$$p_3^2 = \cancel{m^2} = 2 p_1 \cdot p_2 - 2 p_1 \cdot p_4 - 2 p_2 \cdot p_4 + \cancel{p_2^2}$$

$$p_{12} - p_{14} - p_{24} = 0$$

$p_{12} = \omega m$

$p_{14} = \omega \omega' (1 - \cos\theta)$

$p_{24} = \omega m$

↓ solve for ω'

$$\boxed{\omega' = \frac{\omega}{1 + \frac{\omega}{m}(1 - \cos\theta)}}$$

shifted γ freq. as fn. of scattering angle.

↓

originally due to Compton

just E & p cons.

no dynamics

$$\left(\frac{1}{\omega'} - \frac{1}{\omega} = \frac{1}{m}(1 - \cos\theta)\right)$$

$= \Delta\lambda$

Change in wavelength positive!

photon always loses energy in this frame

↓

not true in other frames cf "inverse Compton scattering"

→ ←

→ ←

photon can gain energy from moving electron!

↓

important in astrophysics & cosmology

↓

SZ effect

CMB → hot gas

$$\frac{1}{4} \sum |M|^2 = 2e^4 \left(\frac{\omega}{\omega'} + \frac{\omega'}{\omega} + 2m\left(\frac{1}{\omega} - \frac{1}{\omega'}\right) + m^2 \left(\frac{1}{\omega} - \frac{1}{\omega'}\right)^2 \right)$$

$$= 2e^4 \left(\frac{\omega}{\omega'} + \frac{\omega'}{\omega} - 2(1 - \cos\theta) + (1 - \cos\theta)^2 \right)$$

$$= 2e^4 \left(\frac{\omega}{\omega'} + \frac{\omega'}{\omega} - \sin^2\theta \right)$$

Nice!

$$\frac{d\sigma}{d\cos\theta} \text{ (lab frame)} = \frac{1}{32\pi} \left(\frac{\omega'}{\omega}\right)^2 \frac{1}{m^2} \frac{1}{4} \sum |M|^2$$

$$\boxed{= \frac{\pi \alpha^2}{m^2} \left(\frac{\omega'}{\omega}\right)^2 \left(\frac{\omega'}{\omega} + \frac{\omega}{\omega'} - \sin^2\theta \right)}$$

Klein–Nishina formula (1929)

Early success of QED!!

Low energy limit:

$m \to \infty, \ \omega' \to \omega$

$$\frac{d\sigma}{d\cos\theta} \to \frac{\pi\alpha^2}{m^2}\left(1+\cos^2\theta\right)$$

Thomson formula
classical radiation off free electron

high energy limit

$\omega, \omega' \to \infty$

Go to CM frame

$p_4 = (\omega, \omega\sin\theta, 0, \omega\cos\theta)$ $\qquad E = \sqrt{m^2+\omega^2} \approx \omega + \frac{m^2}{2\omega}$

$p_1 \ (\omega,0,0,\omega) \qquad p_2 \ (E,0,0,-\omega)$ $\qquad p_{12} = \omega(E+\omega)$

$p_3 = (E, -\omega\sin\theta, 0, -\omega\cos\theta)$ $\qquad p_{24} = \omega(E+\omega\cos\theta)$

$$\frac{1}{4}\sum|M|^2 \approx 2e^4\left(\frac{E+\omega\cos\theta}{E+\omega} + \frac{E+\omega}{E+\omega\cos\theta}\right)$$

$$\approx 2e^4\left(\frac{1+\cos\theta}{2} + \frac{2}{\frac{m^2}{2\omega^2}+1+\cos\theta}\right)$$

Huge enhancement at $\theta = \pi$ $\to$ backward scattering

↓

Careful analysis of spin & polarization
dominated by "helicity flip" process
$e^-_R\gamma_L \to e^-_L\gamma_R$ and $e^-_L\gamma_R \to e^-_R\gamma_L$

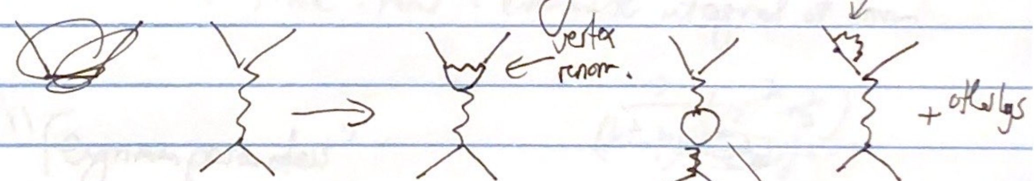
Can use this to e.g. produce polarized electron beams
(polarized light on unpolarized e^- → backscatters predominantly polarized e^-)

Renormalization

New Topic

Going to higher orders in pert'n theory — loops!

e.g. consider Rutherford scattering



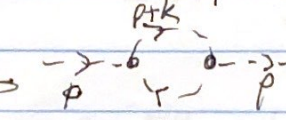
Have to perform loop integrals!

↳ they are divergent!

How to regularize?

What to do about infinities?

Consider simpler example first



$$= g^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2-m^2)((p+k)^2-m^2)}$$