4/1 Two-Dimensional Vortex Melting

• Superconducting Length-Scales and Vortex Lattices

Discovery of Superconductivity

Roots traced back to Michael Faraday
(not for study of electromagnetism, but of liquification of chlorine)

Cl liquifies at -34° C $\rightarrow$ high pressure in sealed vessel

$\downarrow$

quest to liquify elements

He, H hardest

Omnes influenced by van der Waals

Theory of ideal gas ($PV = nRT$)

↓

failed to predict liquification

of gases

(ignored intermolecular forces → no condensation)

van der Waals related T_{liq} to strength

of intermolecular forces

⇓

Omnes wanted to test

prediction

↓

Liquification of He

Use experimental technique to study outstanding ?? of the day.

$$\lim_{T \to 0} \rho(T) = ?$$

Three different theories

$$R = \frac{\rho L}{A}$$

Kelvin : freezing of electrons $\rightarrow$ no current flow

Matthiessen : R would flatten out due to finite # scattering centers (impurities)

Dewar: $\rho = \frac{m}{ne^2\tau}$ lattice vibrations freeze out

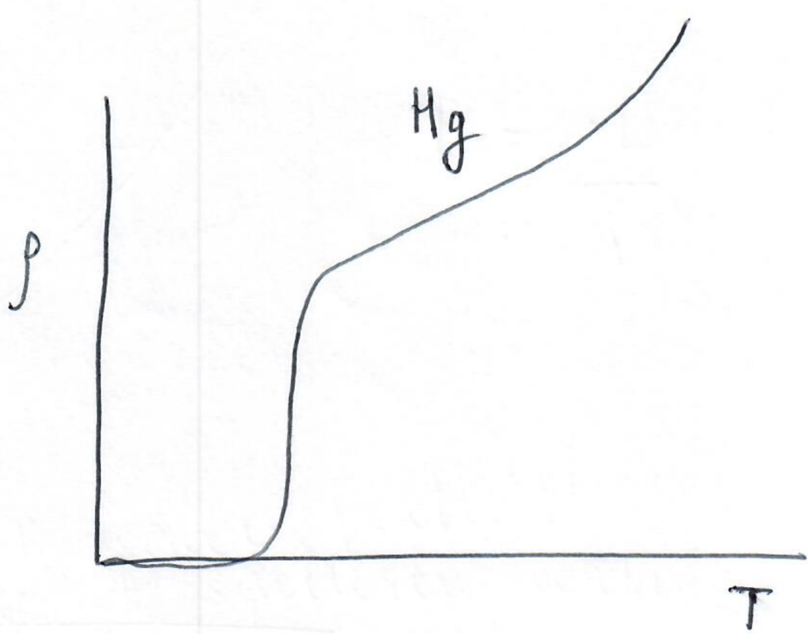
scattering reduced

$T \to \infty$.

Impurities limiting factor $\longrightarrow$ Hg.

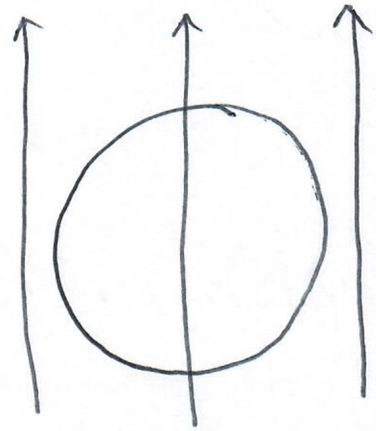
1911

Hg cooled below boiling point of He (4.2 K)



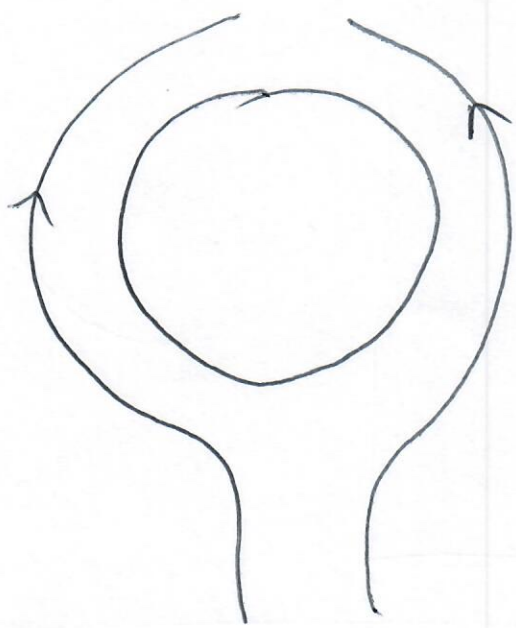
Originally thought was short circuit (but reversible!)

1933 Meissner - Ochsenfeld



$T > T_c$

Perfect Diamagnet!



$T < T_c$

irreversible

<u>Not</u> perfect conductor

$$\vec{E} = \rho \vec{j} \Rightarrow \rho = 0,\ \vec{j} \text{ finite} \Rightarrow E = 0.$$

$$\vec{\nabla} \times \vec{E} = \frac{\partial \vec{B}}{\partial t} \stackrel{E=0}{\Rightarrow} \frac{\partial \vec{B}}{\partial t} = 0 \qquad \times$$

$\vec{j} = \sigma \vec{E}$ (Ohm's Law) $\qquad$ (<u>not</u> $B = 0$)

Superconductor = perfect diamagnet
(not perfect conductor)

Key point: surface currents whose fields exactly cancel original ones.

$\Downarrow$

λ London penetration depth

Flux not expelled entirely

(B decreases exponentially in SCs)

$\exists$ small region near the surface where the field penetrates

- ξ coherence length

Length-scale over which superconducting electrons are coherent

Estimate using the uncertainty principle

$$\Delta x \, \Delta p \sim \hbar$$

$$\Downarrow$$

$$\Delta x \sim \frac{\hbar}{\Delta p}$$

Only electrons within kT_c of Fermi level can participate in SC at $T \sim T_c$

$$\Delta p \sim \left(\frac{kT_c}{\varepsilon_F}\right)(m v_F) \sim \left(\frac{kT_c}{v_F}\right)$$

$$\downarrow$$

$$\Delta x \sim \left(\frac{\hbar v_F}{k_B T_c}\right) \rightarrow \xi \sim \left(\frac{\hbar v_F}{k_B T_C}\right)$$

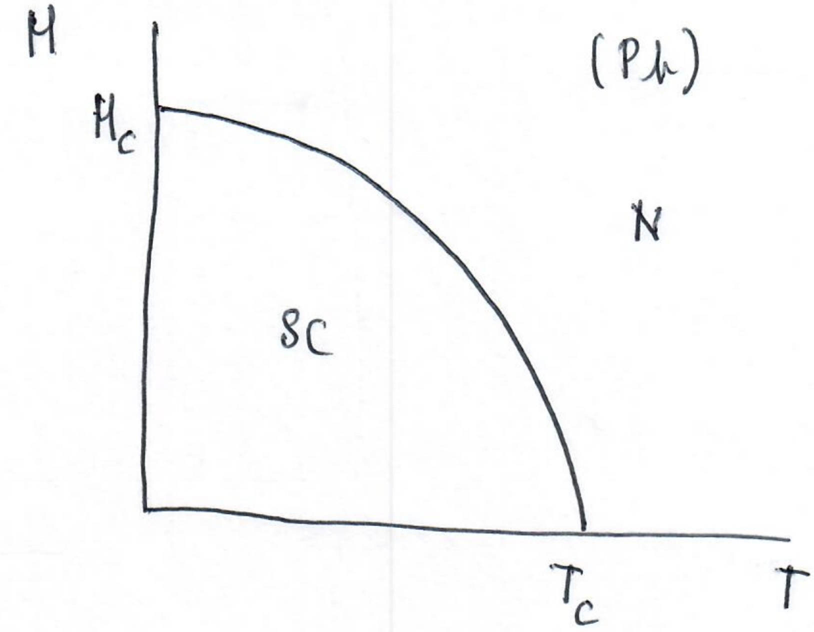
$$\xi \sim \frac{\hbar v_F}{\Delta}$$

Type I vs. Type II Superconductor

Different responses to magnetic field

Type I.

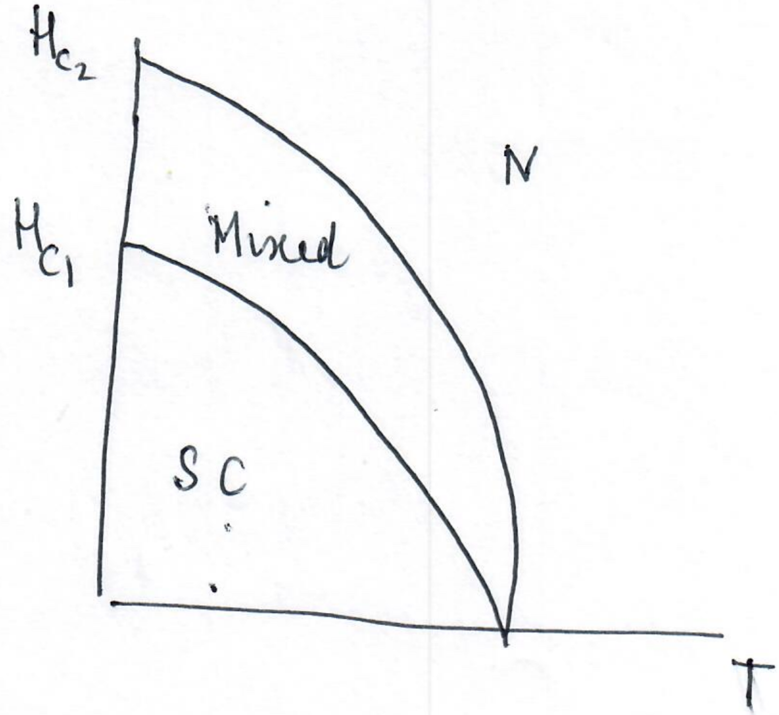
$\xi > \lambda$



SC excludes the field completely until it is destroyed; then field penetrates completely

Type II.

$\lambda > \xi$



l short $\Rightarrow$ ξ short

Alloys $\Rightarrow$ ξ short $\Rightarrow$ II.

(Pb + 2% In)

Mixed state

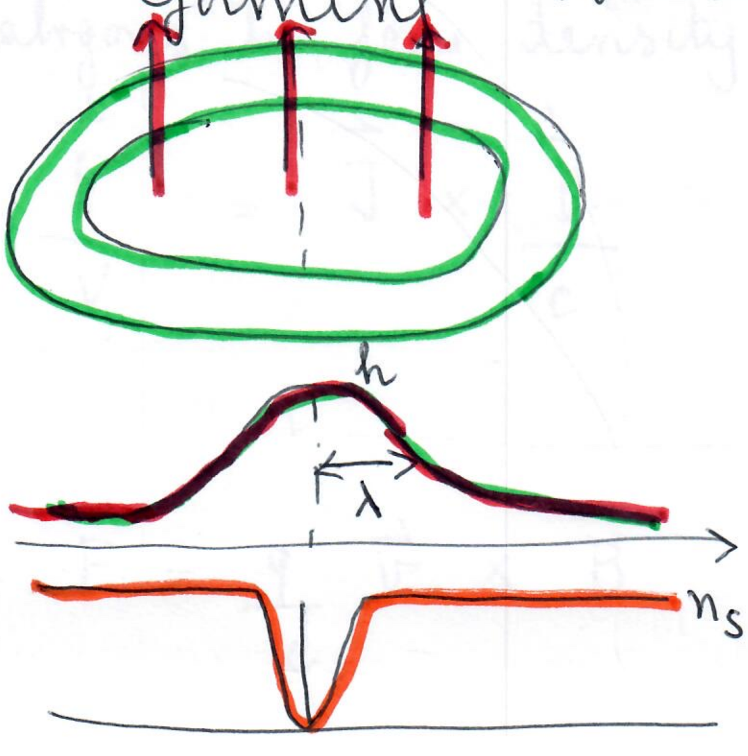
Normal cores surrounded by superconducting current

$\xi < \lambda \Rightarrow$ surface energy < 0 between N and S states

$\Downarrow$

energy reduction by forming vortices

1 vortex line



density of SC electrons

- Flux Motion

E field $\Rightarrow$ vortices move

$\Downarrow$ $J \neq 0$

Lorentz force / length

$$f = J \times \frac{\Phi_0}{c} \qquad \Phi_0 = \frac{hc}{2e}$$

$$\left(\begin{array}{c} \text{analogous to force density} \\ \dfrac{\vec{F}}{V} = \vec{J} \times \dfrac{\vec{B}}{c} \\ \text{or} \\ \vec{F} = \dfrac{q}{c}\, \vec{v} \times \vec{B} \end{array} \right)$$

$J < J_c$ Vortices are pinned

$J > J_c$ Vortices move

Low T_c superconductor (Nb)

Hexagonal "Abrikosov" lattice

Motion of individual vortices restricted due to "rigidity" as in solid

(vortex motion $\rightarrow$ Joule heating)

C_{66} shear modulus

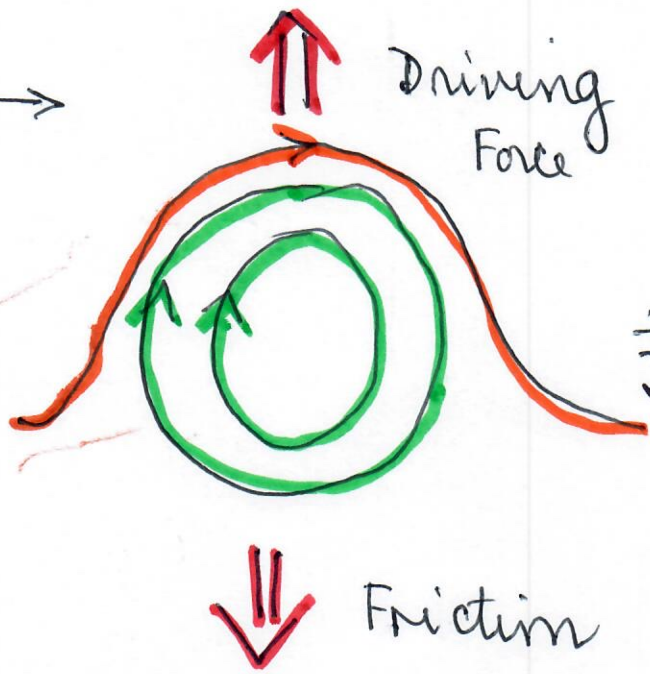
resistance to shearing forces.

Solid $\xrightarrow{H}$ liquid transition

Vanishing of C_{66}.

Magnus force →

(force acting on spinning tennis ball)



$$\vec{F} = \vec{J} \times \frac{\hat{\Phi}_0}{c}$$

$$F_f = -\eta \vec{v}$$

Strength of $\frac{\text{driving}}{\text{friction}}$ forces

$$\frac{F_M}{F_f} \propto \frac{\rho_{transverse}}{\rho_{longitudinal}}$$

$T \uparrow$ small friction

$T \to 0$ should increase dramatically.

Flux flow resistance.

1 vortex

$$\vec{F} = \vec{J} \times \frac{\hat{\Phi}_0}{c} \quad \Rightarrow \quad \vec{E} = \hat{\Phi}_0 \times \frac{\vec{v}}{c}$$

electric

field $\quad \vec{E} \parallel \vec{J}$

(Bardeen - Stephen model:
Power dissipation mostly from normal cores)

Flux flow resistance of SC in mixed phase

$$R_{ff} = \frac{V}{I}$$

Steady state $\quad \vec{E} = \vec{B} \times \frac{\vec{v}}{c}$

$$F_L = F_f$$

$$\frac{J\Phi_0}{c} = \eta v$$

$$\boxed{\rho_{ff} = \frac{E}{J} = \frac{B\,\Phi_0}{\eta c^2}}$$

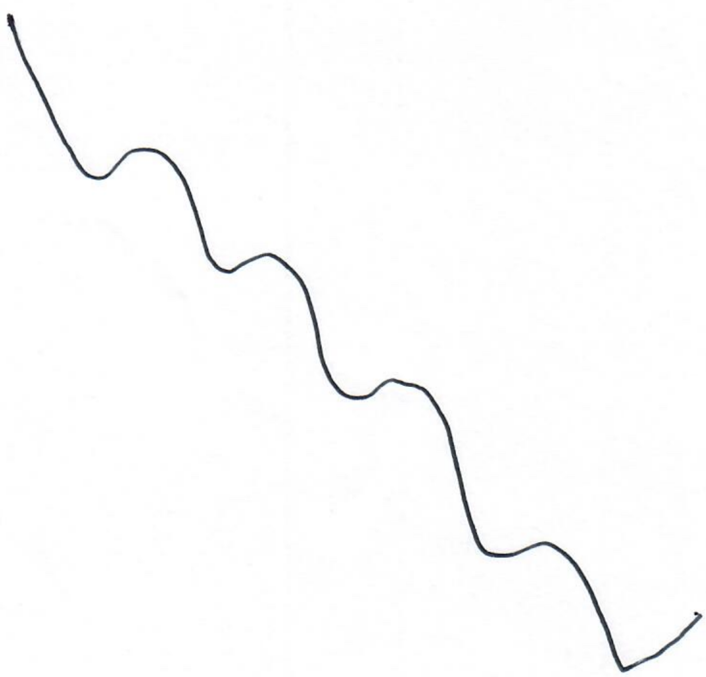
Thermally Activated Flux Flow (TAFF).

Flux "creep"

flux lines "jump" from one pinning center to next

$$\boxed{R_{TAFF} = R_{ff} \exp\left(-\frac{U(J)}{k_B T}\right)}$$

(Anderson - Kim)



Solid

$$U(I) = U_0 \left(\frac{I_c}{I}\right)^{\alpha} \qquad \alpha \sim 1$$

$$\left(R_{TAFF} \to 0 \ , \ I \to 0\right)$$

Liquid

short-range positional / orientational order

$$U \neq U(I)$$

$$R_{TAFF} \neq R_{TAFF}(I) \qquad I \to 0.$$

Mermin - Wagner Theorem

Phase fluctuations in different dimensions

$$H = \frac{K}{2} \int (\nabla \theta)^2 \, d^d r$$

Energetics of long-wavelength fluctuations

$$H \sim \frac{K}{2} \frac{L^d}{L^2}$$

$d < 2$ no LRO

$d = 2$? ?

$d > 2$ yes LRO

2D No LRO

Algebraically decaying correlations are "allowed"