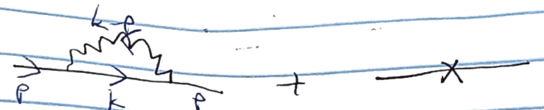
Next consider electron self energy

$$\Delta_e(\not p) = \frac{1}{\not p - m - \Sigma(\not p)}$$

$\Sigma(\not p) =$  $+ \;\; —\!\!\times\!\!—$

$$\cancel{\frac{e^2}{8\pi^2}\int_0^1 dx\,\big((2-\epsilon)(1-x)\not p + (4-\epsilon)m\big)\left(\frac{1}{\epsilon} - \frac{1}{2}\frac{\Delta}{\mu^2}\right)}$$

$$-(Z_2-1)\not p - (Z_m - 1)m \qquad \cancel{\Delta = x(1-x)p^2 + x m^2}$$

$$\Sigma(\not p) = e^2\int\frac{d^4k}{(2\pi)^4}\frac{\gamma^\mu(\not k+m)\gamma_\mu}{(k^2-m^2)(k-p)^2} = 2e^2\int_0^1 dx\int\frac{d^4\ell}{(2\pi)^4}\frac{x\not p - 2m}{(\ell^2-\Delta)^2}$$

superficial degree of divergence $\sim\Lambda$

$$= -\frac{\alpha}{2\pi}\int_0^1 dx\,(2m - x\not p)\left(\frac{2}{\epsilon} + \log\frac{\tilde\mu^2}{(1-x)(m^2-p^2x)+x m_\gamma^2}\right)$$

$$+ \underbrace{(Z_2-1)}_{\delta_2}\not p - \underbrace{(Z_m-1)}_{\delta_m} m$$

photon mass IR cutoff

can take to zero at end

Renorm conditions: $\Sigma(\not p = m) = 0$

$\Sigma'(\not p = m) = 0$

fixes $Z_2 = 1 - \frac{\alpha}{2\pi}\left(\frac{1}{\epsilon} + \text{finite}\right)$

$Z_m = 1 - \frac{2\alpha}{\pi}\left(\frac{1}{\epsilon} + \text{finite}\right)$

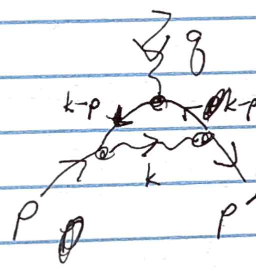
Mass renorm not $\sim\Lambda$!

cf hierarchy problem

consequence of ~~related~~ chiral symmetry

$\delta m \sim m$ "technical naturalness"

Finally, vertex function



$$= e^3 \int \frac{d^4k}{(2\pi)^4} \left(\gamma^\rho \frac{\not k - \not p + m}{(k-p)^2 - m^2} \gamma^\mu \frac{\not k - \not p' + m}{(k-p')^2 - m^2} \gamma^\nu \right) \cancel{\frac{k_\rho k_\nu - k^2 \eta_{\rho\nu}}{k^4}}$$

again vanishes by Ward ident.

need 3 Feynman params

$$\int dF_3 = 2\int_0^1 dx_1\, dx_2\, dx_3\, \delta(x_1+x_2+x_3-1)$$

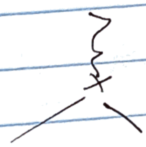
divergent?

$$\frac{k^5}{k^6} dk \longrightarrow \text{log divergent.}$$

$$= \frac{e^3}{8\pi^2} \left(\left(\frac{1}{\varepsilon} - 1 - \frac{1}{2}\int dF_3 \log \frac{D}{\tilde\mu^2} \right) \gamma^\mu + \frac{1}{4} \int dF_3 \frac{\tilde N^\mu}{D} \right)$$

$$D = x_1(1-x_1)p^2 + x_2(1-x_2)p'^2 - 2x_1x_2\, p\cdot p' + (x_1+x_2)m^2 + x_3 m_\gamma^2$$ ← IR reg.

$\tilde N^\mu =$ - - - - finite

 $= (Z_1 - 1)e \longrightarrow Z_1 = 1 - \frac{e^2}{8\pi^2}\left(\frac{1}{\varepsilon} + \text{finite}\right)$

~~Note $Z_1 = Z_2$~~

Note! $Z_1 = Z_2$ (at least see infinite parts)
true more generally

Guaranteed by gauge inv'ce

$\bar{\Psi}(i\partial\!\!\!/ - \gamma e A)\Psi \qquad \Psi \to e^{i\theta}\Psi, \; A_\mu \to A_\mu - i\frac{1}{e}\partial_\mu \theta$

rescale $A \to \frac{1}{e} A$

$-\frac{1}{4e^2}F^2 + \underset{Z_1}{\bar{\Psi}}(i\partial\!\!\!/ - \underset{Z_2}{\gamma^\mu A_\mu})\Psi \qquad \Psi \to e^{i\theta}\Psi$

$A_\mu \to A_\mu - i\partial_\mu\theta$

e only appears in front of $F^2 \longrightarrow$ vac pol $\longleftrightarrow$ e renorm.

So if $Z_1 \neq Z_2$ would spoil gauge transf!

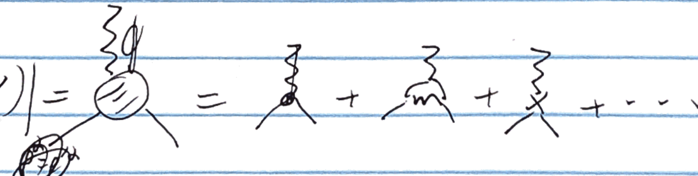
$\longrightarrow$ gauge guarantees charge ratios preserved under renorm! e.g. $\frac{Q_e}{Q_p} = -1$

or $\frac{Q_u}{Q_e} = -\frac{2}{3}$

etc.

~~Renorm Condits~~ Anomalous Magnetic Moment

Vtx: Renormalization Condition

$V(p,p') =$ 

$V(p,p')|_{p^2=p'^2=m^2,\; q^2=0} \longrightarrow$ unlike ϕ^3 thy, physical!