~~$p' = p + q$~~ ~~$q = p' - p$~~

~~$m^2 = m^2 + 2p\cdot q$~~ ~~$q^2 = 0 = 2m^2 - 2p\cdot p'$~~

Rest frame $(E, -p) \leftarrow \bullet \, (m,0,0,0) \rightarrow (\sqrt{p^2+m^2}, p)$

$p + \sqrt{p^2+m^2} = m \Rightarrow p = 0$

So $q = 0$ and $p' = p$.

But still possible!

$$\boxed{\bar{u}(p')\, V^\mu(p,p')\, u(p) \Big|_{\substack{p' \to p \\ p^2 = m^2}} = e\,\bar{u}(p')\gamma^\mu u(p) = 2ep^\mu}$$ (spinor ident)

Consider more general vtx fn $p^2 = p'^2 = m^2$, $q^2 \neq 0$

$\bar{u}(p')\, V^\mu(p,p')\, u(p)$ what could it be?

$p^\mu, p'^\mu, \gamma^\mu$

↳ any $\not{p}$ or $\not{p}'$ hits $\bar{u}$ or u & gives m by Dirac

$p'^2 = m^2$

$p^2 = m^2$

$p'\cdot p \to q^2$

$$A(q^2)\,p^\mu + B(q^2)\,p'^\mu + C(q^2)\,\gamma^\mu$$

$A = B$ by Ward identities (no proof)

$$A(q^2)\,(p + p')^\mu + C(q^2)\,\gamma^\mu$$

$\rightarrow$ only 2 form factors! All orders result!

Another form: Gordon identity

$S^{\mu\nu} = \frac{i}{4}[\gamma^\mu, \gamma^\nu]$

$$\bar u(p')(p'+p)^\mu u(p) = \bar u(p')(2m\gamma^\mu + 2iS^{\mu\nu} q_\nu)u(p)$$

$$\boxed{\bar u' V^\mu u = e\,\bar u(p')\left(F_1(q^2)\gamma^\mu - \frac{i}{m}F_2(q^2)S^{\mu\nu}q_\nu\right)u(p)}$$

vanishes as $q \to 0$!

or

Renorm cond: $\boxed{F_1(0) = 1}$

Detailed calculation: $\boxed{F_2(0) = \frac{\alpha}{2\pi}} + O(q^2)$

Anomalous Magnetic Moment of e^-

Think of $\mathcal{H}_{eff} = \bar\Psi V^\mu A_\mu \Psi$ as effective quantum Hamiltonian

$S^{\mu\nu} q_\mu A_\nu \to S^{\mu\nu} F_{\mu\nu}$

Let's evaluate $\langle e|\mathcal{H}_{eff}|e\rangle$ in background mag field

$A^0 = 0$, $\vec A = (0, Bx, 0)$

$\downarrow$ go to low energy limit $q^2 \to 0$, $p, p' \to (m,0,0,0)$

$\mathcal{H}_{eff} = e\bar\Psi\gamma^\mu F_1(0)\Psi A_\mu$ ($F_1(0) = 1$)

$-\frac{ie}{m}F_2(0)\bar\Psi S^{\mu\nu} A_\mu q_\nu \Psi$ ($A_\mu q_\nu \to F_{\mu\nu}$)

$-eB\,\bar\Psi\left(\gamma^2 + \frac{\alpha}{2\pi m}S^{12}\right)\Psi$

Use G. ident

$$\gamma^\mu \to \frac{(p+p')^\mu}{2m} - \frac{i}{m} S^{\mu\nu} q_\nu$$

So

$$\bar{u}\, V^\mu u = e\,\bar{u}\left(F_1 \frac{(p+p')^\mu}{2m} - \frac{i}{m} S^{\mu\nu} q_\nu (F_1 + F_2) \right) u$$

and

$$\mathcal{H}_{eff} = e\,\bar{\Psi}\, V^\mu \underbrace{A_\mu}_{\text{zero momentum}} \Psi \;\cancel{-\,eB\,\bar{\Psi}\, F_1 \frac{(p+p')^\alpha}{2m}}$$

$$-\frac{eB}{m}\,\bar{\Psi}\, S^{12}\, \Psi\, (F_1 + F_2)$$

$$\langle e | \mathcal{H}_{eff} | e \rangle = -\frac{eB}{2m}\left(1 + \frac{\alpha}{2\pi} + \mathcal{O}(\alpha^2)\right) \underbrace{\langle e | \bar{\Psi} S^{12} \Psi | e \rangle}_{\text{Spin of electron in z direction } \langle S_z \rangle}$$

Compare

$$E = -\vec{\mu} \cdot \vec{B}$$

gyromagnetic ratio

$$\vec{\mu} = \frac{g e}{2m} \vec{S}$$

$$\boxed{g = 2\left(1 + \frac{\alpha}{2\pi} + \mathcal{O}(\alpha^2)\right)}$$

how big next corr'n?

$\left(\frac{\alpha}{4\pi}\right)^2 \sim 10^{-6}$

- Classic result!

QED predicts $\frac{g-2}{2} = \frac{\alpha}{2\pi} + \mathcal{O}(\alpha^2) \approx 0.00116$

Current ~~exp: 0.002 319 304 361 82 (52)~~

~~th:~~ ~~(wrong)~~

exp: $\left(\frac{g-2}{2}\right)_{exp} = 0.001159\,65218\,091\,(26)$

th: $0.001159\,652\,181\,643\,(764)$

(up to α^5!)

agreement to 10 sig figs!

most accurately tested prediction in all of physics!

cf muon mag moment

(th) $0.001\,165\,918\,04\,(51)$

(exp) $0.001\,165\,920\,61\,(41)$

$\updownarrow$ 4σ discrepancy!

→ receives non-negligible QCD contrib, not known well

↑ FNAL & Brookhaven

↓ Longstanding discrepancy of SM

hint of NP?