Midterm (2026)

1. $\boxed{25\text{ pt}}$ Consider an elastic band as a thermodynamic system.

Then $E = E(\underset{\uparrow \atop \text{entropy}}{S}, \underset{\uparrow \atop \text{length}}{L}, N)$ ← # particles

and $dE = T\,dS + \underset{\uparrow \atop \text{force of tension}}{f}\,dL + \underset{\uparrow \atop \text{chemical potential}}{\mu}\,dN$

Finally, the equation of state is given by: $f = K\left(\frac{L}{N}\right)T$.

(a) Derive the Gibbs-Duhem equation for this system.

(b) Find $\left(\frac{\partial E}{\partial L}\right)_T$.

2. $\boxed{25\text{ pt}}$ Consider a system made of N classical independent magnetic dipoles of magnitude μ. The dipoles have a moment of inertia I for rotations perpendicular to the dipole axis. The dipoles are subject to an external magnetic field $\vec{B} = B\hat{z}$.

(a) Evaluate the canonical partition function Z of this system.

(b) Find the average magnetization per spin, $m(B,T)$. Discuss its $T\to\infty$ (high-T) limit.

(c) Find the internal energy per spin, $u(B,T)$, and discuss its $T\to\infty$ and $T\to 0$ limits.

<u>Hint</u> The kinetic energy for a single dipole is given by:

$$K = \frac{I}{2}\dot{\theta}^2 + \frac{I}{2}\sin^2\theta\,\dot{\varphi}^2 \qquad (1)$$

The potential energy is:

$$\mathcal{U} = -\mu\cos\theta\, B. \qquad (2)$$

(θ, φ) = spherical coordinates