$

$$\boxed{i \frac{\partial}{\partial t} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_N \end{pmatrix} = (\alpha_i p_i + \beta m) \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_N \end{pmatrix}}$$

"Dirac eqn"

$$\alpha_x^2 p_x^2 + \alpha_y^2 p_y^2 + \alpha_z^2 p_z^2 + \beta^2 m^2$$

$$+ \; p_x p_y (\alpha_x \alpha_y + \alpha_y \alpha_x) + \cdots$$

$$+ \; m p_x (\alpha_x \beta + \beta \alpha_x) + \cdots$$

$\{A, B\} = AB + BA$

"anticommutator"

eigenvalues ± 1

$$\Rightarrow \quad \alpha_x^2 = \alpha_y^2 = \alpha_z^2 = \beta^2 = 1$$

$$\{\alpha_i, \alpha_j\} = 0 \quad i \neq j$$

$$\{\alpha_i, \beta\} = 0$$

α_i, β must be Hermitian

can show $\mathrm{Tr}\, \alpha_i = \mathrm{Tr}\, \beta = 0$