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A Field Guide to Spin Liquids

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Abstract

Spin liquids are collective phases of quantum matter that have eluded discovery in correlated magnetic materials for over half a century. Theoretical models of these enigmatic topological phases are no longer in short supply. In experiment there also exist plenty of promising candidate materials for their realization. One of the central challenges for the clear diagnosis of a spin liquid has been to connect the two. From that perspective, this review discusses characteristic features in experiment, resulting from the unusual properties of spin liquids. This takes us to thermodynamic, spectroscopic, transport, and other experiments on a search for traces of emergent gauge fields, spinons, Majorana fermions, and other fractionalized particles.

1. THE QUEST FOR SPIN LIQUIDS

The search for spin liquids as fundamentally new states of matter is a long-running quest (1–4). Their occurrence in insulating magnets appears to be greatly facilitated by frustrated interactions for which the corresponding classical spin systems display a large ground state degeneracy, because the local energetics cannot be minimized in a unique way (5–10). For a long time, the central and defining concept involved was a negative one—a (ground) state without any magnetic order—in contrast to the prevailing phases with spontaneous symmetry breaking characterized by local order parameters. The rejuvenated interest in resonating valence bond (RVB) physics through the discovery of high-temperature superconductors (11) focused attention on topological properties (12–15).

Initially, theoretical ideas were centered around wave functions, e.g., of the RVB type, but it took considerable time before microscopic Hamiltonians realizing such states at isolated points (16) or extended bona fide quantum spin liquid (QSL) phases were established (2). The advent of exactly soluble model Hamiltonians (16, 17) has led to an unprecedented understanding of ground- and excited-state properties of QSLs. Nowadays, the theory community has developed a remarkable capacity to invent elaborate schemes with a plethora of different phenomenologies but arguably with little guidance from experiment. At the same time, though the target space of interesting models has exploded, the arsenal of methods for their detection has not grown commensurably. Nonetheless, there has been a sustained materials physics effort covering a huge number of magnetic compounds, and many promising candidate systems have been unearthed, some of which have benefitted from an intense research program that has clarified their properties in considerable detail.

The aim of this review is to contribute toward redressing this balance. In particular, we discuss the rich phenomenology of spin liquids in order to connect to past and future experiments.

Before we embark on this, we would like to begin with a few words about how it fits with the broader research landscape of modern condensed matter physics. The search for spin liquids forms part of the grand challenge of understanding the existence, scope, and nature of physics beyond the standard theory of Landau and spontaneous symmetry breaking. As such, they fall in the field of topological condensed matter physics under the headings of long-range entangled phases and topological order.

To this date, we have one outstanding established class of experimental systems (at least in terms of materials science and beyond one dimension) exhibiting a topologically ordered quantum phase with fractionalized excitations: the fractional quantum Hall effect. The Laughlin state and its even more elaborate brethren (18, 19) have attracted much attention, most recently fueled by the dream of realizing a quantum computer topologically protected against decoherence (20). This is nicely reviewed in Reference 21.

QSLs have the added attraction of accessing the vast space of possible materials provided by the combinatorial richness of the periodic table, and the presence of sometimes large exchange energy scales (hence, larger temperature scales), as well as a high degree of tunability and being amenable to experimental probes, e.g., through the application of magnetic fields and the use of neutron scattering. The central goal for the foreseeable future therefore is an unambiguous identification of a QSL phase. For this, new experiments, including new probes, may be needed, accompanied by a reliable theoretical analysis framework.

Our aim here is to support this quest, but not by providing a complete introduction to quantum spin liquids for the expert. Rather, we present a compendium of ideas to provide a broader overview, which can also act as a guide for the newcomer. Many reviews are available with material that we do not cover here, e.g., comprehensive reviews on spin liquids (22) and frustrated magnetism more generally (23); discussions of spin ice (24); introductory reviews of QSLs (4, 25–27); an overview over spin ice (28) and Kitaev QSLs (29) from a fractionalization perspective, as well as a nice review of classical emergent Coulomb gauge fields (30); and works focusing on the materials aspects of pyrochlores (31), herbertsmithite (32), and Kitaev material candidates (33, 34).

Physics is an experimental science. However, though discoveries are mostly driven by experiments, the resulting insights are naturally preserved in the language of theory. The history of the search for spin liquids is therefore naturally intertwined with what phenomena one excludes and includes under this heading. Before we move on to the core of this field guide, we provide a brief review of the background taxonomy.

The remaining sections are then devoted to spin liquid phenomenology. The ultimate ambition—to provide a textbook, not unlike standard solid-state physics textbooks on conventional phases, on the behavior of such topological phases—is a step too far for us, and we select phenomena that we feel are particularly instructive and/or realistically attainable.

1.1. What Is a Spin Liquid?

What is certainly true is that the meaning of the term has shifted over the years. This is not an uncommon state of affairs, driven not only by the human tendency to adapt a definition to the requirements of the moment but also by the fact that as the understanding of the subtleties of the phenomenon advances, refinements to the concepts follow.

A constitutive concept for a spin liquid is the absence of magnetic order of a system of interacting spins at temperatures smaller than the interaction scale. This encodes the idea of a phase beyond the Landau paradigm (which covers all forms of magnetic order), as well as the intuition that a liquid should be different from a solid. In this sense, other forms of ordering of the spin degrees of freedom—such as nematic orders (35, 36)—also a priori disqualify a system from being classified as a spin liquid.

This negative definition, about the absence of something, continues to be the most ubiquitous and intuitive. Besides its practical limitation, to which we return below, it is nowadays considered to be too broad. For instance, it includes models—interesting in their own right—that are considered somewhat too simple. One is a quantum paramagnet, such as the kagome lattice Ising model in a transverse field, which is connected continuously to a high-temperature (classical) paramagnetic phase. Another is the Shastry–Sutherland model, whose two spins per structural unit cell form a dimer at low temperature, producing a simple inert state that again is straightforwardly connected to a high-temperature paramagnet but that can also display protected edge excitations of triplons at low temperatures (37). One calls this broader class of disordered magnets cooperative paramagnets to distinguish them from magnets disordered by thermal fluctuations at high temperatures, although this nomenclature is by no means universally used.

A more modern, positive definition involves listing conditions that a phase should meet to qualify as a spin liquid. This derives from advances in our understanding of what phases beyond the Landau paradigm can look like and applies these to spin systems.

Akin to the Mermin–Wagner theorem (38), which forbids spontaneous breaking of continuous symmetry (SSB or spontaneous symmetry breaking) at finite temperatures in dimensions $d \leq 2$, there is a rigorous result for spin systems with short-range interactions. For systems with half-odd integer spin per unit cell, hence proper Mott insulators, and without symmetry breaking, the Lieb–Schultz–Mattis theorem states (39) that the ground state is either unique with gapless excitations or degenerate with a gap to excitations. It establishes to some level of mathematical rigor (40, 41) the possibility of gapless QSLs or gapped ones with topological order.

Perhaps the crispest definition is to demand that the magnet should at low temperatures be described by a topological field theory, such as the Chern–Simons theory (42). On one hand, the a priori ruling out of gapless spin liquids and spin liquids with some additional ordered degrees of freedom (43) is very restrictive. On the other hand, this exclusion is not arbitrary—trying to braid quasiparticles in the presence of gapless Goldstone modes of a ferromagnet, like for SU(2) quantum Hall Skyrmions (44), does present an obstacle for envisaged quantum computation experiments (21).

However, in practice, to qualify as a spin liquid, it may be enough to bear in mind that some subset of the degrees of freedom should look essentially topological. This is more or less the attitude we take for the remainder of this review. For the purposes of a field guide, we therefore look for fractionalized excitations and emergent gauge fields. The latter two are intimately linked because standard spin flip excitations always lead to integer changes of the total spin. Therefore, excitations labeled by fractions of such quantum numbers, e.g., quasiparticles carrying half-integer spin, need to be created in even numbers and, once they are separated, we can think of the emergent background gauge field as taking care of the global constraint. The connection holds in higher-dimensional spin liquids, which are the focus of our field guide, but in $d = 1$ fractionalization may appear in a much simpler way via domain wall excitations of ordered states.

1.2. How to Tell One, as a Matter of Principle...

The characterization of a topological state of matter proceeds most easily via its global properties. Given the importance that numerical simulations have played in advancing the field, some of these—while appearing rather complex—are comparatively straightforwardly diagnosed numerically.

The topological order discovered by Wen & Niu posits that topological states have a degeneracy that depends on the genus of the surface they live on (45). For instance, the Laughlin state at filling fraction $\nu = 1/3$ is nondegenerate on a sphere, and threefold degenerate on a torus. This is intimately connected with the existence of fractionalized quasiparticles. When a pair of Laughlin quasiparticles of charge $\pm e/3$ is created from a ground state, and one member of the pair moves around a noncontractible loop of the torus before annihilating the other, the system moves from one ground state to another. Only once three such particles, and hence an electron with unit charge, have made such a trajectory does the system return to the original ground state. Indeed, the connections between quantum Hall physics and quantum spin liquids can become remarkably detailed, such as in transfers of wave functions between the two settings (see, e.g., 46).

Observing the topological ground-state degeneracy can be challenging (47, 48), but numerical methods like density matrix renormalization group are well-suited for extracting a quantifier just as reliable for gapped QSLs. In analogy to long-range order of conventional phases the long-range entanglement can serve as an order parameter of topological phases (49–51). The entanglement entropy of a ground-state wave function can be calculated from a reduced density operator with one part of the total degrees of freedom of a bipartitioned system traced out (with a smooth boundary of length L separating the two regions). For gapped phases it follows a universal scaling form,

$$S = cL - \gamma + \dots . \qquad 1.$$

The first term with a nonuniversal prefactor c is the area law common to all gapped phases, but the second term γ quantifies the long-range entanglement (it is independent of the length of the boundary of the partition). It is only nonzero in a topological phase and is directly related to the emergent gauge structure of the QSL phase, e.g., in a Z_2 QSL $\gamma = \ln 2$ (51). More detailed analyses can then yield information on the properties of fractionalized excitations and edge spectra (52).

1.3. . . .And in Practice

Above, we called these diagnostics comparatively straightforward because the experimental situation is considerably less promising. The entanglement entropy—in particular, any subleading contribution to it—does not correspond to any natural measurement on a many-body quantum spin system. Also, putting even a two-dimensional magnet on a manifold of nontrivial topology sounds like a thought experiment par excellence, even more so than the idea of diagnosing a spin stiffness for a conventionally ordered magnet by twisting boundary conditions.

The core aim of this review is to address precisely the question of how to move forward from here. Lacking a silver bullet or smoking gun (or whatever alternative martial metaphor one prefers to use) in experimental reality, one needs to make do with the probes that exist (or can be realistically invented), and think about how best to employ and combine them for an unambiguous identification of spin liquids.

1.4. The Role of Universality

Before we turn to this in more detail, we would like to raise an additional ideological point of fundamental importance that may at times be somewhat underappreciated when making contact between theory and experiment. The tension arises from the need in theory to devise precise definitions. One of the most successful of these is the idea of universality, which is intimately related to the success of the developments of the concepts of SSB and encoded in the renormalization group half a century ago. Phases and phase transitions have properties that are independent of microscopic details of the Hamiltonian—these properties are called universal.

The question then is: How much of this universality is visible—and where/how—in an actual experiment? The fundamentalist answer is, a priori, nothing. A case in point is the existence of Goldstone modes accompanying the breaking of a continuous symmetry, in the limit of long wavelengths and low frequencies. Obviously, the limit of low frequencies will be cut off by a finite energy resolution—not only because of Heisenberg's uncertainty relations—of any conceivable experimental probe. On top of these, innumerable other limitations, many of them based on nothing less than the second law of thermodynamics, will always be with us. We have to live with them [but can at times even turn them to our advantage (see Section 5)].

In practice, this is not just a peripheral complaint. As we argue below, many of the most striking manifestations of spin liquid behavior in fact are nonuniversal in the sense that they could be altered without leaving the phase; or conversely, that proximate phases may exhibit the behavior we are interested in, essentially just as characteristically as the pristine version.

As a poster child for this, we would like to adduce the fractionalized Heisenberg chain (see **Figure 1*a***). The agreement between theory and experiment is striking, up to considerable detail of the structure factor at high energies, including subtle intensity variations with wavevector and frequency. However, none of these are universal. Close inspection of the universal part of the spectrum at low frequencies reveals the opposite (58): Due to the residual coupling between neighboring chains, they undergo an ordering transition into a different phase with different, "conventional" universal behavior of a long-range ordered three-dimensional magnet.

A fundamentalist "universalist" perspective therefore leaves two unpalatable alternatives: One has either the low-temperature, conventional ordered phase or, above the ordering transition, a phase continuously connected to a boring high-temperature paramagnet. Both miss the remarkable, and in our minds convincing, evidence for fractionalization in practice in this compound. As is so often the case, while the world view of the fundamentalist is deceptively simple, much of what makes real life interesting lies in the gray areas, the appreciation of which requires an open mind.

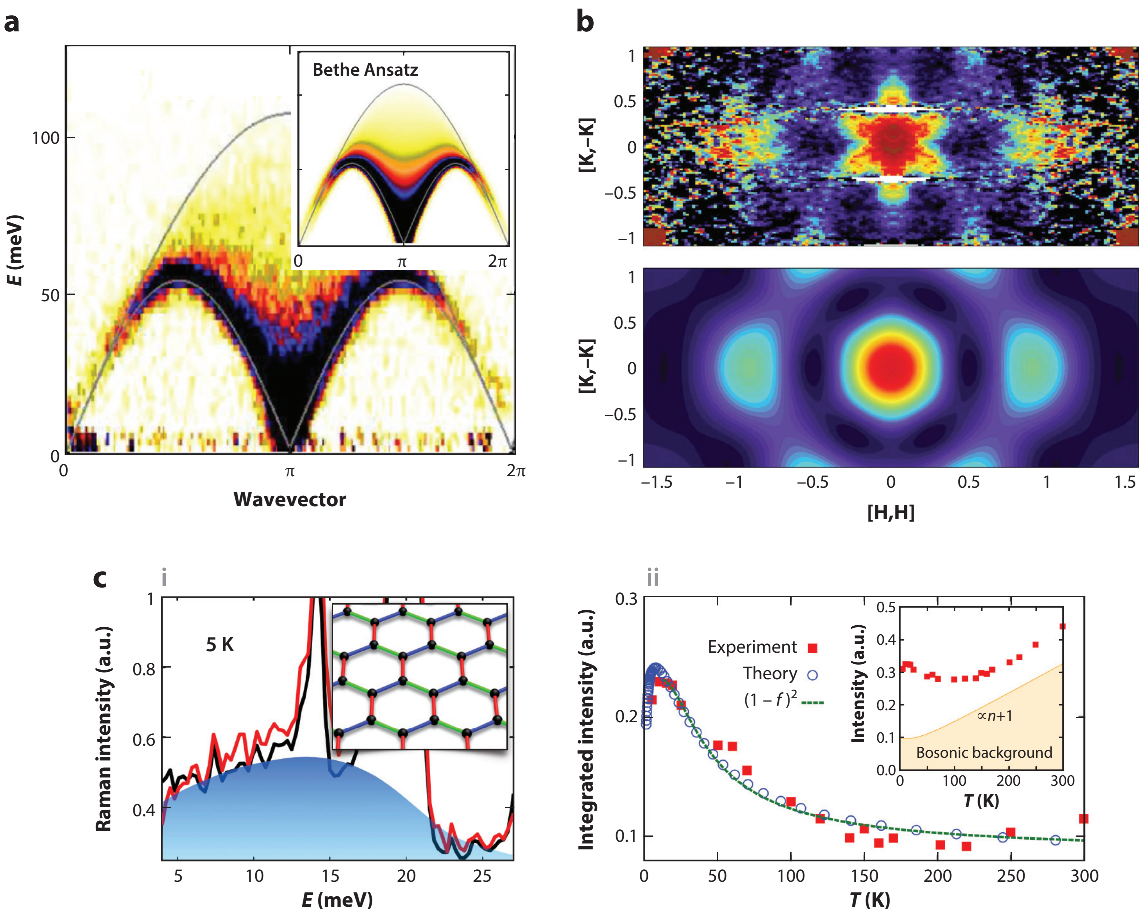

Figure 1

Comparison of inelastic scattering experiment and theory. (*a*) Fractionalized spinons in the $S = 1/2$ Heisenberg chain material $KCuF_3$ (53, 54). The boundaries of the scattering continuum are given by the kinematically allowed combinations of a pair of almost free spinons, into which a single spin flip can decay. The intensities reflect, among various experimental details, the concomitant matrix elements. (*b*) Finite frequency neutron structure factor of the proximate spin liquid in α-$RuCl_3$ (55). Although the ground state of this material exhibits magnetic zig-zag order with the requisite magnon excitations at low energies, the excitation spectrum at higher energies (*top panel*) qualitatively resembles the exact solution of the nearest-neighbor Kitaev quantum spin liquid (*bottom panel*), particularly with regard to a robust (also in temperature) and broad maximum at the zone center. (*c*) Fermionic nature of excitations in $RuCl_3$ as evidenced in Raman scattering (56, 57). Data extracted from experiment (*i*) after background subtraction can again be accounted for within the Kitaev model (*ii*). It can be fit to a functional form including fermionic thermal occupation function, f, for the pair of excitations created by the photon, which is an indication of the fermionic nature of the emergent quasiparticles in the Kitaev spin liquid.

2. HOW TO START LOOKING...

A time-honored way of making a first cut at the diagnosis of spin liquidity in a candidate material is via studies of thermodynamic properties (see Section 3) in part because these are relatively easy to carry out locally in a laboratory. The first chore is to establish the absence of magnetic ordering in a strongly interacting (low-temperature) regime.

A popular measure for its existence is the frustration parameter $f = |\Theta_{\rm CW}|/T_{\rm f}$ (59, 60), which is defined as the ratio of two quantities. One is the Curie(–Weiss) temperature extracted

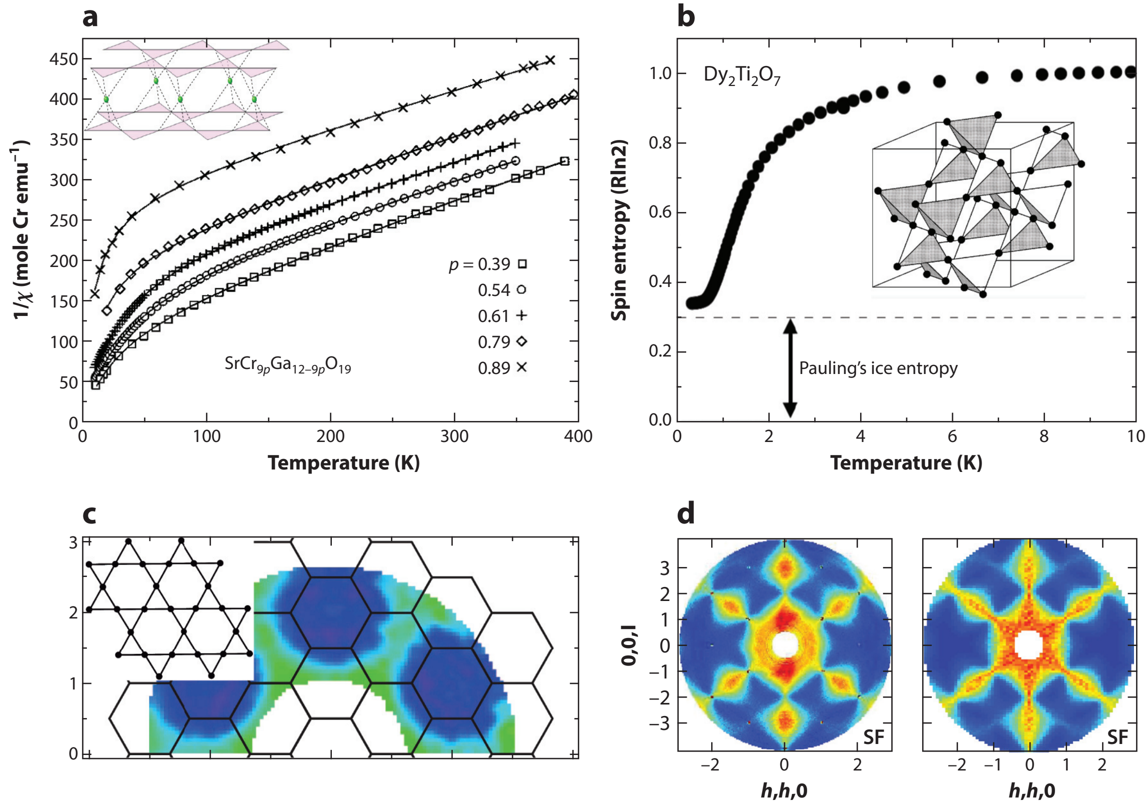

Figure 2

Signatures of frustration and spin liquidity across experimental compounds—schematic lattice structures are shown in the insets. (*a*) The Curie–Weiss temperature can be obtained by extending a linear fit to the high-temperature inverse susceptibility until it intersects the horizontal axis. For various levels of dilution in $SCGO_x$, the featureless regime of the susceptibility extends below this temperature, thus defining the cooperative paramagnetic regime. (*b*) Frustrated magnets exhibit a strong spectral weight downshift. Shown is the residual entropy of spin ice and the frustrated pyrochlore lattice (modeled after Reference 25), which appears to persist down to the lowest temperatures measured. (*c*) Short-range correlations in real space translate into broad features in inelastic neutron scattering in reciprocal space, shown for the kagome lattice material herbertsmithite (62). (*d*) An emergent U(1) gauge field gives rise to characteristic pinch-point correlations. This is shown for spin ice, an Ising magnet on the pyrochlore lattice (*left panel*, neutron scattering experiment; *right panel*, theory for the corresponding SF). Panel *a* adapted from Reference 61, panel *c* adapted from Reference 62, and panel *d* adapted from Reference 63. Abbreviations: SCGO, $SrCr_{9p}Ga_{12-9p}O_{19}$; SF, spin flip channel.

from a straightforward fit to the high-temperature susceptibility, which to first order in a high-temperature expansion is given by $\chi = C/(T - \Theta_{CW})$. This expression applies to an insulating magnet the size and nature of whose magnetic moments determine the Curie constant C, and whose interactions determine the size of Θ_{CW}. The second quantity, T_f, is the location of any nonanalyticity (divergence, cusp, etc., ...) in χ, indicating a residual ordering tendency or spin freezing which is commonly encountered in frustrated magnets. The regime in temperature $\Theta_{CW} \gg T > T_f$, the cooperative paramagnetic regime, is then a natural place to start looking for a spin liquid, and it is well defined provided f is sufficiently large (see **Figure 2*a*** for a classic example).

If appropriate measurements are possible and available (e.g., thanks to the availability of a neutron source), the absence of ordering can be confirmed by verifying that no magnetic Bragg peaks appear when cooling down the system. The challenge in practice lies in the need to eliminate the presence of less obvious ordering tendencies (such as multipolar or distortive order), which are more elusive in neutron scattering, and also not to miss any features in the specific heat, which is nonspecific in the type of orderings it picks up other than requiring the corresponding nonanalytic features of the phase transition to be discernible above its smooth background temperature dependence. Also, it may be polluted by incidental phase transitions, such as those of the lattice that have little bearing on its magnetism.

Further valuable insights can be gleaned in spectroscopic experiments (see Section 4 and **Figure 1**). These can provide considerably more detailed information than the purely macroscopic thermodynamic ones: Inelastic neutron scattering has made tremendous technological progress in the past few years, and now it routinely provides data in $d + 1$ (wavevector + frequency) space. Other probes, such as Raman scattering, and NMR lack wavevector dependence and provide a local response averaged over the full system. Crucially, alternative probes each couple differently to the magnetic degrees of freedom, and therefore, they come with different selection rules for probing the quasiparticles (or nonquasiparticle excitations) of the material, thereby providing complementary evidence, as we discuss below.

One central motivation for the use of finite-frequency probes lies in the fact that the ground states of topological systems are at first sight unspectacular. Although gapless spin liquids should generically come with algebraically decaying ground-state correlations, gapped spin liquids (just like the Laughlin charge liquid) really look featureless. It is the fractionalized quasiparticles that provide a local indication of the topological physics involved: Instead of the magnons in an ordered magnet, one looks for spinons, holons, monopoles and the like. (An alternative to studying such finite-frequency responses lies in enlisting the help of disorder to nucleate these excitations already in the ground state; Section 5.)

For such excitations, kinematic considerations can play an important role: Broad responses for a scattering experiment involving the creation of several particles, thereby weakening the restrictions imposed by energy and momentum conservation, are taken as a prime indicator of novel spin liquid physics. In the case in which the excitations of the emergent gauge field are very heavy—as in the case of Kitaev's QSL, where they do not move at all—they can effectively remove momentum conservation as a kinematic constraint, as their energy is barely momentum dependent (64) (**Figure 2*c***). In other cases, there may be direct evidence for the emergent gauge field, as is provided by the pinchpoints in spin ice (28) (see **Figure 2*d***).

2.1. . . .And Where

The search for spin liquid compounds has been going on for a long time, and many compounds have yielded interesting insights and phenomena. To conclude this introduction, we mention some materials that have shaped our own thinking. This article tries to synthesize the resulting insights, rather than discuss and model each system—or indeed, all systems—in specific detail.

Kagome-based lattices are prime examples for geometric frustration and very prominent for quasi-two-dimensional compounds. The jarosites (65) represent a family of compounds with varying degrees of further-neighbor interactions and disorder. Beyond this, two particularly well-studied systems are volborthite (66), which is now believed to have an important spatial anisotropy (67, 68), and herbertsmithite (69, 70). The latter is in some ways an outstanding candidate (32). Next in line are triangular compounds, in which a family of organic systems ("dmits") are particularly prominent (71), as discussed in the thermodynamics section below.

In the past few years, candidate materials (72, 73) for Kitaev spin liquids have received intense attention (29, 33, 34), with a combination of mapping out the Hamiltonian and understanding the resulting consequences keeping a large community busy. These include the Kitaev iridates A_2IrO_3 with A = Na, Li (74, 75), as well as α-$RuCl_3$ (76, 77) (see **Figure 1*b*,*c***).

Historically important—as the founding material of highly frustrated magnetism—has been SCGO ($SrCr_{9p}Ga_{12-9p}O_{19}$) (59), a kagome-triangle-kagome trilayer, which may alternatively be viewed as a slab of the pyrochlore lattice of corner-sharing tetrahedra. The pyrochlore lattice in turn hosts the Ising magnet known as spin ice (24, 28) most prominently, or $\{Dy/Ho\}_2Ti_2O_7$, the only generally acknowledged fractionalized magnetic material in three dimensions. Many other compounds exist on this lattice, such as a more quantum version with *Pr* ions (78), as well as a large class of spinel compounds, including the well-studied Cr spinels (79) with, like SCGO, isotropic $S = 3/2$ moments. Diluting a quarter of sites of that lattice in turn yields the hyperkagome lattice, with $Na_4Ir_3O_3$ being the most prominent exponent (80). Newcomers are constantly added to this list, most recently $Ca_{10}Cr_7O_{28}$ (81), $Ba_3NiSb_2O_9$ (82), $YbMgGaO_4$ (83), 1T-TaS_2 (84), Cu_2IrO_3 (85), and $H_3LiIr_2O_6$ (86).

3. THERMODYNAMICS AND TRANSPORT

A key target for diagnosing spin liquids is the experimental identification of fractionalized excitations at low energies. Thermodynamic and transport measurements are complementary in this endeavor, the former probing the necessary low-energy density of states (DOS) and the latter the mobility of the excitations. The absence of standard Goldstone modes from conventional symmetry breaking phases, e.g., spin waves of an ordered magnet, can be deduced from the absence of nonanalyticities in thermodynamic observables. In practice many material candidates have preemptive symmetry-breaking instabilities leading to nonanalyticities from subleading interactions. It prevents a true low-temperature liquid phase, for example, due to weak interlayer couplings of quasi-two-dimensional materials, but as long as the frustration parameter f is small enough, the correlated paramagnetic regime at intermediate temperatures is a good starting point in the search for QSL physics.

Candidate spin liquids often exhibit a spectral weight downshift of the specific heat as part of their refusal to order. This can in the most extreme cases go as far as apparent violations of the third law of thermodynamics. This happens in spin ice (**Figure 2*b***), where upon cooling a residual entropy is measured, indicating that even at the lowest experimentally accessible temperatures, the system continues to explore an exponentially large number of states. This sets cooperative paramagnetism apart from, say, dimensionality-induced destruction of ordering. Although purely one-dimensional spin systems such as a $S = 1/2$ Heisenberg antiferromagnetic chain do not order at any nonzero temperature, they nonetheless are close to an ordered state and often lose most of their entropy already upon cooling through Θ_{CW}.

3.1. Thermodynamics

Even though characteristic correlations of a QSL are only expected at temperatures well below Θ_{CW}, the high-temperature thermodynamic response of a candidate material already contains useful information. Details of a microscopic description are obtained by a comparison of the temperature scale at which (and how) the magnetic susceptibility deviates from Curie–Weiss behavior to that calculated in a high-temperature expansion of a putative spin Hamiltonian (87, 88). The angular magnetic field dependence of χ with respect to crystal orientation is in principle able to detect spin-anisotropic interactions (89). The goal is to invert macroscopic measurements to microscopic descriptions. However, this is possible only as long as the low-energy spin Hamiltonian and the magnetic *g*-tensor are sufficiently simple.

A defining feature of QSLs are fractionalized magnetic excitations that, after subtracting other contributions to the heat capacity (mainly from phonons, but at times, in particular at the lowest energies, nuclear spins), can be probed via the low temperature dependence of thermodynamic observables. In a gapless QSL, information about a low-energy power-law DOS, $N(\omega) \propto \omega^{\alpha}$, can be readily extracted because the specific heat is able to directly probe the exponent α:

$$\frac{C_{\mathrm{V}}}{T} = \frac{1}{T}\frac{\partial}{\partial T}\int \mathrm{d}\omega\omega N(\omega) n(\omega, T) \propto T^{\alpha}. \qquad 2.$$

Hence, in conjunction with the dimensionality of the system, detailed information about the low energy dispersion can be inferred, such as the presence of emergent Fermi surfaces, Dirac points or nodal lines, all of which have been proposed in model QSLs (90–92). For example, the linear-in-temperature specific heat of dmit shown in **Figure 3*a*** indicates the presence of a Fermi surface, which is corroborated by the observation of a linear-in-temperature longitudinal heat conductivity (94) (see Section 3.2). Of course, the procedure rests on assumptions like the presence of weakly interacting quasiparticles whose thermal distribution only depends on their individual energies, for example, $n(\omega, T)$ being the Fermi–Dirac or Bose–Einstein distribution functions.

Another caveat is that these power laws are not necessarily fixed to simple integers or fractions. For instance, in a honeycomb system with slowly varying bond disorder, the resulting hopping problem is equivalent to particles in a random gauge field. Famously, this problem gives rise to a DOS with a power increasing continuously with disorder strength (98). Such strains may quite conceivably be present in, say, organic systems with relatively low lattice rigidities.

Another intrinsic complication for a straightforward interpretation of thermodynamic data could be the presence of very different energy scales, making it hard to estimate the right scaling regime. For example, the Dirac spectrum of the honeycomb Kitaev QSL with $N(\omega) \propto \omega$ would simply predict $C_{\mathrm{V}}/T \propto T$, which turns out to be only observable at extremely low temperatures, but instead over a large temperature window a metallic like $C_{\mathrm{V}}/T \propto T^{0}$ appears (99, 100). The reason is that spin flip excitations fractionalize into Majorana fermions and flux excitations. The latter have a small gap that is only a fraction of the total magnetic energy scale. At all but the lowest temperatures the presence of thermally excited fluxes destroys the Dirac spectrum of the Majorana fermions, changing the low-energy DOS to a roughly constant $N(\omega) \propto \omega^{0}$. This particular example shows, on the one hand, the difficulties of drawing reliable conclusions from thermodynamic measurements. On the other hand, it highlights how a close comparison of microscopic calculations and experimental data could be used in principle to extract complementary information about the fractionalized excitations of a QSL.

3.2. Longitudinal Transport: Thermal Versus Charge

In the absence of mobile charge carriers in magnetic insulating materials thermal transport experiments can probe the mobility of elementary excitations. Ideally the heat flow along a temperature gradient contains information about the velocity, $v_{\mathbf{k}}$, and mean-free path, l_{MFP}, of fractionalized quasiparticles of a QSL with dispersion $E_{\mathbf{k}}$, e.g., as obtained for the longitudinal thermal conductivity κ in a semiclassical Boltzmann calculation

$$\kappa = \frac{\partial}{\partial T}\int \mathrm{d}^{d}k\, l_{\mathrm{MFP}} E_{\mathbf{k}} |v_{\mathbf{k}}| n(E_{\mathbf{k}}, T). \qquad 3.$$

In many low-dimensional QSL candidate materials, such purely magnetic contributions have been observed, e.g., in triangular (94, 101), kagome (102), and Kitaev honeycomb (103, 104) systems.

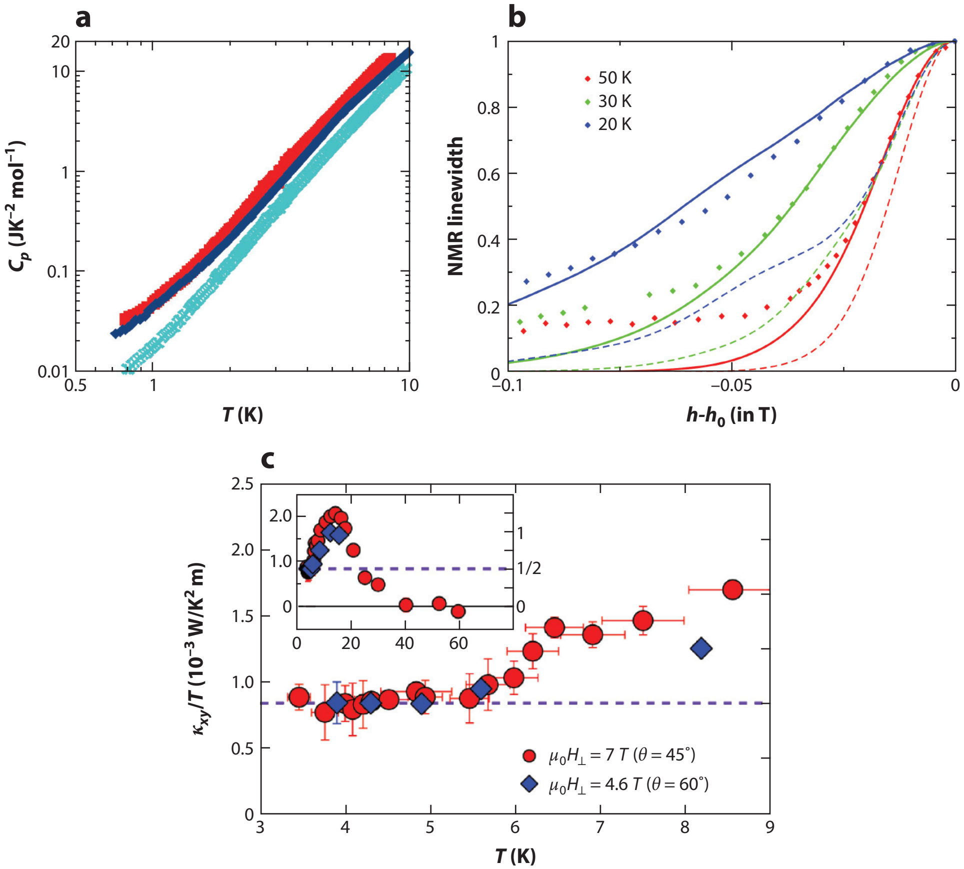

Figure 3

(*a*) The gaplessness of a spin liquid should be manifest in thermodynamic measurements. Shown is the linear specific heat (93) for three different triangular organic compounds "dmits," which is also reflected in a residual linear-in-temperature longitudinal heat conductivity (94) (not shown). (*b*) Chemical disorder, such as vacancies, can lead to defects charged under the emergent gauge field. SCGO is believed to host such defects, known as orphan spins, which surround themselves with spin textures, the shape of which is given by Gauss's law, which also implies interactions between them. This gives rise to a nontrivial magnetization distribution, which shows up in the linewidth of the NMR experiments (95, 96). This in turn allows an estimate of the effective disorder strength in the samples under consideration. (*c*) Evidence for a quantized thermal Hall effect in the Kitaev candidate material α-$RuCl_3$: As a function of magnetic field, a low-temperature plateau of κ_{xy}/T at half the quantized value (*dashed line*) of the integer QHE emerges, which indicates the possible presence of chiral Majorana edge states (97). Abbreviations: NMR, nuclear magnetic resonance; QHE, quantum Hall effect; SCGO, $SrCr_{9p}Ga_{12-9p}O_{19}$.

However, in general it is hard to separate out the spurious phonon contribution because, in the presence of sizeable spin–phonon couplings, magnetic excitations scatter from phonons and vice versa, which makes it hard to disentangle the two. These problems could in principle be overcome by studying directly the spin current transport that is measurable via the inverse spin Hall effect as demonstrated with insulating ordered magnets (105). This has recently been proposed for QSL materials (106), but more theoretical work and experiments are needed to show whether spin transport measurements can be turned into a new tool for studying QSLs.

3.3. Bulk-Boundary Correspondence and Quantization of Currents

A crowning achievement would be the observation of quantized transport signatures directly related to topological invariants to rank alongside the famously quantized Hall conductivity of the fractional quantum Hall effect. Again, for insulating magnets, these cannot be charge transport, but nevertheless a quantization of the thermal Hall effect (107), κ_{xy}, has been predicted in certain types of QSLs with broken time-reversal symmetry (108). The origin of the quantization (17) can be illustrated for a chiral QSL with ν chiral edge modes (their one-dimensional dispersions labeled by momentum q connecting zero energy with the gapped bulk states) and at temperatures below the bulk gap Δ. The current is a priori simply determined by the product of energy, thermal occupation (here for fermionic spinons obeying Fermi–Dirac statistics), and their velocity,

$$I_{xy} = \nu \int_0^{\Delta} \epsilon(q) n(\epsilon) v(q) \frac{\mathrm{d}q}{2\pi} = \nu \int_0^{\infty} \epsilon(q) \frac{1}{1+e^{\epsilon(q)/T}} \frac{\mathrm{d}\epsilon}{\mathrm{d}q} \frac{\mathrm{d}q}{2\pi} = \nu \frac{\pi}{24} T^2. \quad 4.$$

Experiments have observed signatures of a thermal Hall effect in disordered magnetic insulators, e.g., in kagome Volbortite (102), pyrochlore compound $Tb_2Ti_2O_7$ (109), and the Kitaev candidate, α-$RuCl_3$ (110). Especially promising is the very recent observation of the quantized prefactor of the temperature dependence in magnetic field-tuned α-$RuCl_3$ (97) (see **Figure 3**). A quantized response of this kind might arise naturally in the non-Abelian QSL phase of Kitaev's honeycomb model, which appears in an appropriately oriented magnetic field. However, for an unambiguous confirmation and interpretation in terms of chiral edge modes, it is necessary to carefully disentangle it from more prosaic types of heat transport via acoustic phonons (111, 112). In addition, not every spin liquid comes with a quantized transport coefficient, and at any rate, different spin liquids may not be distinguishable in this way alone. Nevertheless, the resulting problems of uniqueness and completeness of a classification scheme can perhaps be deferred until a time that such quantized transport has unambiguously been detected in at least two compounds.

3.4. Unconventional Phase Transitions

Despite the identification of a distinguishing characteristic of a QSL via its topological properties—the long-range entanglement of its ground state—this can be of limited practical use as even some of the most paradigmatic states—among them classical spin ice, the Z_2 gauge theories, or the Kitaev honeycomb model—are only zero-temperature phases, and no phase transition occurs when cooling down from the simple high-temperature paramagnet. Nevertheless, the behavior at short length and timescales [and here short could mean logarithmic in system size (113)] can still be governed by the fractionalized excitations of the zero temperature QSL (57).

In other systems, phase transitions out of topological phases do occur, and can be of autonomous importance. Since the first nonsymmetry-breaking phase transition beyond the Landau paradigm identified by Wegner for lattice gauge theories (114, 115), many other interesting proposals have been made. We mention these only in passing because pinning these down in detail is even more challenging than identifying the relevant phase itself.

The basic attraction of such transitions is that they reflect the exotic nature of the emergent degrees of freedom. For instance, the Kasteleyn transition (116) is an asymmetric transition on account of the string-like nature of an emergent U(1) gauge field: When a string has a negative free-energy-per-unit length, its probability is suppressed by a Boltzmann factor whose argument is linear in systems size as it needs to span the entire system. Hence, no strings—not even in the form of fluctuations—are present until the sign of its free energy changes, upon which there is a totally conventional continuous onset of string density. In spin ice, where the topological spin state is most reliably established, (a thermally rounded version of) this transition has indeed been observed (117–119).

Spin ice also hosts a liquid–gas transition with a critical endpoint of the emergent magnetic monopoles as zero-dimensional defects in a three-dimensional topological phase (120). These form a Coulomb liquid that can then be treated with methods imported from electrochemistry such as Debye–Hückel theory and its extensions (121–123). This is an instance of nontrivial collective behavior of the emergent degrees of freedom, which in itself remains a largely unexplored aspect of the field (Section 5).

Much beautiful theory has been developed regarding such unconventional phase transitions, including the identification of unusual signatures such as anomalously large exponents. A particular case in point is the possibility of deconfined quantum criticality (124, 125), in which the critical point with deconfined excitations separates two symmetry-breaking confined phases (2).

4. SPECTROSCOPY

The absence of Bragg peaks in zero-frequency measurements probing static correlations is an alternative indicator for ruling out conventional symmetry-breaking phases in spin liquid candidate materials. The generic situation of the correlations not only lacking a long-range ordered component but also being numerically short range means that, in reciprocal space, all features are broad.

Inelastic scattering experiments at nonzero frequency have the big advantage of also probing excited states beyond the asymptotic low-energy regime of thermodynamic measurements. The apparent disadvantage that these are nonuniversal is remedied by the prospect of identifying concrete spin liquids in actual experiments. Many experimental probes with ever increasing frequency resolution are available. Each of these has its well-developed strengths, and also its well-known set of shortcomings, to discuss, all of which would go beyond the scope of a simple review such as this, so we emphasize the points specific to spin liquids in the following.

First, just like in the thermodynamic probes, one thing to fundamentally look out for is an unusual temperature dependence of dynamical correlations (126) (see **Figure 1*c*** for a recent example). There should again be a cooperative paramagnetic regime in which interactions are strong but response functions change little as the temperature is lowered.

4.1. Inelastic Neutron Scattering and Quasiparticle Kinematics

The method of choice for measuring the basic spin correlation functions—both static and dynamic—is inelastic neutron scattering whose cross-section is given by

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega\mathrm{d}E} \propto F(q)\left(\delta_{\alpha\beta} - \frac{q^\alpha q^\beta}{q^2}\right)\sum_{\mathbf{r_i},\mathbf{r_j}} e^{i\mathbf{q}\cdot(\mathbf{r_i}-\mathbf{r_j})}\int \mathrm{d}t\mathrm{d}t' e^{-i\omega(t-t')}\langle S^\alpha_{\mathbf{r_i}}(t)S^\beta_{\mathbf{r_j}}(t')\rangle. \qquad 5.$$

From the sum rules connecting static correlations to frequency integrated dynamical ones, it is apparent that the absence of static Bragg peaks in spin liquids goes along with the spectral weight being found elsewhere at nonzero frequencies.

A central role in pursuing spin liquids and their concomitant fractionalization is played by selection rules. Simply put, if scattering involves a two-body process—e.g., a neutron spin flip creating a magnon—the twin constraints of energy and momentum conservation can lead to a sharp response in the form of a single line of energy versus wavevector transfer, $\omega(q)$, which represents the magnon dispersion relation. This simple situation is fortuitous in that the selection rules for neutron scattering off magnons permit precisely such a matrix element. The combined facts that neutrons are well matched to length/energy scales and quantum numbers of magnons underpins some of their phenomenal success in the field of magnetism.

A broader response can therefore have many different origins. First, there may not be such simple scattering processes available, such as there are in Raman scattering, where zero wavevector transfer, $q = 0$, requires the creation of multiple magnetic excitations. Second, there may not be a simple magnon available but rather only fractionalized excitations that must be created together, thereby rendering the scattering process a many-particle one. Third, there may not be a quasiparticle description of the low-energy spectrum in the first place, so that no dispersion relation $\omega(q)$ exists even as a matter of principle. This latter situation, especially with view to gapless spin liquids, is perhaps the least understood at this point in time.

Hence, though the existence of a broad dynamic scattering response is a good indicator of a correlated paramagnetic regime, it is itself not a sufficient piece of evidence for fractionalized excitations of a spin liquid. Nevertheless, given the large amount of information available in a full $\omega(q)$ map, it opens the possibility of providing a more microscopic modeling of the nonuniversal features discussed above.

For the Heisenberg chain, the kinematics of fractionalization is most beautifully illustrated already in the three parabolas that denote the possible energy-momentum combinations of two independent domain walls obtained from flipping a single spin in an ordered chain. The rather nontrivial intensity map—the kinematically allowed processes take place with very different intensities—follows from an actual enumeration of the matrix elements involved (see **Figure 1*a***).

In high dimensions, an analogous picture may, e.g., apply to a Z_2 spin liquid where a triplet excitation can be decomposed into a pair of $S = 1/2$ spinons, which would appear as monomers in a quantum dimer model (127). For this to be straightforwardly visible, it would be necessary for the individual spinon to act like a coherent free particle, which is likely the case in the limit of low spinon excitation energies. However, it is at present unclear—and one of the most interesting questions—over what energy range such long-lived well-defined fractionalized quasiparticles will exist.

This case of symmetric fractionalization into two particles is a particularly simple scenario. In addition, many other emergent particles are possible, and it is a priori straightforward to come up with mean-field parton constructions with a wide variety of different fractionalization schemes (90). One that is not uncommon is to end up with a spin flip corresponding to the creation of a gauge-charged particle, and an excitation of the emergent gauge field itself, e.g., as happens in the Kitaev models (128, 129). This fractionalization can be very asymmetrical: The flux may be very heavy, that is to say, have a very small bandwidth. It can thus take up an arbitrary amount of momentum at almost constant energy and thereby render momentum conservation essentially inoperative (64). This then leads to features in reciprocal space that are so broad that it is hard to infer much about the dispersion of the light particle. Nevertheless, key information about the energy of flux excitations and the DOS of the light fractionalized particles can in principle be inferred from the inelastic neutron scattering INS response (128–130).

4.2. Light Scattering

Light can interact with the purely magnetic low-energy degrees of freedom of Mott insulators via the virtual hopping processes that determine the magnetic exchange constant themselves. This has been shown, for example, to lead to a nonzero electric polarizability. It gives rise to an AC optical conductivity in Mott insulators (131), signatures of which have been analyzed for many QSLs (132–135). Furthermore, optical absorption can directly couple via magnetic dipole excitations to spin flips and, thus, probe the zero-momentum structure factor. Such experiments on an α-$RuCl_3$ have been recently interpreted as indications of a magnetic-field-induced QSL (136–138).

Alternatively, higher-order photon processes can induce virtual electron–hole pairs causing double spin flip excitations (139). Such a dynamical Raman response of kagome (140, 141) and Kitaev QSLs (142, 143) has been analyzed theoretically. Interestingly, the difference in matrix elements compared with those of INS permits a more direct coupling to certain types of fractionalized quasiparticles (57), and the polarization dependence of this zero momentum probe contains additional information (144). Hence, Raman measurements on pyrochlores (145), herbertsmithite (146), and two- (56) and three-dimensional (147) Kitaev candidate materials have been interpreted in terms of the spinon DOS of QSLs, but it is difficult to separate the inevitable phonon contribution to Raman scattering; we return to this point below in Section 4.5. Finally, with a further increase in energy resolution, resonant inelastic X-ray scattering (RIXS) will be a promising new tool for probing QSL excitations including their momentum dependence (148, 149).

4.3. Local Probes

Nuclear magnetic resonance (NMR) experiments probe the local magnetic fields of the spin degrees of freedom in insulators via the hyperfine interaction with nuclear levels. They are a powerful tool in the study of spin liquids (150). For example, an NMR frequency, which remains sharp and does not split when cooling to low temperature, rules out the presence of static magnetism with inequivalent magnetic sites or a static disordered state. Even more information is obtained from the relaxation time $1/T_1T$, which is directly sensitive to the local magnetic susceptibility (in the zero-frequency limit) related to the magnetic DOS. In a gapped QSL an Arrhenius-type behavior is expected (151), but gapless QSLs would again lead to characteristic power-law behavior as a function of temperature similar to the specific heat but without the parasitic phonon contributions.

An alternative probe of local magnetic fields is the relaxation of spin-polarized muons deposited in a candidate material. These μSR experiments can reliably distinguish between the presence of static moments due to conventional long range order, which leads to long-lived oscillations of the polarization, or dynamical moments of spin liquids, which lead to a quick decay without oscillations (152). The main advantage of μSR is its high sensitivity but a straightforward interpretation of the data may not be available because the positively charged muon interacts with the lattice, altering the local magnetic environment. Nonetheless, by treating the muon's strong Coulomb interaction carefully, along the lines of standard first-principles methods (153), much precise information can be extracted.

Unfortunately, experiments with both local and controlled spatial resolution are missing for magnetic insulators. For weakly correlated electronic materials, thanks to scanning tunneling microscopy (STM) and angle-resolved photo emission spectroscopy (ARPES), a hallmark signature of topological systems—the bulk-boundary correspondence—could be confirmed shortly after its prediction, e.g., of the surface Dirac cone of three-dimensional topological insulators (154) or of Majorana zero energy modes in superconducting wires (155). Of course, for a direct confirmation of topological surface states in spin liquid candidates without charged quasiparticles similar measurement tools are highly desirable. In that context it will be promising to explore new directions, for example, spin noise spectroscopy, scanning SQUID (superconducting quantum interference device) magnetometry (156), Raman microscopy (157), or inelastic STM (158) on spin liquid candidate thin films all with spatial resolution.

4.4. Bound States of Fractionalized Quasiparticles

Contrary to the intuition developed from the discussion of INS experiments, fractionalized quasiparticles need not have a broad, fully continuous spectrum. Instead, they may form bound or localized states, whose quantum numbers may closely resemble those of the unfractionalized spin flip. We remind the reader that the statement of deconfinement of fractionalized particles refers to the energy cost of their separation being bounded; this does not preclude the possibility of discrete composite states with a finite binding energy. Again, in $d = 1$ there is a celebrated and experimentally established instance of this (159), when at the magnetic-field-tuned critical point the exchange field leads to a discrete part of the spectrum from bound pairs of domain wall excitations. In higher dimension, analogous phenomena have been theoretically proposed. In spin ice, like in the $d = 1$ case, application of a field can lead to bound states of monopoles with a characteristic spectrum (160).

More exotically, the possibility of the gauge field degree of freedom being involved in a bound state is present in non-Abelian spin liquids. This is analogous to the case of a $p_x + ip_y$ superconductor (161), in which vortices host Majorana fermion bound states, e.g., the two Majoranas of a pair of vortices lead to a fermionic bound state whose energy goes exponentially to zero with vortex separation.

The long-term goal is the controlled manipulation of the degenerate manifold of states for braiding the vortices in the context of topological quantum computation (21). Slightly less ambitious would be the observation of such, for example, a flux-Majorana bound state in Kitaev QSLs, which has been shown to lead, for example, to a sharp contribution in the spin structure factor (129, 130) in the non-Abelian phase of the Kitaev honeycomb model. There, a spin flip introduces a pair of nearest-neighbor fluxes binding a pair of Majoranas below the gapped continuum response. Alternatively, already in the Abelian but anisotropic Kitaev phases (162, 163), the leading response can be a sharp delta-function corresponding to the addition of a pair of emergent gauge fluxes.

4.5. Statistics

The quantum statistics of quasiparticles is a very fundamental property—it affects the many-body DOS even for noninteracting particles. This is evidenced by the (conventional) Fermi sphere and its suppressed heat capacity and, thus, in principle accessible in thermodynamic measurements already (Section 3). The temperature dependence of dynamical scattering experiments contains information about the thermal distribution functions that are qualitatively different for quasiparticles with different quantum statistics, e.g., Fermi–Dirac versus Bose–Einstein. For example, in the context of Raman experiments on the Kitaev candidate material α-$RuCl_3$ a close comparison of the T dependence of the high-energy Raman response and experimental data arguably points to the presence of spin fractionalization in terms of fermionic excitations (57).

In general, fractionalized quasiparticles in topological phases can have unusual exchange statistics of anyons due to the relative phases picked up when emergent particles of different type are interchanged. Such braiding operations are particularly of interest given their much-appreciated potential in procuring a framework for fault-tolerant topological quantum computing (20) mentioned above. Directly probing the exchange statistics of emergent particles in a quantum spin liquid is a tall order, which at present—given the difficulties in doing the same even in the much more controlled setting of quantum interferometers in the quantum Hall effect—seems not too close at hand.

However, there are nonetheless qualitative features to look out for. For instance, a scattering process that creates a set of fractionalized particles via a local interaction is sensitive to the statistics of the particles generated together, as their relative wave function influences the matrix elements for the process in question: For a point-like interaction, the creation of two fermions, say, is inhibited by the vanishing of their wave function as the pair moves close together. Hence, under rather general conditions for gapped QSLs the onset of the INS response is dominated by the long-range statistical interaction between Anyonic quasiparticles leading to a universal power-law dependence as a function of frequency (164). Similarly, a promising direction will be ambitious experiments for noise spectroscopy directly probing statistics or measuring emergent quantum numbers of fractionalized quasiparticles.

5. DISORDER AND DEFECT PHYSICS

The presence of disorder—defects, vacancies, impurities, and the like—is an unavoidable fact of life in condensed matter systems. Indeed, in many candidate spin liquids, understanding the role of disorder is a crucial step toward the identification of the physics involved (see e.g., 69, 70, 78, 165).

Besides being a nuisance, however, disorder can also be used as a probe. The basic idea is that disorder can make fractionalization physics visible in the ground state that would otherwise require probing excitations. As a simple illustration, consider the following picture. A vacancy in a two-dimensional system can be thought of as inserting a microscopically tiny hole into the plane—in a sense, it changes the topology of the plane into that of an annulus. This hole can then have effective degrees of freedom. One instance could occur in a non-Abelian spin liquid, where (well-separated) vacancies can host Majorana zero modes at zero energy, as described in Section 4.5. Also, not unlike impurities in a semiconductor, a vacancy can host a localized fractionalized excitation—such as a magnetic monopole—that would otherwise require an activation energy in the bulk (166).

In this sense, disorder physics is closely related to our discussion of dynamical probes of fractionalized degrees of freedom upon replacing dynamical probes by probes of the disorder sites. Local probes can then be used to resolve the signal coming from the defects. Whereas in a bulk probe a signal from a small density of defects, unless it is singularly large, is easily swamped by the bulk signal, a local probe like NMR can detect the defect response at a frequency separated from that of the bulk. Again, in one dimension, edge defects have provided a beautiful picture of the physics of the gapped Haldane chain (167). This kind of study has been carried out in some detail for gauge-charged vacancy degrees of freedom (95) following detailed NMR measurements on SCGO (96) (see **Figure 3*b***). These orphan spins (61) establish an oscillating spin texture that decays following a Coulomb law; modeling this has led to the conclusion that the level of disorder in SCGO is likely higher than that determined from the nominal stoichiometry only. The spin liquid response to disorder can thus be used to inverse-infer properties of the composition of the material itself.

The response to disorder can also be very intuitive. For a spinon Fermi surface, one can develop an analogy to the response of a Fermi liquid to disorder: An impurity can induce Friedel oscillations, leading to a disturbance surrounding the impurity modulated at the Fermi wavevector, which in turn can give rise to RKKY-type interactions between impurities (168).

As mentioned above, collective defect physics is a huge field in its own right that is, so far, relatively little studied. A systematic understanding of the many-body state of a finite density of defects embedded in a topological spin liquid is a particularly promising direction for the discovery of surprising new phenomena (165, 169–173). This topic is beyond the scope of the present article and arguably merits a stand-alone treatment.

6. DISCUSSION AND FUTURE DIRECTIONS

After several decades of searching for quantum spin liquids in magnetic materials, we are still awaiting the unambiguous sighting of this elusive state of matter. Encouragingly, recent years have seen a flurry of discoveries of new candidate materials and novel indicative signatures of liquidity, which raise the hopes that this long search will come to a successful conclusion in the not too distant future. One of the great attractions of finding topological phases in magnetic systems is their high tunability. For instance, magnetic fields of a few tens of teslas in strength are becoming increasingly routinely available. They potentially push proximate spin liquid candidates (77) in the desired direction (174) and the impressive advances in the energy-wavevector resolution of neutron experiments, for example, enable a visualization of the field-induced melting of conventional long-range magnetism (175). A magnetic field not only can add a Zeeman term to the Hamiltonian but also acts in the presence of spin-orbit interactions as a versatile probe, even mimicking an effectively staggered field on different sublattices that can change the effective dimensionality of the emergent gauge field (28).

In the absence of uniquely and individually compelling features of spin liquidity in a particular material, an investigation of how a particular feature (gap size, mode dispersion, continuum bandwidth, bound-state energy) changes as a function of tuning parameter (external fields, pressure, composition, strain) will be a crucial ingredient for assessing the validity of a particular interpretation of experimental data in terms of a spin liquid under the merciless action of Ockham's razor. In that context, a joint effort of both theory and experiment continues to be called for in order to pin down the rich phenomenology of spin liquids.

Besides such more detailed model-based input, methodological progress is also on the horizon. High on the wish list are improved local probes, especially with a resolution approaching the lattice scale, which is currently elusive for, say, SQUID-based devices measuring local-field distributions. These could then be used to probe boundary modes or impurity susceptibilities even in insulating magnets in the hope of emulating their huge success in electronic systems with charge degrees of freedom via STM or ARPES for surface states. Another exciting development for probing fractionalized excitations lies in the realm of nonequilibrium techniques, e.g., pump-probe measurements (176, 177) or spin echo and noise spectroscopy. The nonequilibrium physics of topological phases is a nascent field that will surely hold numerous surprises for the patient explorer.

Our field guide is also subject to continual extension because of the continuous discovery of new materials. These may arise in the form of bulk materials, metal organic frameworks (178), or metallic Kondo systems (179), or in artificial settings such as nanostructured/thin film samples. In addition, there remains the promise (180, 181) of one day realizing new phases of spin systems in analog cold atomic quantum simulators.

In conclusion of this little field guide, it is worth recalling that in physics, the most interesting phenomena are often the ones that are not anticipated at all so that the most basic suggestion remains to produce and analyze experimental data with an open mind.

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Errata

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