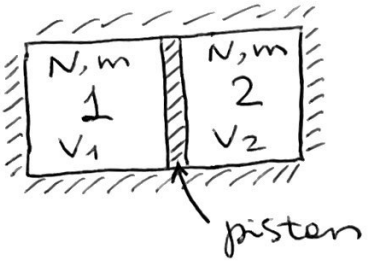
HW #4

[20 points per problem]

(1.) A cylinder is separated into two compartments by a free-sliding piston:



Two ideal Fermi gases are placed into the compartments: compartment 1 has particles with mass m, spin $S = \frac{1}{2}$, while compartment 2 has particles with mass m, spin $S = \frac{3}{2}$. Both gases have the same total number of particles N.

Find the ratio of the two volumes, $\frac{V_2}{V_1}$, at equilibrium with $T = 0$ and $T = \infty$.

(2.) Consider 2D ideal Bose gas.

(a) Find $\log \Sigma(z, V, T)$ as a 1D integral.

Σ ↑ grand-canonical partition function

(b) Find $\frac{P}{k_B T}$ ← pressure as a function of

$$g_2(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^2} \quad \text{and} \quad \lambda = \sqrt{\frac{2\pi\hbar^2}{m k_B T}}$$

↑ thermal wavelength

Hint: expand the integral in (a) in powers of z.

(c) Find the average number of particles per unit area $\frac{N}{L^2} = n$ as a function of z and T; insert the equation to find $z = z(n, T)$.