Yes! If theory is renormalizable

(no proof here)

Thm sort: $\mathcal{L}$ renom. $\longleftrightarrow$ all couplings mass dim ≥ 0.

- superrenorm " " " > 0
- renorm $= 0$
- nonrenorm < 0

So actually ϕ^3 in $d=4$ is superrenorm $\rightarrow$ only finite # diagrams diverge

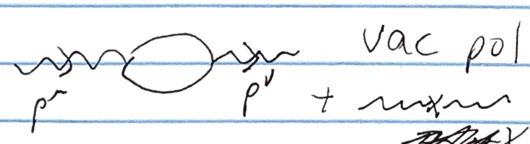
What about QED?

$e\bar{\Psi}A_\mu\gamma^\mu\Psi \rightarrow e$ is dim'less $\rightarrow$ renormable

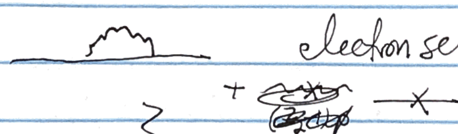
divergences @ every order in pert'n theory but cancel w/ finite # ct.

Now let's sketch how works in QED...

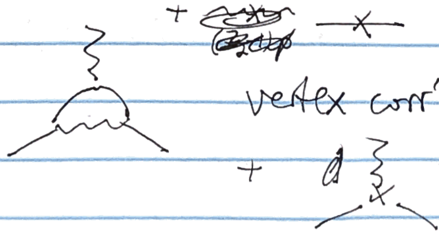
$\mathcal{L}_{QED} = -\frac{1}{4}Z_\gamma F_{\mu\nu}^2 + Z_2\bar{\Psi}i\partial\!\!\!/\Psi - Z_1 e\bar{\Psi}A\!\!\!/\Psi - Z_m m\bar{\Psi}\Psi$ @ 1-loop



vac pol $\Pi^{\mu\nu}(p^2)$



electron self E $\Sigma(p)$



vertex corr'n $V^\mu(p)$

Will skip all calculational details

ward identity!

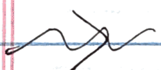
$$\Pi^{\mu\nu}_{loop} = -\frac{e^2}{2\pi^2}\left(p^2\eta^{\mu\nu} - p^\mu p^\nu\right)\int dx\, x(1-x)\left(\frac{2}{\varepsilon} + \log\frac{\tilde{\mu}^2}{m^2 - p^2x(1-x)}\right)$$

$$\left(\tilde{\mu}^2 \equiv 4\pi e^{-\gamma}\mu^2\right)$$

$$= -\left(p^2\eta^{\mu\nu} - p^\mu p^\nu\right)\Pi_{loop}(p^2) = -p^2 P^{\mu\nu}\,\Pi_{loop}(p^2)$$

$$\Delta^{\mu\nu}_{photon}(p^2) = \frac{P^{\mu\nu}}{p^2} + (\quad)$$

$$\Pi^{\mu\nu}_{ct} = -\left(p^2\eta^{\mu\nu} - p^\mu p^\nu\right)(Z_\gamma - 1)$$

 $= \dfrac{P^{\mu\nu}}{p^2} = \eta^{\mu\nu} - \dfrac{p^\mu p^\nu}{p^2}$

(HW)

$$P^{\mu\nu}P_{\rho\nu}{}^{\sigma}$$

$$= \left(\eta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)\left(\eta_{\nu}{}^{\sigma} - \frac{p_\nu p^\sigma}{p^2}\right)$$

$$= \eta^{\mu\sigma} - \frac{p^\mu p^\sigma}{p^2} = P^{\mu\sigma}$$

$$\Delta^{\mu\nu}_{photon} = \frac{P^{\mu\nu}}{p^2} - \frac{P^{\mu\rho}}{p^2}\Pi_{\rho\sigma}\frac{P^{\sigma\nu}}{p^2} + \cdots$$

$$= \frac{P^{\mu\nu}}{p^2} - \frac{P^{\mu\nu}}{p^2}\Pi(p^2) + \cdots$$

$$= \frac{P^{\mu\nu}}{p^2\left(1+\Pi(p^2)\right)}$$

$$\boxed{\text{OS renorm condition: } \Pi(p^2=0) = 0}$$

no mass renormalization! (γ mass "protected" by gauge sym)

photon should be "canonically normalized" on shell

mode expansion w/ wave fn e^{ikx} & not Ze^{ikx} ...

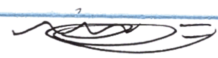
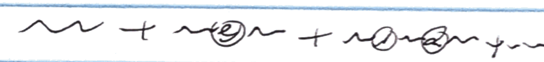
$$\Rightarrow \boxed{Z_\gamma = 1 - \frac{e^2}{6\pi^2}\left(\frac{1}{\varepsilon} - \log\frac{m}{\tilde{\mu}}\right) + \cdots}$$

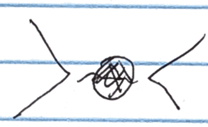
~~Coupling Running coupling~~

Then $\Pi^{ren} = \Pi^{1\text{-loop}} + \Pi^{ct} = \Pi^{1\text{-loop}} + (Z_\gamma - 1)$

$$= \frac{-2\alpha}{\pi} \int dx \, x(1-x) \log \frac{m^2 - p^2 x(1-x)}{m^2}$$

<u>Physical interpretation</u>

 $= \dfrac{1}{p^2(1+\Pi(p^2))} =$ 

 $\rightarrow$ Coulomb screening!

$\dfrac{1}{p^2} \xrightarrow{FT} \dfrac{1}{r}$ Coulomb potential (can make more precise using TOPT Born approx $\langle k | V | \tilde{k} \rangle = V(k - \tilde{k})$ p is momentum transfer)

$\dfrac{1}{p^2(1+\Pi(p^2))} \xrightarrow{FT} V(r)$ <u>screened coulomb pot'l</u>

<u>Small p limit</u>: $\Pi^{ren} \approx +\dfrac{2\alpha}{\pi} \int dx \, x(1-x) \dfrac{p^2 x(1-x)}{m^2} \approx \dfrac{\alpha}{15\pi} \dfrac{p^2}{m^2}$

$$\frac{e^2}{p^2\left(1 + \frac{\alpha}{15\pi}\frac{p^2}{m^2}\right)} = \frac{e^2}{p^2}\left(1 - \frac{\alpha}{15\pi}\frac{p^2}{m^2}\right) = \frac{e^2}{p^2} - \frac{e^4}{60\pi^2 m^2}$$

$$\downarrow$$

$$-\frac{e^2}{4\pi r} - \frac{e^4}{60\pi^2 m^2} \delta(r)$$

NRQM "Uehling Term"

Contributes to Lamb shift $\rightarrow$ shifts $L=0$ (S-wave) energies of Hydrogen $2P_{1/2}$ / $2S_{1/2}$

Large p limit: $\int dx\, x(1-x) = \frac{1}{6}$

$$\Pi = \frac{-\alpha}{3\pi} \log\left(\frac{-p^2}{m^2}\right) \qquad (p^2 < 0 \text{ for } t\text{-channel})$$

$$V(p) = \frac{4\pi\alpha}{p^2\left(1 - \frac{\alpha}{3\pi}\log\left(\frac{-p^2}{m^2}\right)\right)} \simeq \frac{4\pi\alpha_{eff}(p)}{p^2}$$

$$\boxed{\alpha_{eff}(p) = \frac{\alpha}{1 - \frac{\alpha}{3\pi}\log\left(\frac{-p^2}{m^2}\right)}}$$

First example of Running coupling!

α_{eff} changes w/ momentum transfer (due to charge screening)

as $|p^2|$ increases, α_{eff} ~~decreases~~ increases
decreases, decreases

QED coupling becomes weaker in IR
stronger in UV

"Landau Pole"

when $\log\left(\frac{-p^2}{m^2}\right) \sim \frac{3\pi}{\alpha}$ $\quad p^2 \sim m^2 e^{\frac{3\pi}{\alpha}}$

α_{eff} diverges $\qquad \frac{3\pi}{\alpha} \sim 10^3$

$p \sim m e^{\frac{3\pi}{2\alpha}} \sim 10^{280}$ MeV

QED is technically not UV complete (way above M_{pl} so OK, plus grand unif.)