Kitaev Spin Liquid

Exactly Soluble (decomposition of Majorana fermions)

↑ we will discuss them shortly

Fractionalization of spin can be "tracked"

↙ static $\mathbb{Z}_2$ gauge field with a Δ

↙ KSL in ground-state flux sector

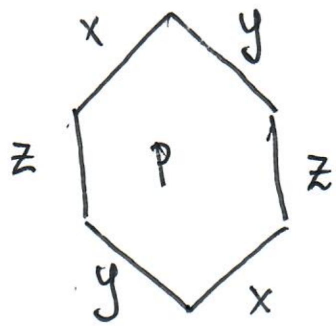
↘ (Majorana) fermion degree of freedom

Transitions out of QSL

⇓

"confinement" of Majorana fermions

Kitaev Model



$$H = -J_x \sum_{x \text{ links}} S_i^x S_j^x - J_y \sum_{y \text{ links}} S_i^y S_j^y - J_z \sum_{z \text{ links}} S_i^z S_j^z$$

$$W_p = 2^6 \prod_{j \in p} S_j^\gamma$$

$\gamma = x, y, z$ type bond.

$$[H, W_p] = 0, \quad [W_q, W_p] = 0 \quad \forall q, p.$$

Now for a flavor for this model and what we learn from it...

(1) Majorana representation of spins

To describe spins in a system <u>without</u> long-range order we use

fermion operators $\{ a_{\uparrow}, a_{\uparrow}^{+}, a_{\downarrow}, a_{\downarrow}^{+} \}$

$\Downarrow$ real and imaginary parts

One complex fermion mode

$$c_1 = (a + a^{+}) = c_1^{+}$$

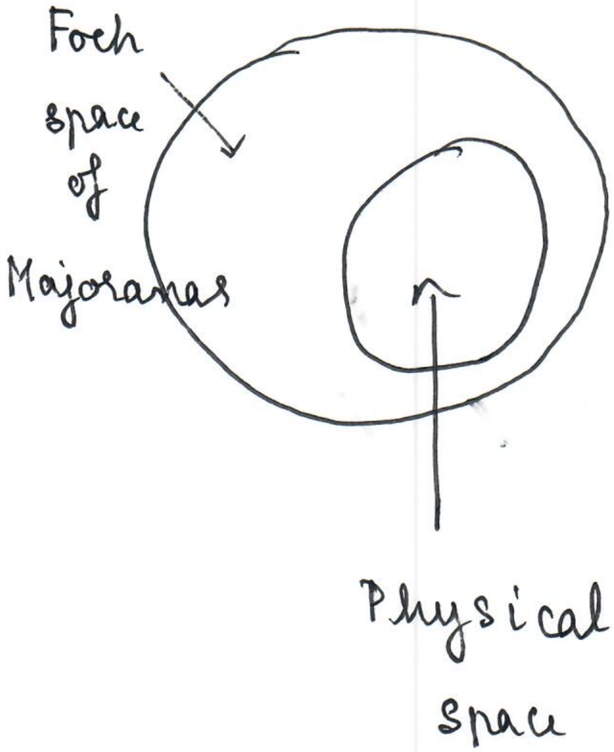
Majorana operators

$$c_2 = i(a - a^{+}) = c_2^{+}$$

$$\{c_i, c_j\} = 2\delta_{ij}$$

Fermionic Fock space is <u>doubled</u>

$$\{|\uparrow\rangle, |\downarrow\rangle\} = \{|00\rangle_{\uparrow\downarrow}, |11\rangle_{\uparrow\downarrow}, |01\rangle_{\uparrow\downarrow}, |10\rangle_{\uparrow\downarrow}\}$$



Need a projector onto physical space

Redundant representation!

For each vertex, we define

$$b^x = a_\uparrow + a_\uparrow^\dagger \qquad c = -i(a_\downarrow - a_\downarrow^\dagger)$$

$$b^y = -i(a_\uparrow - a_\uparrow^\dagger)$$

$$b^z = a_\downarrow + a_\downarrow^\dagger$$

Fermionic representation of spins

$\Downarrow$

$$S_j^\gamma = \frac{i}{2} b_j^\gamma c_j$$

We can define a bond operator

$$u_{ij}^\gamma = b_i^\gamma b_j^\gamma$$

eigenvalues $\pm i$

(commutes w/ H)

All subject to constraint

$$b_j^x b_j^y b_j^z c_j = 1$$

$\Downarrow$

commutation conservation $\Rightarrow$ b_i^γ's constrained

u_{ij}^γ emergent gauge field

(2) Solve model

$$H = -\frac{1}{4} \sum_{\langle ij \rangle_\gamma} K_\gamma \, u_{ij}^\gamma \, c_i \, c_j$$

Quadratic

Procedure :

(1) Fix $W_p = \pm 1 \quad \forall p$

(2) Find u_{jk}'s

(3) Solve for $E(\{W_p\})$

(4) Repeat for all configurations

Find minimal energy

③ Dispersion in the ground-state

- Uniform phase
  ($W_p = -1$ everywhere)

- Coherent propagation of
  Majorana fermions