Azimuthal symmetry — indep of ϕ

ϕ

$$\frac{d\sigma}{d\cos\theta} = \frac{1}{16\pi E_{cm}^2}\left(- \sim \right)^2$$

- Check: no singularities in θ $(t, u < 0 \ \forall \theta)$

  $\rightarrow$ finite xsec.

  $$\sigma = \int_{-1} d\cos\theta \, \frac{d\sigma}{d\cos\theta}$$

Feynman Rules for QED

- Photon propagator

$$\sim\sim = \langle 0 | T A_\mu(x) A_\nu(y) | 0 \rangle \xrightarrow{\text{mom space}} \frac{\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}}{p^2}$$

needed in transverse gauges $\partial^\mu A_\mu = 0$

- Fermion propagator

$$\longrightarrow = \langle 0 | T \, \bar{\Psi}_a(x) \bar{\Psi}_b(y) | 0 \rangle \Rightarrow \frac{1}{\not{p} - m} = \frac{\not{p} + m}{p^2 - m^2}$$

green's fn

$(\not{p}\not{p} = p^2 \mathbb{1}_4)$

$\gamma^\mu \gamma^\nu g_{\mu\nu}$

$\Psi(x) = \int a_s(p) u_s(p) e^{-ipx} + b_s^\dagger(p) v_s(p) e^{ipx}$

$\bar{\Psi} = \int a_s^\dagger(p) \bar{u}_s(p) e^{ipx} + b_s(p) \bar{v}_s(p) e^{-ipx}$

- External photon

  $\bullet\!\sim\sim$ p $\quad \epsilon_\lambda(p)$ in

  $\sim\sim\!\bullet$ $\quad \epsilon_\lambda^*(p)$ out

- External fermion

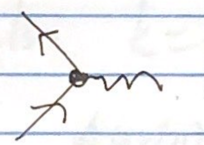
  incoming electron $\bullet\!\longrightarrow$ mom, $\rightarrow$ charge $\quad u_s(p)$

  in pos $\bullet\!\longrightarrow$ " , $\leftarrow$ charge $\quad \Psi(x)|0\rangle \quad \bar{v}_s(p)$

  out e^+ $\longrightarrow\!\bullet$ $\quad v_s(p)$ $\quad$ out e^- $\longrightarrow\!\bullet$ $\quad \langle 0 | \bar{\Psi}(x) \quad \bar{u}_s(p)$

• Vertex

$$H_{int} = e\int d^3x\, \bar{\Psi}(x)\gamma^\mu A_\mu \Psi(x)$$



$= e\gamma^\mu$

Can show:

$$\boxed{\begin{aligned} b_s^\dagger(p) &= \int d^3x\, e^{ipx}\,\bar{\Psi}(x)\gamma^0 u_s(p) \\ d_s^\dagger(p) &= \int d^3x\, e^{ipx}\,\bar{v}_s(p)\gamma^0 \Psi(x) \end{aligned}}$$

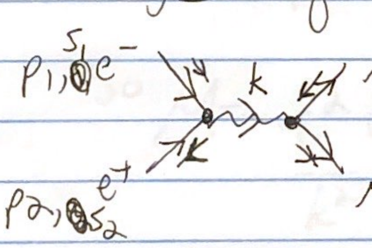
explains External state FR.

Let's do some real-life physical examples!

1. Simplest ex: $e^+e^- \to \mu^+\mu^-$ (imagine QED w/ 2 fermions)

Only 1 diagram

$\mathcal{M} =$



$\mathcal{M}(p_1,p_2,p_3,p_4)_{s_1 s_2 s_3 s_4}$

$$= e^2\, \bar{u}_{s_3}(p_3)\gamma^\mu v_{s_4}(p_4) \left(\frac{\eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}}{k^2} \right) \bar{v}_{s_2}(p_2)\gamma^\nu u_{s_1}(p_1)$$

What next?

wat do $\to$ need $|\mathcal{M}|^2$ $\to$ in general a mess

$\to$ simplifies if one sums over final state spins and averages over initial state spins

generally most expt'ally accessible.

$$\cancel{\frac{1}{4}\sum_{spins}|M|^2 = \int e^4\, \bar{u}_{s_3}(p_3)\gamma^\mu v_{s_4}(p_4)\, \frac{\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)}{k^2}}$$

Also $k = p_1 + p_2 = p_3 + p_4$ (need: $\gamma^\dagger = \gamma^0\gamma\gamma^0$)

$$\not{p}\, u_s(p) = -m u_s(p) \qquad \bar{u}_s(p)\not{p} = -m\bar{u}_s(p)$$

$$\not{p}\, v_s(p) = +m v_s(p) \qquad \bar{v}_s(p)\not{p} = +m\bar{v}_s(p)$$

HW $(\bar{u}\gamma^\mu u)^* = \bar{u}\gamma^\mu u$

So
$$\bar{v}(p_2)\not{k}\, u(p_1) = \bar{v}(p_2)(\underbrace{\not{p}_2}_{+m} + \underbrace{\not{p}_1}_{-m})u(p_1) = 0$$

Sim $\bar{u}(p_3)\not{k}\, v(p_4) = 0$

So
$$M = \frac{e^2}{k^2}\bar{u}_{s_3}(p_3)\gamma^\mu v_{s_4}(p_4)\, \bar{v}_{s_2}(p_2)\gamma_\mu u_{s_1}(p_1)$$

$$\frac{1}{4}\sum_{spins}|M|^2 = \frac{e^4}{4k^4}\left(\bar{u}_{s_3}(p_3)\gamma^\mu v_{s_4}(p_4)\,\cancel{\bar{v}_{s_2}(p_2)\gamma_\mu u_{s_1}(p_1)}\right) \cdot \bar{v}_{s_4}(p_4)\gamma^\nu u_{s_3}(p_3)\Big)\left(\bar{v}_{s_2}(p_2)\gamma_\mu u_{s_1}(p_1)\bar{u}_{s_1}(p_1)\gamma_\nu v_{s_2}(p_2)\right)$$

~~So need~~ $= \frac{1}{4}\mathrm{Tr}\left(\gamma^\mu v\bar{v}(p_4)\gamma^\nu u\bar{u}(p_3)\right)\mathrm{Tr}\left(\gamma_\mu u\bar{u}(p_1)\gamma_\nu v\bar{v}(p_2)\right)$

So need $\sum_s u_s\bar{u}_s(p)$, $\sum_s v_s\bar{v}_s(p)$

"completeness relations"

$$\boxed{\begin{aligned}\sum_s u_s \bar{u}_s(p) &= \not{p} - m \\ \sum_s v_s \bar{v}_s(p) &= \not{p} + m\end{aligned}}$$

Check: annihilated by $\not{p}+m$ & $\not{p}-m$ on both sides respectively

$$\frac{1}{4}\sum |M|^2 = \frac{e^4}{4k^4} Tr\left(\gamma^\mu(\not{p}_4+m)\gamma^\nu(\not{p}_3-m)\right) \times Tr\left(\gamma_\mu(\not{p}_1-m)\gamma_\nu(\not{p}_2+m)\right)$$

Need "trace identities"

$$\begin{cases} tr\,\gamma^\mu = 0 \\ tr\,\gamma^\mu\gamma^\nu = 4\eta^{\mu\nu} \\ tr(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = 4(\eta^{\mu\nu}\eta^{\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho}) \end{cases}$$

$$= \frac{4e^4}{k^4}\left(p_1^\mu p_2^\nu + p_2^\mu p_1^\nu - (p_1\cdot p_2 + m_e^2)\eta^{\mu\nu}\right)$$

$$\cdot \left(p_{3\mu}p_{4\nu} + p_{3\nu}p_{4\mu} - (p_3\cdot p_4 + m_\mu^2)\eta_{\mu\nu}\right)$$

$\vdots$

$$= \frac{2e^4}{s^2}\left(t^2 + u^2 + 4s(m_e^2+m_\mu^2) - 2(m_e^2+m_\mu^2)^2\right)$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 E_{CM}^2}\frac{|\vec{p}_3|}{|\vec{p}_1|}\cdot\frac{1}{4}\sum|M|^2$$